Almost local generation of Einstein-Podolsky-Rosen entanglement in nonequilibrium open systems

Rebecca Schmidt,* Jürgen T. Stockburger, and Joachim Ankerhold†
Institut für Theoretische Physik, Universität Ulm, Albert Einstein-Allee 11, 89069 Ulm, Germany
(Received 28 May 2013; published 19 November 2013)

The generation of entanglement in an open system is studied in a model consisting of two independent Gaussian parties sharing a conventional heat bath environment. Reservoirs of this type appear in a broad class of condensed phase systems and themselves are insufficient to generate entanglement from initially separable states. Already local driving by an external classical field, however, is sufficient to promote this system-bath interaction to a source of entanglement. The presence or absence of the effect depends on the specific pulse shape of the external control, which we determine through optimal-control techniques.

DOI: 10.1103/PhysRevA.88.052321 PACS number(s): 03.67.Bg, 03.65.Ud, 02.30.Yy, 03.65.Yz

I. INTRODUCTION

Entanglement is one of the most fascinating manifestations of quantum nonlocality in systems with more than one degree of freedom. It has thus been investigated as a key resource for quantum information in case of both discrete [1,2] and continuous variables [3–7]. Both fundamental properties of bipartite Gaussian entanglement and dynamical processes producing entangled states have been investigated extensively [8–13], with varying roles assigned to a dissipative environment and to external control fields [14–17].

The nonlocal interaction required for the generation of entanglement [2] is sometimes postulated as an explicit feature of the system either in the form of a direct constant [9,11,18] or time-varying coupling [14] between two parties. Alternatively, in the absence of direct coupling, entanglement can be generated by relying on the effective static coupling arising from a nontranslational invariant interaction of two parties with a common reservoir [9,13,15]. This latter type of coupling may be strong even in the case of weak dissipation at finite frequencies.

In this work, we consider a more subtle dissipative mechanism which will not generate entanglement by itself. However, when leveraged by suitably chosen classical driving fields, the combined effect of driving and dissipation can result in entanglement. We do not assume environments which require specific engineering [19], but rather are known as standard models for decoherence in solid-state devices [20] and for quantum Brownian motion [21] with a translational invariant system-bath coupling. When eliminating the bath degrees of freedom, the impact of the reservoir only appears in the form of fluctuating forces and velocity-dependent friction. Remarkably, even local parametric driving is sufficient to promote the reservoir from an entanglement-degrading to an entanglement-promoting feature. This effect, as we will demonstrate, is quite sensitive to the exact time dependence of the driving fields, which we determine through optimal-control techniques [22].

We find that a weakly damped bipartite harmonic system in a Gaussian state (Fig. 1) can be driven into an entangled state by local driving. The immediate effect of local driving on modes A and B is single-mode squeezing. Dissipation in the reduced dynamics of the bipartite system transforms this single-mode squeezing into two-mode squeezing (when viewing symmetric and antisymmetric modes) or entanglement (when viewing local modes A and B). This effect is found over a wide temperature range. Suitable pulse shapes yielding significant entanglement can be found even for the case of driving only mode A. These findings are of potential relevance for current experiments with superconducting circuits. They also open new possibilities for teleportation without an explicit state transfer.

II. MODEL AND OPEN SYSTEM DYNAMICS

We consider a compound system with Hamiltonian $H = H_S(t) + H_I + H_R$, where two harmonic oscillators of equal mass $M$ and frequency $\Omega$ form a distinguished system with $H_S(t) \equiv H_A(t) + H_B(t) = \sum_{j=A,B} \frac{p_j^2}{2M} + \frac{M\Omega^2}{2}q_j^2 + \frac{u_j(t)}{2}q_j^2$. (1)

