Bitter decoration of vortex patterns in superconducting Nb films with random, triangular, and Penrose arrays of antidots

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We imaged Abrikosov vortex patterns in thin Nb films with random, periodic (triangular), and quasiperiodic (Penrose) arrays of antidots. Vortex positions were visualized by Bitter decoration for a range of applied fields $B$, antidot radii $r$, and densities $n_p$ after field-cooling through the transition temperature $T_c$ to a base temperature $T \approx 2$ K. The observed vortex patterns correspond to snapshots of vortex positions at the time of decoration. The effectiveness of antidots as artificial pinning sites for vortices is found to be sensitive to several factors: array geometry, antidot size and density, and applied field. Overall, the triangular lattice provides the most effective pinning landscape, with antidots trapping the highest proportion of vortices, but for a wide range of parameters the Penrose lattice is equally effective. For a quantitative analysis, we determined the occupation number $n$ (average number of vortices trapped per antidot) from each image. This revealed a significantly more complicated dependence of antidot occupation on applied field and/or antidot density than that predicted by simple models considering pinning by an isolated antidot. In particular, upon increasing the antidot density $n_p$, we find a marked increase in $n$ for triangular arrays, which we attribute to the additional repulsion from interstitial vortices, pushing more vortices into antidots with decreasing antidot separation. This effect is also present but less pronounced for Penrose arrays, which can be explained by the variation of antidot spacing inherent to the Penrose geometry and accordingly more options for accommodating interstitial vortices.

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I. INTRODUCTION

Nanoengineering of superconducting structures and devices provides a very effective way to fine-tune their properties, for example by controlling the size and shape of a superconductor or through the introduction of artificial pinning centers for Abrikosov vortices. In the latter case, much attention has been focused on thin films containing periodic arrays of holes (antidots) or magnetic dots, as these are shown to significantly enhance the critical current $I_c$ at certain values of the applied magnetic field $H$. Namely, sharp maxima in $I_c(H)$ or magnetization $M(H)$ are observed at applied fields corresponding to exact matching between the periodicity of the antidot lattice and the vortex lattice. One can define matching fields $B_m$ ($B$ is the magnetic induction) for which the density of vortices $n_v = B/\Phi_0$ matches the density of pinning sites $n_p$, i.e., $n_v = mn_p$, with the first matching field $B_1 = n_p\Phi_0$ corresponding to $n_v = n_p$, i.e., one magnetic flux quantum $\Phi_0 = h/2e$ per pinning site. Since $I_c(H)$ and $M(H)$ are governed by the underlying vortex configurations, a number of numerical studies investigated these in thin superconductors with square and triangular lattices of pinning sites under various matching conditions (see, e.g., Refs. 11–14), and some of the predicted matching configurations were seen directly in imaging experiments.

More recently, it was proposed that pinning due to arrays of artificial pinning sites can be further enhanced by creating lattices with many built-in periods, such as a fivefold Penrose lattice or other quasiperiodic tilings, so that the critical current is increased not just at matching fields but over broad field intervals. Quasiperiodic pinning arrays were realized experimentally by using Nb thin films with magnetic dots or antidots, or Pb and Al thin films with magnetic dots. Transport measurements on superconducting thin films with quasiperiodic fivefold Penrose pinning arrays indeed showed significant critical current enhancement not just at exact matching fields $B_m$ but also in between such fields, which was attributed to effective trapping of vortices by many constituent periodic sublattices. A direct comparison between antidot arrays arranged in a fivefold Penrose lattice (PL) and a triangular lattice (TL) revealed that matching peaks were much less pronounced for the PL’s, while the absolute values of $I_c$ for the PL’s were still found to be lower than the peak values for the TL’s. Moreover, an appreciable enhancement of $I_c$ for the PL’s over the TL’s was only observed extremely close to the critical temperature $T_c$ (at normalized temperature $t \equiv T/T_c \approx 0.998$) and for fields below $B_1$. As the temperature was decreased below $t = 0.995$, the peaks in $I_c(B)$ both for the PL’s and TL’s gradually diminished (transforming into plateaus and kinks), with the decline setting in at higher $T$ for the PL’s.

