Quantum phase transitions in fermionic systems

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Outline

1. **Introduction**
   - Understanding metals
   - Fermi liquids
2. **Break-down of a Fermi liquid**
   - Luttinger liquids
   - BCS-BEC crossover
   - Gauge fields
   - Quantum critical point
3. **Quantum phase transitions in fermionic systems**
   - Heavy fermions
   - Organic superconductors
   - High temperature superconductors
4. **Summary**

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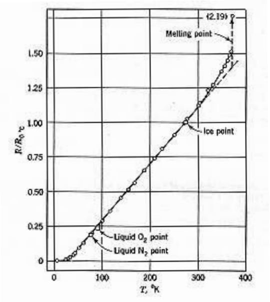
Understanding metals

First puzzle

Understanding metals

First puzzle

Resistivity vs. temperature



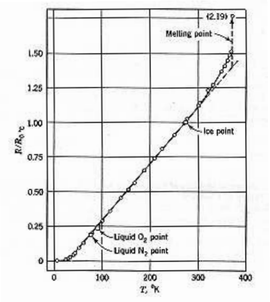
$\rho \sim T^2$

More degrees of freedom participate in conduction?

Understanding metals

First puzzle

Resistivity vs. temperature



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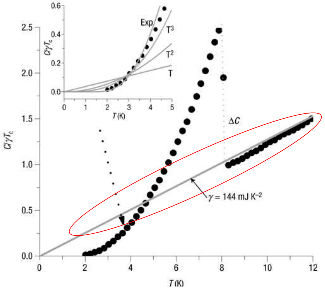
More degrees of freedom participate in conduction?

Specific heat vs. temperature



$c_V \sim T$

Less degrees of freedom are excited

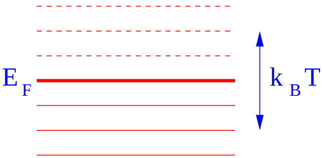
Understanding metals

Fermi-Dirac statistics

Fermi-Dirac statistics

Specific heat of a Fermi gas

$$c_V \sim N(E_F)\, k_B T$$



Understanding metals

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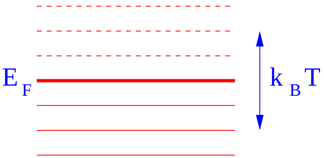

Pauli paramagnetism

$$\chi = \frac{\partial M}{\partial H} = N(E_F)\, \mu_B$$

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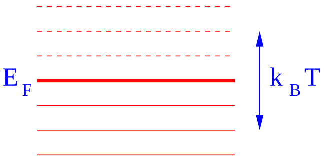

Pauli paramagnetism

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Second puzzle: electrons are charged particles
$\hookrightarrow$ **Coulomb interaction**

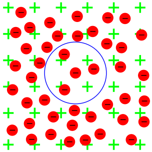
Quasiparticles

Fermi liquids

Quasiparticles

Weak interacting particles:

$$\begin{aligned} Q &= e \\ S &= \frac{1}{2} \end{aligned}$$



Fermi liquids

Quasiparticles

Weak interacting particles:

$$
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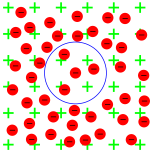
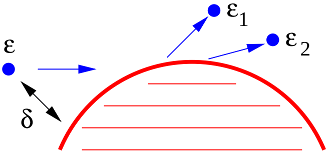


Scattering rate close to a Fermi surface



$$
\begin{aligned}
\varepsilon &= E_F + \delta & \varepsilon_1 &= E_F + \delta_1 \\
\varepsilon' &= E_F - \delta' & \varepsilon_2 &= E_F + \delta_2
\end{aligned}
$$

Fermi liquids

Quasiparticles

Weak interacting particles:

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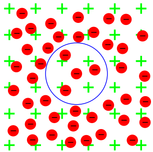