The last term in Eq. (1) represents local parametric driving of strength $u_j(t)$. The oscillators interact with a common reservoir which has the conventional form of a thermal reservoir used in a quantum Brownian motion context [21,23], i.e.,

$$H_R = \sum_k \frac{p_k^2}{2m_k} + \frac{m_k\omega_k^2}{2}x_k^2,$$ (2)

$$H_I = (q_A + q_B) \sum_k c_k x_k + (q_A + q_B)^2 \sum_k \frac{c_k^2}{2m_k\omega_k^2}.$$ (3)

The reduced dynamics depends on the properties of the reservoir only through the inverse thermal energy $\beta = 1/k_B T$ and through the spectral density $J(\omega) = \pi \sum_k c_k^2/(2m_k\omega_k)(\omega - \omega_k)$, formed from the parameters in Eqs. (2) and (3). The last part in $H_I$ (known as the “counterterm” in the context of the Caldeira-Leggett model) should not be misread as an interaction term for the modes; it ensures a vanishing net effect of the reservoir on the dynamics if adiabatic elimination is performed. If the reservoir is traced out from the exact, full dynamics, only reservoir fluctuations and velocity-dependent memory friction affect the system dynamics. In the present

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* Present address: Department of Physics, Dartmouth College, Hanover, New Hampshire, USA.

† rebecca.schmidt@uni-ulm.de
two-mode model and in contrast to previous work \[13,15\], the memory friction thus includes “mutual drag” between the oscillators, which is the only feature of the model that qualifies as a source of quantum nonlocality \[12\].

While the complete density operator of system and reservoir obeys the standard Liouville-von Neumann equation, the treatment of the dynamics of the relevant reduced density is a formidable challenge. This is particularly true in case of low temperatures and a priori unknown control signals where conventional perturbative expansions like Born-Markov master equations fail. In this situation stochastic Liouville von-Neumann equations (SLNs) have been proven as formally exact and numerically powerful tools to capture on the same footing the non-Markovianity of the reduced time evolution and arbitrary external driving fields also for nonlinear systems. They are based on a stochastic representation of the Feynman-Vernon influence functional such that the physical reduced density is obtained by averaging the time evolution according to the SLN over proper noise realizations \[24\].

In case of a continuous degree of freedom and an Ohmic-type reservoir with

$$ J(\omega) = \frac{\eta \omega}{1 + \omega^2/\omega_c^2} $$

parameterized by a coupling constant \(\eta\) and high-frequency cutoff \(\omega_c\), the SLN dynamics of the density \(\rho_{AB,\xi}\) is governed by

$$ \dot{\rho}_{AB,\xi} = -\frac{i}{\hbar} [H_A + H_B, \rho_{AB,\xi}] + \frac{i}{\hbar} \xi(t) [q_A + q_B, \rho_{AB,\xi}] $$

$$ -\frac{i}{\hbar} \frac{\eta}{2} [q_A + q_B, \{p_A + p_B, \rho_{AB,\xi}\}] . $$

The \textit{physical} reduced density operator \(\rho_{AB}\) is obtained after averaging over sufficiently many realizations, i.e., \(\rho_{AB} = \text{E}[\rho_{AB,\xi}]\). We emphasize that this procedure provides the exact result independent of temperature, damping strength, and driving force.

Particularly for Gaussian modes, the reduced density matrix is fully determined by the first and second cumulants of the elements \(x_j, j = 1, \ldots , 4\) of the vector \(x = (q_A, p_A, q_B, p_B)\) of phase-space operators. Since first moments can always be adjusted by local unitary operations, they cannot affect entanglement properties. The central quantity is thus the covariance matrix \(\sigma\) with \(\sigma_{ij} = \frac{1}{2} \langle x_i x_j + x_j x_i \rangle - \langle x_i \rangle \langle x_j \rangle\). In the \(2 \times 2\) block structure \[8\],

$$ \sigma = \begin{pmatrix} \alpha & \gamma \\ \gamma^T & \beta \end{pmatrix} , $$

where \(\alpha, \beta\) correspond to the covariance matrices of the respective subunits A and B, while \(\gamma\) (transpose \(\gamma^T\)) carries the mixed cumulants and thus nonlocal information.