To fully understand the role of symmetry of the pinning array in pinning enhancement, one needs to determine the exact vortex positions in samples with quasiperiodic pinning arrays and their evolution with the applied field and temperature. This problem was addressed in numerical studies and in one scanning Hall probe (SHP) study on Penrose arrays with magnetic dots. SHP microscopy (well below $T_c$) enabled the imaging of vortex patterns corresponding to peaks in transport measurements [resistance $R(B)$ slightly above or $I_c(B)$ slightly below $T_c$ (Refs. 21 and 22)] and revealed the formation of a symmetry-induced giant vortex. All the above studies, both theoretical and experimental, focused on low fields $B \lesssim B_2$, so the underlying vortex configurations for higher
matching fields are unknown even in theory. Furthermore, there have been no experimental comparative studies of the vortex patterns in quasiperiodic, periodic, and random pinning arrays, and no imaging experiments have been reported so far on either quasiperiodic or random antidot pinning arrays.

Here we present direct observations by Bitter decoration of vortex patterns in thin Nb films with periodic (TL's), quasiperiodic (fivefold PL's), and random configurations of antidots. We determine the occupation number $n$ (i.e., the average number of vortices captured per antidot) for different antidot sizes and densities. We find that overall the periodic triangular lattice provides the most effective pinning landscape, with the antidots trapping the highest proportion of vortices both at matching and nonmatching fields. However, for certain parameters, e.g., relatively small antidots or low fields, TL's and PL's are equally effective. Furthermore, the occupation number is found to be sensitive not only to the antidot size but also to the antidot spacing and the applied field.

II. SAMPLES AND MEASUREMENT TECHNIQUES

Experiments were carried out on Nb films (thickness $d = 60$ nm) deposited by dc magnetron sputtering on several Si substrates (chip size $10 \times 10$ mm$^2$). Patterning by e-beam lithography and the lift-off technique produced on each chip up to 30 four-quadrant (4Q) structures with an area $1 \times 1$ mm$^2$ (each quadrant with an area 200 $\times$ 200 $\mu$m$^2$). Each 4Q structure contains three different configurations of antidots (TL, PL, and random arrangement) and a plain film (without antidots) in the fourth quadrant. Within each 4Q structure, $n_p$ (and hence $B_1$) and $r$ are the same. Different 4Q structures differ by antidot density $n_p$ and radius $r$. Below, we show data for samples with three different $r$: “small” (74 $\pm$ 6 nm), “medium” (164 $\pm$ 14 nm), and “large” (227 $\pm$ 12 nm). The errors in $r$ are due to averaging over antidot structures produced in different lithography and deposition runs, as well as to nonideal shapes of the antidots; see, e.g., Fig. 1. The plain film was used as a reference, e.g., to estimate demagnetization and to calculate the average magnetic induction $B = \Phi_0 N_v/A$ from the number of vortices $N_v$ per area $A$.

Details of the geometric construction of the fivefold Penrose lattice are described elsewhere.\textsuperscript{21} Here we just mention that it consists of two types of rhombuses with equal side lengths (lattice parameter) $a_{PL}$: “thick” and “thin,” with angles $(2\theta, 3\theta)$ and $(\theta, 4\theta)$, respectively ($\theta = 36^\circ$). The smallest antidot spacing is $a_{PL}/\tau$ along the short diagonal of the thin rhombuses [with the “golden mean” $\tau \equiv (1 + \sqrt{5})/2 \approx 1.618$]. The antidot density in the PL is $n_p = \tau^2/[a_{PL}^2(1 + \tau^2) \sin \theta]$, while for the triangular lattice (with lattice parameter $a_{TL}$) it is $n_p = 2/[a_{TL}^2 \sqrt{3}]$. For most experiments described below, the antidot density was fixed at $n_p = 0.47 \, \mu$m$^{-2}$ ($B_1 = 0.97$ mT), corresponding to a lattice parameter $a \approx 1.6$ $\mu$m for both PL’s and TL’s. In a separate experiment, the antidot density was varied between 0.47 $\mu$m$^{-2}$ ($B_1 = 0.97$ mT) and 0.12 $\mu$m$^{-2}$ ($B_1 = 0.25$ mT), corresponding to a variation in $a$ from $\approx 1.6$ $\mu$m to $\approx 3.2$ $\mu$m.