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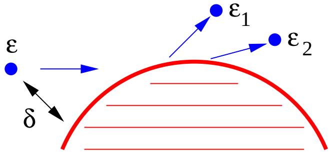

$$\varepsilon = E_F + \delta \qquad \varepsilon_1 = E_F + \delta_1$$
$$\varepsilon' = E_F - \delta' \qquad \varepsilon_2 = E_F + \delta_2$$

$$\Gamma \leq \int_{E_F}^{E_F+\delta} d\varepsilon_1 \, N(\varepsilon_1) \times \int_{E_F}^{E_F+\delta} d\varepsilon_2 \, N(\varepsilon_2) \times N(\varepsilon_1 + \varepsilon_2 - \varepsilon) \sim [N(E_F)]^3 \, \delta^2$$

Weak interaction near E_F

Fermi liquids all over

Fermi liquids all over

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Fermi liquids

Fermi liquids all over

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- **^{3}He** $\longrightarrow \sim 100 mK$

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Fermi liquids

Fermi liquids all over

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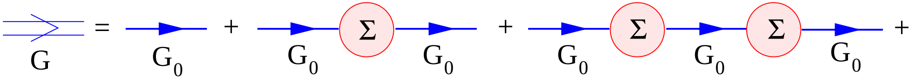
- **Neutron stars** $\longrightarrow \sim 10^9 - 10^{12} K$

Fermi liquids

Quasiparticle weight



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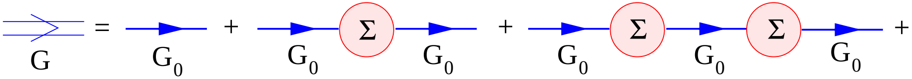

Propagator of a particle

$$
\begin{aligned}
G &= G_0 + G_0 \Sigma G_0 + G_0 \Sigma G_0 \Sigma G_0 + \cdots \\
&= G_0 \left(1 + \Sigma G\right) = \left(G_0^{-1} - \Sigma\right)^{-1}
\end{aligned}
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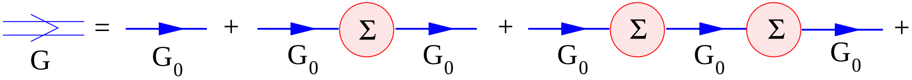

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Fermi liquids

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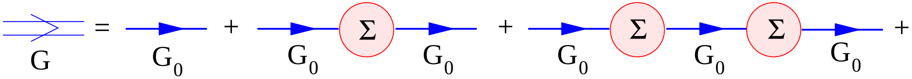

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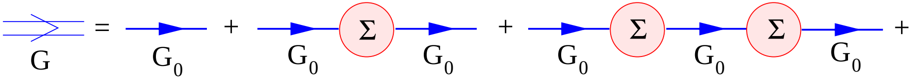
Quasiparticle weight



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\hookrightarrow G_{\mathrm{coh}}(\mathbf{k}, t) &= \int_{-\infty}^{\infty} d\omega \, \mathrm{e}^{-i\omega t} \frac{z(\mathbf{k})}{\hbar\omega - \epsilon_{\mathbf{k}} + i\Gamma} = z(\mathbf{k}) \exp\left[-i\left(\epsilon_{\mathbf{k}} - i\Gamma\right) t\right]
\end{aligned}
$$

$z(\mathbf{k})$: **Quasiparticle weight**

Quasiparticle weight and spectroscopy

Quasiparticle weight and spectroscopy

- **Angle-resolved photoemission spectroscopy (ARPES)**
  Spectral function
  $A(\mathbf{k}, \hbar\omega) = -\frac{1}{\pi} \mathrm{Im}\, G(\mathbf{k}, \omega) = \frac{z(\mathbf{k})\,\Gamma}{(\hbar\omega - \epsilon_{\mathbf{k}})^2 + \Gamma^2}$

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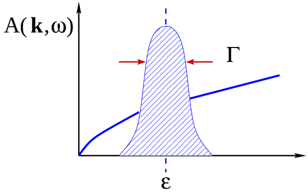