Now, a well-defined measure for entanglement in bipartite Gaussian systems is given by the logarithmic negativity \[8,25\]

$$ E_N = \max \{0, -\ln(\tilde{\nu}_-)\}, \tag{7} $$

with \(\tilde{\nu}_-\) being the smallest symplectic eigenvalue of the partially transposed density matrix. Entanglement only exists if \(E_N > 0\) while \(E_N = 0\) corresponds to purely classical and/or local quantum correlations. Note that \(\det \gamma < 0\) is necessary for \(E_N\) to be positive \[26\].

Without external driving, any initially separable two-mode state will remain separable indefinitely in the scenario considered here. This applies in particular to the steady state, which can be calculated nonperturbatively exactly \[21\]. In case of ground states as initial states, the resulting expression

$$ \tilde{\nu}^2_{-st} = \frac{(q_A + q_B)^2 (p_A + p_B)^2}{(\hbar/2)^2} \tag{8} $$

contains an uncertainty product of the thermal variances always exceeding the ground-state limit (cf. Fig. 2) so that \(\det \gamma > 0\). It is to be noted, however, that this does not preclude discord as a further type of nonclassical correlations \[27,28\].

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III. OPTIMAL CONTROL OF ENTANGLEMENT

Although we have stated that interaction with a common reservoir alone is not sufficient to generate entangled states, it nevertheless plays a key role in the generation of entanglement through an external control signal: Since the system Hamiltonian (1) provides no two-body interactions, it nevertheless plays a key role in the generation of entangled states, but it requires additional ingredient beyond local controls. In the sequel, we not only show that entangled states can be created by this combination of factors but also perform a numerical search for the maximal entanglement that can be achieved in finite time for this purpose, the recently developed optimal-control formalism for open quantum systems [22] is exploited to maximize the entanglement at a given final time \(t_f\), i.e., we seek a maximum of the functional

\[
F[u_A(t), u_B(t); \sigma(t)] = E_N(\sigma(t_f))
\]

of the control fields \(u_A(t), u_B(t)\). While for further technical details we refer to Ref. [22], it is noted here that the variation of \(F[u_A(t), u_B(t); \sigma(t)]\) with respect to the control fields and to the state and under the constraint that the equations of motion derived from Eq. (5) are to be obeyed provides us with (i) a set of stochastic equations of motion for the cumulants of the so-called costate with an end-time boundary condition (backward propagation) and (ii) an update formula for the control signals. This system of equations (forward propagation, backward propagation, and control update) has to be solved consistently and iteratively.

Practically, maximizing the logarithmic negativity \(E_N = \max(0, -\ln(\tilde{\gamma}))\) and thus searching iteratively for solutions with arguments in the log function decreasing towards zero poses a numerical problem. Namely, for arguments sufficiently close to zero, the optimization procedure becomes extremely sensitive to the noise averaging. It turns out that for the system in question (two identical oscillators, coupled to a common reservoir) minimizing \(\det{\gamma}\) leads as well to the optimization goal but without numerical instabilities.

IV. ENTANGLEMENT GENERATION

In the sequel, we consider an initial preparation with both oscillators in the ground state \(E_N = 0\) [29], and without initial correlations between system and reservoir. This is followed by propagation of the system under the influence of control fields \(u_A(t), u_B(t)\) and interaction with an Ohmic bath with spectral distribution (4). We use dimensionless units with frequencies scaled with \(\Omega\) and lengths scaled with \(\sqrt{\hbar/(M\Omega)}\).

Figure 2 displays bar graphs of optimized covariance matrices (right) compared to ground and stationary states (left). Apparently, substantial off-diagonal correlations are built up, positive and negative valued, so that one indeed has \(\det{\gamma} < 0\). Notably, the same is true for single-site control \(u_B(t) \equiv 0\).