To characterize the electric transport properties of our films, several samples were patterned into cross-shaped bridges for four-point transport measurements. These yielded $T_c = 8.67$ K for the plain film and a slightly reduced $T_c = 8.66$ K for films with antidots. From the measured resistivity in the normal state ($T = 10$ K), we estimate a mean free path $\ell \approx 6$ nm, zero-temperature coherence length $\xi(0) \approx 13$ nm, and zero-temperature London penetration depth $\lambda(0) \approx 71$ nm (see Ref. 21 for details).

The Bitter decoration technique used to image vortices is based on in situ evaporation of 10–20 nm Fe particles that are attracted to regions of magnetic-field gradients created by individual vortices; this enables their visualization (details of the technique are described elsewhere).\textsuperscript{24} Importantly, the technique allows simultaneous observation of vortices and antidots, thus allowing unambiguous answers as to the exact vortex positions with respect to the antidots. All experiments were done by field-cooling in external fields ranging from $\mu_0 H = 0.4$ to 3.5 mT (perpendicular to the thin-film plane), and Fe particles were evaporated at a base temperature $T \approx 2$ K.\textsuperscript{25} Each decoration experiment yielded a snapshot of vortex positions either in just one or simultaneously in several four-quadrant structures. This was particularly useful for the investigation of the effect of antidot density on vortex capture, as we were able to span several matching conditions at the same $H$ (see below).
III. RESULTS

A. Bitter decoration: Fixed antidot density

Figure 1 shows typical high-magnification images of vortices in films with a triangular [Fig. 1(a)] and Penrose [Fig. 1(b)] lattice of antidots with different \( r \) and the same \( n_p \). The bright rings of higher density of Fe particles around the antidots indicate the presence of vortices trapped inside, while the interstitial vortices are seen as clusters of white Fe particles. Bitter decoration does not allow us to measure the amount of flux contained in a given antidot, i.e., we cannot clearly distinguish between antidots containing, e.g., \( \Phi_0 \) and \( 2\Phi_0 \). The relatively low contrast in these images (compared to, e.g., vortex images in mesoscopic disks\(^2\)) is due to the large effective penetration depth (Pearl length) \( \Lambda(T) = \frac{\lambda^2(T)}{d} \approx 0.15 \, \mu m \) [with \( \lambda(T) = \lambda(0)(1 - t)^{1/2} \) and for \( T \approx 4 \) K as the estimated temperature at the time of decoration\(^{25}\)]. Several decoration experiments for medium and large holes (radius \( r \approx 160 \) nm and \( \approx 200 \) nm) at fields \( B \leq B_1 = 0.97 \) mT showed no interstitial vortices in films with both triangular and Penrose antidot configurations, i.e., all vortices were captured by the antidots (although some interstitials were always seen in samples with random antidots). Due to the particularly low contrast in small fields\(^26\) below \( B_1 \), it was not possible to clearly discriminate occupied and unoccupied antidots, which made it impossible to test the predictions made in Refs. 17 and 18 for vortex patterns below \( B_1 \).

As the field was increased above \( B_1 \), interstitial vortices started to appear, as well as significant differences between different antidot configurations in terms of the proportion of vortices in interstitial positions. This is illustrated in Fig. 2, which shows typical images obtained in the same experiment at the third matching field, \( B = 3B_1 \). It is clear that, at this field value, the TL is the most effective in capturing vortices. Just 0.3% of all available vortices here are found outside the antidots. In contrast, for the PL many more vortices, 23%, are found in interstitial positions, even though the antidot size and density are exactly the same. Randomly distributed antidots provide the least efficient pinning landscape—here the density of vortices in large patches seen between clusters of antidots is roughly the same as in the reference plain film. Due to the limited resolution of the lift-off procedure, closely spaced antidots in random configurations often merged, producing large holes of irregular shapes as seen in Fig. 2(c).