Further insight is gained through a transformation to normal modes \(Q_\pm = (q_A \pm q_B)/\sqrt{2}, P_\pm = (p_A \pm p_B)/\sqrt{2}\) which transforms the system Hamiltonian (1) into

\[
\tilde{H}_S = \sum_{\sigma = \pm} \frac{P_\sigma^2}{2} + \frac{1}{2} \left(1 + \frac{u_A + u_B}{2}\right) Q_\sigma^2 + \frac{u_A - u_B}{2} Q_+ Q_-
\]

(10)

Then, for \(u_A(t) \equiv u_B(t)\), entanglement \(E_N > 0\) requires

\[4 \det{\gamma} = \Delta_P \Delta_Q - \left[(Q_+ P_+ - Q_- P_-)\right]^2\]

(11)
to be negative, where \(\Delta_X = \langle X_2^2 \rangle - \langle X_1^2 \rangle, X = P, Q\). Apparently, this is only possible if the evolution of the two normal modes is not degenerate. Lifting this degeneracy is the decisive role the heat bath plays in this setting. However, external driving is needed as well in order to render the normal-mode states dissimilar enough to result in entangled A and B states.

Entanglement generation is thus a cooperative effect of local driving and global dissipation in the present setting—either factor by itself is neutral or detrimental to entanglement. A tailored control pulse changes the nonlocal effects of the heat bath from a destructive influence on quantum resources [12] to an asset promoting entanglement.

The correlators in Eq. (11) can be obtained from the Wigner functions plotted in Fig. 3 for the final state under an optimized driving protocol. For symmetric control (left), \(u_A \equiv u_B\), a strongly squeezed antisymmetric mode results, while the symmetric mode is close to a thermal state. This leads to opposite signs of the terms \(\Delta_P\) and \(\Delta_Q\); therefore the right-hand side of Eq. (11) is negative, and the local modes \(q_A\) and \(q_B\) are entangled. For a more quantitative analysis, it is convenient to represent the correlation matrix elements \(\langle Q_\pm^2 \rangle, \langle P_\pm^2 \rangle, \langle Q_\pm P_\pm \rangle\) by a different parameter set: squeezing parameters \(r_\pm\), squeezing angles \(\phi_\pm\), and width parameters \(a_\pm\).

To introduce these parameters, we note that for a single mode the matrix elements of \(\sigma\) [Eq. (6)] can be parametrized as

\[
\begin{align*}
\sigma_{qq} &= a^2 (\cosh 2r + \cos 2\phi \sinh 2r), \\
\sigma_{qp} &= -a^2 \sin 2\phi \sinh 2r, \\
\sigma_{pp} &= a^2 (\cosh 2r - \cos 2\phi \sinh 2r),
\end{align*}
\]

(12)

FIG. 3. (Color online) Reduced Wigner functions for the antisymmetric \(Q_-\), \(P_-\) (top) and symmetric \(Q_+, P_+\) (bottom) normal modes at \(t_f = 6\pi\) for two-site (left) and single-site (right) control. The green line indicates the ground-state width \((\sqrt{\hbar/(2m\Omega)})\); parameters are as in Fig. 2.
with \( \sigma_{qq} \sigma_{pp} - \sigma_{qp}^2 = a^4 \) [5]. Alternatively, the width parameter \( a \) can be replaced by an effective temperature parameter through

\[
a^2 = \frac{1}{2} \coth \frac{\beta}{2}.
\]

For the case of symmetric driving, the two-mode covariance matrix factorizes when transformed to normal modes \( Q_\pm \). It is therefore determined by the parameter set \( r_\pm, \varphi_\pm, a_\pm \). Using this parameter set and reverting to local modes, the quantity \( \det \gamma \) takes the form

\[
\det \gamma = \frac{1}{4} [a_+^4 + a_-^4 - 2a_+^2 a_-^2 \{ \cosh 2r_- \cosh 2r_+ - \cosh(2\varphi_- - 2\varphi_+) \sinh 2r_- \sinh 2r_+ \}].
\]

Accordingly, the condition \( \det \gamma < 0 \) translates thus into

\[
cosh 2r_- \cosh 2r_+ - \cosh(2\varphi_- - 2\varphi_+) \sinh 2r_- \sinh 2r_+ > \frac{1}{2} \left( \frac{a_+^2}{a_-^2} + \frac{a_-^2}{a_+^2} \right). \tag{15}
\]

This way, one observes a similarity between the states considered here and two-mode Einstein-Podolsky-Rosen (EPR) states: EPR states can be seen as factorized pure states of squeezed symmetric and antisymmetric modes, with \( r_+ = -r_- \) and identical squeezing angles [30]. The states produced in our numerical simulations show strong squeezing in the antisymmetric mode but not in the symmetric mode, which is close to a thermal state as will be shown in Sec. V. They might therefore be labeled “semi-EPR” states.

Even using more rudimentary single-site control (\( u_A \neq 0, u_B \equiv 0 \)), we find that the cooperative effect between driving and dissipation persists, although the similarity to EPR states is diminished; see Fig. 3 (right). Moreover, additional correlations between the symmetric and antisymmetric modes are needed to fully characterize the quantum state.

The gradual (nonmonotonic) buildup of entanglement over time is shown for the cases of symmetric and single-site control in Fig. 4 (top), with a rapid final approach to values of \( E_N(t_f) \approx 2.33 \) (single-site control) and \( E_N(t_f) \approx 4.37 \) (symmetric control). Simulations at lower temperatures show further improvements.

The numerical values of the entanglement measure may be related to the number of states involved, allowing a rough comparison to qubit-based entanglement. For two two-level-systems (qubits), \( E_N = 1 \) corresponds to a Bell state with maximal entanglement. Furthermore, for two-mode squeezed vacuum states with squeezing parameter \( r \) and negativity \( E_N(r) = 2r \), the number of excited states in each mode dominantly contributing to the entanglement can be estimated as \( m \approx \exp[E_N(r)/2] \). While, as discussed above, the situation here is different due to the dissipative \( Q_\pm \) mode, one may at least estimate that symmetric (single-site) control involves about \( m \approx 40 \) \( (m \approx 5) \) states in the \( Q_\pm \) mode.

At this point, the question may arise as to which extent the generation of entanglement depends actually on the profile of the driving pulse. Are fields determined via optimal control really necessary or can similar results also be achieved with much simpler monochromatic pulses? To study this issue, we consider driving signals of the form \( u_A(t) \equiv u_B(t) = u_0 \sin^2(vt) \) for \( v = 1, 2 \). In all cases, entanglement is not created within the time window for which optimal control leads to substantial nonlocality. Thus, to get a deeper insight into the mechanism that allows optimal control to approach its goal, we consider in the sequel a perturbative analysis.

V. PERTURBATIVE TREATMENT

We assume a symmetric drive \( u(t) \equiv u_A(t) = u_B(t) \) such that the two normal modes \( Q_\pm \) experience identical parametric driving with only the \( Q_+ \) mode interacting with the reservoir. Furthermore, for Gaussian modes the fundamental solutions of the classical equation of motion play a prominent role as also known for the fluctuations of a harmonic oscillator [21]. By generalizing results for periodic parametric driving [31], one then arrives at a perturbative description.