The above picture of antidot effectiveness becomes much more complicated when the size of the antidots is taken into account as well. The results of eight separate decoration experiments done at different fields \( H \) and antidot radii \( r \) are summarized in Fig. 3. In most of these experiments, three 4Q structures containing antidots of three different sizes were decorated simultaneously; the antidot density \( n_p \) was the same throughout (\( n_p \approx 0.47 \, \mu m^{-2} \)). In Fig. 3, we plot the occupation number \( n \), i.e., the average number of vortices captured per antidot, versus normalized field \( B/\Phi_0 \). Here, \( n \) is calculated as \( n = (BA - N_i\Phi_0)/A\Phi_0n_p \), where \( N_i \) is the number of interstitials in the area \( A \). It is clear that the greater effectiveness of the TL is only apparent when the antidots are sufficiently large (\( r \approx 230 \) nm). For medium-sized antidots (\( r \approx 160 \) nm), the differences in \( n \) between PL’s and TL’s are very small, and for the smallest antidots (\( r \approx 70 \) nm), there is no difference at all within our experimental accuracy. Furthermore, the antidot occupation \( n \) saturates at \( \approx 1 \) and 2 for small and medium antidots, respectively, and at \( \approx 3 \) for large antidots (TL’s only). At first glance, this may seem surprising because even for the smallest antidots \( r > \xi(4 \, K) \approx 18 \) nm, and one would expect the saturation number to be\(^{27} \) \( n_s = r/2\xi(T) \approx 2, 4, \) and 6 for small, medium,
FIG. 3. (Color online) Occupation number \( n \) as a function of field \( B \) (normalized to fixed \( B_1 \), i.e., with fixed \( n_p = 0.47 \, \mu m^{-2} \)) for different antidot size \([r \approx 70 \, nm \, (small), 160 \, nm \, (medium), \text{and} \, 230 \, nm \, (large)]\) and array geometry \([\text{triangular lattice (TL), Penrose lattice (PL), and random (Ran)}]\). The straight dashed line \( n = B/B_1 \) corresponds to full occupation (i.e., all vortices captured by antidots); solid and short-dashed lines are guides to the eye. The inset shows data for random antidot configurations.

and large antidots, respectively. Here \( \xi(T) = \xi(0)(1 - T/\Phi_0)^{-1/2} \) is the temperature-dependent coherence length. However, we recall that the decoration patterns obtained in field-cooling experiments typically correspond to a snapshot of vortex positions “frozen in” at a higher temperature \( T^* \), due to the quickly diminishing vortex mobility with decreasing temperature (corresponding to increasing \( I_c \) as observed in transport experiments). For extra vortices to enter the antidots with decreasing temperature [as \( n_v(T) \) increases], the interstitials have to be mobile to come sufficiently close to the antidots and feel their attractive potential. With this in mind, we use the simple relation \( n_v(T) = r/2\xi(T) \) and our Bitter decoration data for \( n_v \) (which we assume correspond to \( T = T^* \)) and \( r \) to estimate \( \xi(T^*) \), which gives us the same estimate \( \xi(T^*) \approx 40 \, nm \) for all three antidot sizes in Fig. 3 (for TL’s). Then using the temperature dependence for the coherence length \( \xi(T) = \xi(0)(1 - T/\Phi_0)^{-1/2} \), we estimate \( T^* \approx 0.89 T_c \). We note that the use of the relation \( n_v = r/2\xi \) may provide only a rough estimate for \( T^* \), as this relation is, strictly speaking, only valid for a single isolated antidot.\(^{27,28}\)

Deviations from this relation due to antidot configuration (PL’s and random) are already visible in Fig. 3, and the impact of variable \( n_v \) will be discussed below.