A. Classical dynamics

The classical equation of motion for a damped harmonic oscillator parametrically driven by a control field \( u(t) \) reads in dimensionless units

\[
\dot{q}(t) + \eta \dot{q}(t) + \omega(t)^2 q(t) = 0, \tag{16}
\]

where \( \omega(t)^2 = 1 + u(t) \) and \( q \) corresponds to the symmetric mode \( Q_+ \) for \( \eta \neq 0 \) and to \( Q_- \) for \( \eta = 0 \). We now consider weak friction \( \eta \ll 1 \) and assume about the control signal only that it is sufficiently smooth and bounded from below such that \( \omega(t)^2 \gg 0 \). Then, solutions to Eq. (16) can be written in
the form \( q(t) = e^{-|\sigma|^2/2} Q(t) \) where
\[
\tilde{Q}(t) + \omega(t)^2 Q(t) = 0, \tag{17}
\]
with \( \omega(t)^2 = \omega_0^2 - \eta^2/4 \approx \omega_0^2 \) in the weak-friction limit. Relevant observables can then be expressed in terms of the independent solutions \( \phi_\ell(t), \ell = 1, 2 \) of Eq. (17) with initial conditions \( \phi_1(0) = 0, \phi_2(0) = 1, \dot{\phi}_2(0) = 0 \). According to Abel’s identity [32] the corresponding Wronskian then obeys
\[
\phi_2(t)\dot{\phi}_1(t) - \phi_1(t)\dot{\phi}_2(t) = 1. \tag{18}
\]
Further properties about the \( \phi_\ell \) are known only in case of a purely periodic drive, e.g., \( u(t) = \cos(2\omega_0|t|) \) (see Ref. [33]). Even though typical optimal-control signals are not of this simple form (cf. Fig. 5), it is instructive to recall some results: solutions are of the form \( \phi_\ell(t) = [F_\ell(t) - F_\ell(-t)]/[2F_\ell(0)] \) and \( \phi_2(0) = [F_1(t) + F_1(-t)]/[2F_1(0)] \) where the Mathieu function \( F_\ell(t) \) obeys the Floquet exponent \( \nu \) and a periodic function \( p(t) = p(t + \pi/\omega_0) \); orbits of Eq. (16) are found to be unstable if \( |\text{Im}\nu| > \eta/2 \).

### B. Quantum dynamics

In the quantum regime, we focus on the variances \( \sigma_{xy} = \frac{1}{2}\langle xy + yx \rangle - \langle x \rangle \langle y \rangle \) of the conjugate operators \( q,p \) in \( \{q,p\} \) of the canonical ensemble. In the undamped case \( \eta = 0 \), they obey the following set of equations:
\[
\begin{align*}
\dot{\sigma}_{qq}^{(1)} &= 2\sigma_{qp}^{(0)}, \tag{19} \\
\dot{\sigma}_{pp}^{(1)} &= -2\omega(t)^2\sigma_{qp}^{(0)}, \tag{20} \\
\dot{\sigma}_{qp}^{(1)} &= \sigma_{pp}^{(0)} - \omega(t)^2\sigma_{qq}^{(0)}. \tag{21}
\end{align*}
\]
For a ground state as initial state one has \( \sigma_{qq}(0) = \sigma_{pp} = 1, \sigma_{qp} = 0 \) and finds in terms of the classical solutions
\[
\begin{align*}
\sigma_{qq}^{(0)}(t) &= \phi_1(t)^2 + \phi_2(t)^2, \tag{22} \\
\sigma_{pp}^{(0)}(t) &= \phi_1(t)^2 + \phi_2(t)^2, \tag{23} \\
\sigma_{qp}^{(0)}(t) &= \phi_1(t)\dot{\phi}_1(t) + \phi_2(t)\dot{\phi}_2(t). \tag{24}
\end{align*}
\]
The calculation for a dissipative system is much more cumbersome. It can conveniently be performed using techniques from Ref. [31] in terms of the path integral representation of the reduced density operator of the system. For the variances one then obtains \( \sigma_{xy}(t) = \sigma_{xy,0}(t) + \sigma_{xy,\beta}(t) \), where the first transient part depends on the initial state but is independent of temperature while the second nondecaying part depends on temperature only. Approximately, in the weak-friction limit one has \( \sigma_{xy,\eta}(t) \sim \exp(-\eta t)\sigma_{xy}^{(0)}(t) \). Furthermore, the nondecaying parts of the variances take the form [31]
\[
\sigma_{qq,\beta}(t) = a_{qq}(t), \tag{25}
\]
This includes the real part of the reservoir force-force correlation,
\[
K(s) = \int_0^\infty d\omega \frac{\pi}{\sinh(\omega \beta/2)} \cos(\omega s), \tag{26}
\]
and a function \( \varphi(t,s) = \langle \phi_1(t)\phi_2(s) - \phi_1(s)\phi_2(t) \rangle \) which obeys the classical equation (17) with boundary conditions \( \varphi(0) = \phi_1(t), \varphi(t) = \phi_2(t) \) and \( \dot{\varphi}(t) = 0 \). Furthermore, \( d\varphi(t,s)/ds = -1 \) according to Eq. (18).

The momentum variance reads
\[
\sigma_{pp,\beta}(t) \equiv \dot{a}_{pp}(t) = a_{pp}(t) + 2a_{qp}(t)\phi_1(t)^2/\phi_1(t)^2 + a_{qp}(t)\phi_2(t)/\phi_1(t) \]
with
\[
a_{pp}(t) = \int_0^t ds \int_0^s du \frac{\phi_1(s)}{\phi_1(t)} K(s-u)\phi_1(u)/\phi_1(t) e^{-\eta t-(s+u)/2}, \tag{27}
\]
and
\[
a_{qp}(t) = \int_0^t ds \int_0^s du \frac{\phi_1(s)}{\phi_1(t)} K(s-u)\varphi(t,u)e^{-\eta t-(s+u)/2}. \tag{28}
\]

For the mixed variance one finds
\[
\sigma_{qq,\beta}(t) \equiv \dot{a}_{qq}(t) = a_{qq}(t)\phi_1(t)/\phi_1(t) + a_{qp}(t). \tag{29}
\]
For a vanishing drive \( u(t) = 0 \), these results reduce to the known equilibrium variances of the dissipative quantum oscillator which, for weak to moderate friction, differ by slight shifts [21] from the values predicted in the canonical ensemble. The position correlations \( \sigma_{pp,\beta}(t \to \infty) \) are somewhat smaller (quantum Zeno effect) and the momentum correlations \( \sigma_{pp,\beta}(t \to \infty) \) are increased by a correction \( \propto \ln \omega \). The equilibrium squeezing parameter \( \eta \) is thus slightly negative. In the high-temperature limit or for very weak friction, all parameters revert to their standard equilibrium values, which are characterized by \( \eta = 0, \varphi_\beta = 0 \), and width \( \sigma \) is related to the physical reservoir temperature by Eq. (13).

Approximately, this scenario also applies in case of finite driving as we show now. For that purpose, we first mention that the bath kernel takes for very low temperatures the form
\[
K(s) \sim \eta/s^2, s > 1/\omega, \tag{30}
\]
and in the high-temperature regime becomes local in time, i.e., \( K(s) \sim (\eta/\beta)^2 s \). Furthermore, for optimized control over an interval \([0,T]\) with \( T \) covering at least a few periods of the bare oscillator, signals \( u(t) \) typically consist of an initial segment (about at most two periods of the bare oscillator) with large amplitudes and a second segment with low amplitudes and quasiperiodic behavior; see Fig. 5. Correspondingly, the functions \( \phi_1(s), \phi_2(s) \) show an oscillatory behavior similar to the nondriven case for times \( s > \tau \) with \( \tau \gg \tau \) as displayed in Fig. 6. This in turn splits the range of the time integration in Eq. (25) in a short time domain \( s < \tau \) and a range where the damping-dependent