Data for samples with random antidot configurations are shown in the inset of Fig. 3. Here the proportion of vortices in interstitial positions is always much higher than for TL’s and PL’s, as is seen qualitatively in Fig. 2(c). In terms of \( n \), there is no difference between large and medium antidots, while small antidots trap on average significantly less than \( \Phi_0 \) even at \( B = 3B_1 \). These findings are consistent with the expectation that vortex-vortex repulsion should lead to a macroscopically uniform distribution of vortices in the film, i.e., it should prevent trapping of many vortices by large (clusters of) antidots even if their size permits it. As the random antidot configurations will not be considered further below, we would like to stress at this point that Bitter decoration clearly shows their inefficiency in pinning as compared to TL’s and PL’s. This is in line with the transport data, showing always much lower \( I_c \) as compared to TL’s and PL’s.\(^{21}\)

The observed vortex configurations help to elucidate the microscopic mechanism of pinning in arrays with different geometric arrangement of pinning sites. Moreover, they provide further information to explain the lower absolute values of the critical current \( I_c \) at integer matching fields for PL’s (due to more interstitials compared to TL’s) and the inefficiency of random antidot configurations. However, one has to be careful in comparing transport data obtained very close to \( T_c \) (showing matching effects) with images of vortex configurations obtained below the freezing temperature \( T^* \). This applies to the Bitter decoration experiments described here, but also to (most) other vortex imaging techniques.

Indeed, Fig. 3 shows that both for TL’s and PL’s, the large and medium-sized antidots are fully occupied (no interstitials) for all fields \( B < 2B_1 \). With the definition of the normalized critical current \( J_c \) as the ratio of the density of pinned vortices \( nvp \) over \( n_v \) from Ref. 17, one obtains \( J_c = n/(B/B_1) \) and hence \( J_c = 1 \) for full occupation \( n = B/B_1 \); dashed line in

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Fig. 3). As we did not find any indications of modulations in antidot occupation \( n \) below \( 2B_1 \) (as predicted by simulations and observed in transport experiments\(^{23}\)), our observations imply \( J_* = 1 \) at all \( B < 2B_1 \). Furthermore, the fact that we obtain \( J_* = 1 \) near \( B = B_1/2 \) is clearly in contradiction to simulations, which predict either a minimum in \( J_* (B) \) or a flat \( J_* (B) \) with \( J_* \) well below \( 1 \) near \( B = B_1/2 \).\(^{15,18}\) We also note that full occupation up to \( B = B_1 \) was also shown by SHP imaging (at 4.2 K) on PL’s with magnetic dots (see supplementary material in Ref. 23). The absence of modulations in \( J_* (B) \) in the imaging experiments is not surprising. This simply suggests that for small enough fields at \( T \approx T^* \), vortex-vortex interactions are too weak to oppose their trapping in potential wells created by the antidots.

It is interesting to note that the very few interstitials seen for TL’s of large antidots at \( B \approx B_1 \) [Fig. 2(a)] are not distributed uniformly but mostly form dimers, i.e., pairs of vortices sitting equidistant on opposite sides of an antidot. We believe that this is an experimental manifestation of the effect predicted in Ref. 12 (cf. Fig. 14), namely that pinning sites (antidots) can remain unoccupied (or underoccupied) with two vortices sitting on either side of an antidot and caged in by six adjacent antidots filled with vortices. In this case, placing an interstitial vortex between two occupied antidots and creating a highly asymmetric pattern of vortex interactions costs more energy than having two interstitials symmetrically placed around an unoccupied (or underoccupied) antidot. As noted in Ref. 12, this type of defect can disrupt overall ordering of interstitial vortices, so that higher-field fractional matching effects cannot be observed. It is also interesting to analyze the positions of interstitial vortices where their number exceeds one per antidot. For TL’s, this situation was studied in detail in numerical simulations,\(^{11}\) and our results for TL’s with medium-sized holes at \( B \approx B_1 \) [see Fig. 4(a)] agree with their predictions, i.e., the interstitials predominantly sit in the centers of the triangles formed by the antidots.

Our images for PL’s reveal that for a wide range of applied fields (corresponding to up to two interstitials per antidot), interstitial vortices are found predominantly in the centers of thick rhombuses [see Fig. 4(b)], which is in accordance with the theoretical predictions\(^{17,18}\) and SHP imaging on magnetic dot Penrose arrays.\(^{23}\)

### B. Bitter decoration: Variable antidot density

The results for TL’s presented in Fig. 3 imply that, as the applied field increases, new vortices continue to fill the antidots until the saturation is reached, and only after that do vortices start to appear in interstitial positions, in accordance with Refs. 11, 15, and 16. However, deviations from that simple picture are obvious for the PL’s (cf. short-dashed line in Fig. 3), for which the distance between antidots takes different values throughout the lattice.

Indeed, the importance of antidot spacing for a given pinning array was shown by recent numerical simulations based on nonlinear Ginzburg-Landau theory for thin films (\( d \ll \xi, \lambda \)), perforated with square or circular antidots arranged in a square lattice. These simulations showed that the occupation number \( n_p \) and hence the saturation number \( n_\tau \), should depend not only on the antidot size but also on the antidot lattice parameter \( a \propto 1/\sqrt{n_p} \) and the overall vortex density \( n_v = B/\Phi_0 \).\(^{14,29}\) In order to investigate the possible effects of the density of the antidot lattice on the occupation number \( n_\tau \), we carried out a series of decoration experiments on TL’s and PL’s with variable antidot density \( n_p \propto 1/a^2 \) for several values of applied field.

![FIG. 5. (Color online) SEM images of vortex patterns in Nb films with triangular arrays of antidots (\( r \approx 230 \text{ nm} \)) decorated at the same applied magnetic field \( B = 2.3 \text{ mT} \). From (a) to (f), the antidot density decreases from \( n_p = 0.47 \) to 0.12 \( \mu \text{m}^{-2} \), i.e., the lattice parameter \( a \propto 1/\sqrt{n_p} \) increases from 1.6 to 3.2 \( \mu \text{m} \). Yellow dots mark positions of interstitial vortices. The labels indicate \( B_1 = B_1/\Phi_0 \) (upper left) and occupation number \( n \) (upper right).](184520-5)
TL’s and three different values of symbols are interpolated from measured data points (np with different right insets show occupation number n external fields (see text for further details). Upper left insets indicate (large set consists of six 4Q structures with the same antidot size ranging from B different applied fields for each fixed value of S. RABLEN the TL’s [cf. Fig. 6(a)], the antidot occupation vortices. 4 to 3) leads to a sharp increase in the number of interstitial to cover a range of five to seven (integer) matching fields B to 0 Fig. 5(c) to Fig. 5 (by increasing μm−1). This allowed us to cover a range of five to seven (integer) matching fields Bn for each fixed value of B by changing a by a factor of 2.

Three (nominally identical) sets were decorated at three different applied fields B = 1.51, 1.97, and 2.30 mT. Each set consists of six 4Q structures with the same antidot size (large r ≈ 230 nm) and geometry but different antidot density, ranging from nP = 0.12 μm−2 (B1 = 0.25 mT; a ≈ 3.2 μm) to 0.47 μm−2 (B1 = 0.97 mT; a ≈ 1.6 μm). This allowed us to cover a range of five to seven (integer) matching fields Bn for each fixed value of B by changing a by a factor of 2.

Figure 5 shows images from one set (TL’s only) obtained for B = 2.3 mT. Remarkably, even though a ≫ ξ(T∗) ≈ 40 nm for all antidot densities, the relatively small increase in a from Fig. 5(c) to Fig. 5 (by increasing nA/nP from approximately 4 to 3) leads to a sharp increase in the number of interstitial vortices.

For a quantitative analysis, Figs. 6(a) and 6(b) show the occupation number n (from Bitter decoration images) versus nA/nP = B/B1 for TL’s and PL’s, respectively. For the TL’s [cf. Fig. 6(a)], the antidot occupation n shows a clear increase in n for the two higher values of the applied field upon decreasing nA/nP (from 4 to 3), while it stays roughly constant at n ≈ 2 for the smallest applied field. Upon further reducing nA/nP, the occupation number reaches the (dashed) line n = nA/nP (full occupation) and follows this line. For the PL’s [cf. Fig. 6(b)], the increase in n with decreasing nA/nP below 4 is also present but much less pronounced.