FIG. 5. (Color online) Optimized control signal for symmetric two-site control for the parameters in Fig. 2.
expansive dictates the decay. Typically, the contribution of the latter range dominates so that one can write $a_{qp}(t_f) = (q^2)_{\beta} + \epsilon_q(t_f)$ with a smaller time-varying part $\epsilon_q(t_f)$ of order $\eta e^{-\beta t_f} \phi(t_f)$. A similar analysis shows that $a_{pp}$ in leading-order approaches, i.e., $a_{pp}(t_f) = (p^2)_{\beta} + \epsilon_p(t_f)$, while the mixed variances remain smaller than of order 1. Neglecting these small thermal and dynamical corrections, one may insert for the $Q_+$ mode the thermal-state parameters $r_+ = 0, \phi_+ = 0$, and $a_+ = 1/2 \coth(\beta/2)$ into Eq. (15). Observing that the antisymmetric mode $Q_{-}$ is a pure state ($a_- = 1/2$), Eq. (15) then reduces to

$$\cosh 2r_- > \coth \beta. \quad (29)$$

This indicates that sufficiently strong squeezing can cancel the destructive effect of thermal fluctuations. We will study this in detail in the following section.

VI. PERSISTENCE OF ENTANGLEMENT

The numerical results depicted in Fig. 4 (bottom) indeed show that the logarithmic negativity is of order 1 even for inverse temperatures around $\beta = 0.01$. Our analysis in the previous section reveals that the final state of the $Q_+$ mode differs very little from a thermal state for the parameters covered by our numerical simulations. Thus, the condition (29) indicates that raising temperature does not preclude entanglement per se. However, it gives a lower bound on the squeezing required to obtain entanglement at a given temperature, which becomes more and more stringent as temperature is raised. Given an absolute upper bound of the achievable squeezing $r_-$, Eq. (29) indicates the highest temperature at which the condition $\det \gamma < 0$ can be fulfilled. This result is largely independent of the dissipative coupling strength; it remains valid as long as the oscillators are underdamped.

Interestingly, the generated entanglement in the present scenario even persists once the external control is switched off, as illustrated in Fig. 7 for $\beta = 0.1$. Since the $Q_-$ mode does not interact with the reservoir (decoherence-free subspace), while the $Q_+$ is almost thermalized, entanglement is “stored” in the $Q_-$ mode. This feature may be attractive for potential applications.

VII. DISCUSSION

We have shown that dynamical symmetry breaking for two-mode Gaussian parties due to the combined impact of common dissipation and local parametric driving efficiently generates entanglement. This proves that even in conventional thermal reservoirs as they appear, e.g., in solid-state devices, quantum nonlocality can be induced under nonequilibrium conditions.

While in our analysis we focused on reservoirs with Ohmic-type of spectral densities, the general mechanism for the creation of entanglement is mainly independent of the specific form of the reservoirs. The same is true for the form of the parametric drive. Local optimal control allows to achieve substantial logarithmic negativity in finite time and even at high temperature, but it is expected that driving forces with other profiles, e.g., purely periodic driving, may induce entanglement as well. The scenario studied here may thus be applicable to various bipartite systems in condensed-phase devices. Particular examples include two Cooper pair boxes spatially well separated in a “bad” cavity or two impurity fermions embedded in a Bose-Einstein condensate. Potentially, nitrogen-vacancy centers in diamonds provide a test bed to dynamically induce entanglement at high temperatures [34]. Beyond these direct realizations, there are also consequences for quantum teleportation to be explored. Instead of transferring one half of an entangled pair from A to B, it may be possible to create a spatially separated entangled pair in place through a dynamical process involving a (possibly “dirty”) shared medium or reservoir.
ACKNOWLEDGMENTS

The authors like to thank E. Kajari, T. Häberle, and F. Jelezko for valuable discussions. J.A. thanks the Department of Physics and Astronomy of Dartmouth College, Hanover, USA for their kind hospitality. Financial support was provided by Deutsche Forschungsgemeinschaft through AN336/6-1 and SFB/TRR21.

[29] This is in contrast to the situation in Ref. [12], where it is shown that the destructive impact of a bath sometimes does not completely destroy strong initial entanglement.