The sharp rise in n with decreasing a is exactly the prediction (for a square lattice of pinning sites) of Ref. 29, i.e., with decreasing antidot separation, the interstitial vortices are pushed so close together that their mutual repulsion makes it energetically more favorable for the system to push extra vortices into the antidots, thus increasing n. In the inset of Fig. 2 in Ref. 29, a steplike increase in n(a) upon decreasing a is predicted for fixed nA/nP = 2, . . . , 5. In order to facilitate direct comparison with this prediction, in the insets of Fig. 6 we plot n versus 1/√nA/nP ∝ a for two different values of fixed nA/nP = B/B1 (at the third and fourth matching field) for TL’s and PL’s. As our data points in the main graphs are not exactly at B = B3 and B = B4, we show here data points that are close to these conditions as full symbols, and we also plot interpolated data points as open symbols in the insets. For the TL’s [upper left inset in Fig. 6(a)], the increase of the occupation number from 2 to 3 upon decreasing the lattice parameter a is clearly visible. This increase in antidot occupation seems to occur at the same antidot separation for both matching fields B3 and B4. For the PL’s, the increase in n with decreasing a seems to set in at the same value of a ≈ 2 μm; however, this increase is much shallower and is not completed for the lowest values of a shown here. This might be explained by the fact that in the PL’s there is more room for the interstitials to go into the thick rhombuses, and hence the force due to vortex-vortex interaction between interstitials is not yet strong enough to increase the occupation number to 3. This then explains the lower (average) n for the PL’s as compared to the TL’s.

IV. CONCLUSIONS

Our Bitter decoration experiments revealed that the effectiveness of antidots as artificial pinning sites depends crucially on several factors: array geometry, antidot size, antidot density (antidot separation), and applied magnetic field (vortex density). Regarding array geometry, we find that the triangular lattice provides the most efficient pinning, although the Penrose lattice is equally effective for a wide range of parameters, while the random arrangement of antidots is always much less effective. This observation is broadly in agreement with electric transport measurements obtained very close to Tc, except for earlier observations in which the Penrose lattice provided even improved pinning as compared to the triangular lattice below the first matching field. However, we argue that the observation of vortex patterns by imaging after field-cooling to well below Tc provides snapshots of the vortex configuration at the freezing temperature T∗ (in our case ≈0.89 Tc), which cannot be directly compared to matching effects in electric transport measurements, typically performed much closer to Tc. We also note that for the interpretation of the matching effects observed very close to Tc, one should
also consider the hole-induced suppression of $T_c$ (Little-Parks effect) in the wire network regime, as discussed in Ref. 30. In contrast, our Bitter decoration experiments reveal information on the occupation number $n$ (average number of vortices trapped per antidot) at $T^*$, i.e., in the pinning regime. These experiments provide insights into deviations from the simple pinning effect of an isolated antidot\textsuperscript{27,28} due to the presence of a whole array of antidots. Upon increasing the antidot density $n_p$, i.e., reducing the distance between antidots, we clearly find an increase in $n$ for triangular arrays due to the repulsive interaction of vortices, which pushes more vortices into antidots. For Penrose arrays, this effect is less pronounced, which can be due to the reduced symmetry of the Penrose array as well as to more options for accommodating interstitial vortices because of the inherent variations of antidot spacing.

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\textsuperscript{25}Thermal evaporation of Fe particles usually leads to a temporary increase in temperature of the decorated samples, but the increase never exceeded 2 K, leaving the Nb films in the low-temperature limit, $T < 0.5 \mathcal{T}_c$.
\textsuperscript{26}The contrast in Bitter decoration images is influenced by many factors, including the self-field of vortices in a given superconductor, vortex size (effective magnetic penetration depth $\Lambda$), the average vortex density, and the size of Fe particles used for decoration. In situations in which the forces on Fe particles are weak, as in the present experiment (due to the large $\Lambda$), the contrast also depends on the applied magnetic field: As the evaporated Fe particles drift toward the surface of the sample, they are polarized by the external field, which increases their sensitivity to magnetic-field gradients due to vortices.
\textsuperscript{27}G. S. Mkrtchyan and V. V. Schmidt, Sov. Phys. JETP \textbf{34}, 195 (1972).