Fermi liquids

Quasiparticle weight and spectroscopy

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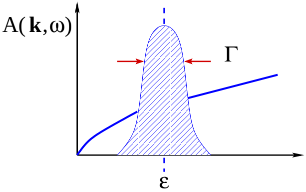
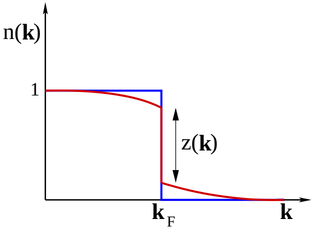


- **Momentum distribution function**

  $G(\mathbf{k},\omega) \longrightarrow n(\mathbf{k})$



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Break-down of a Fermi liquid

Interaction singular in the infrared

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- **1-D: Luttinger liquids**

Break-down of a Fermi liquid

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- **Resonance: BEC-BCS crossover**

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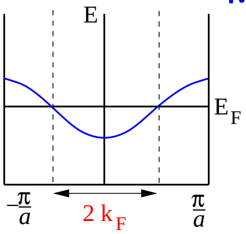
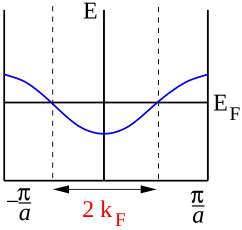
Luttinger liquids

Metals in one dimension

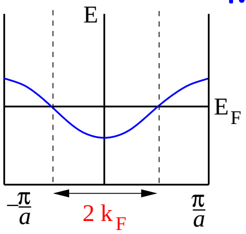




Metals in one dimension

Zero energy excitations for $q = 2k_F$

Luttinger liquids

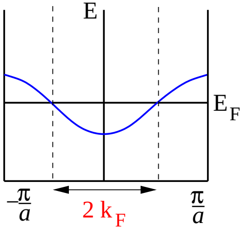


Metals in one dimension

Zero energy excitations for $q = 2k_F$

$\hookrightarrow$ **Diverging response at** $2k_F$

Luttinger liquids



Metals in one dimension

Zero energy excitations for $q = 2k_F$

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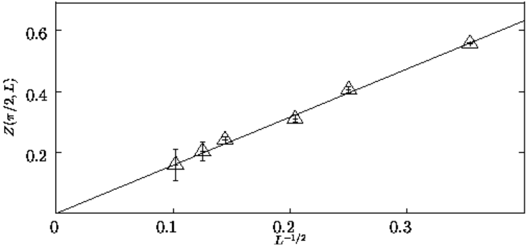
Vanishing quasiparticle weight

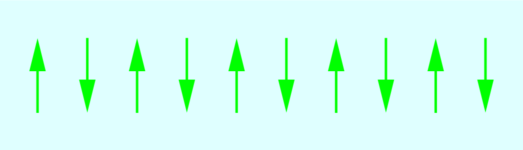


QMC for the t-J model

$z(k) \rightarrow 0$ **in the thermodynamic limit**

New elementary excitations

Hole in a quantum antiferromagnet



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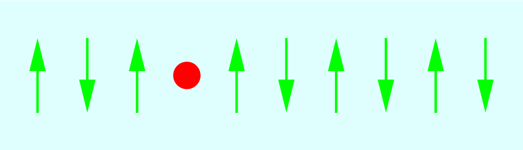

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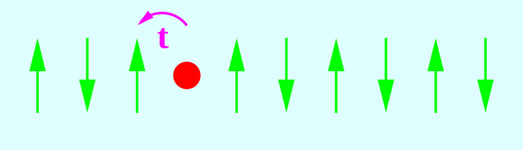

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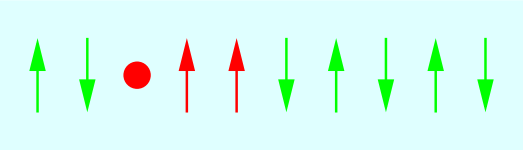

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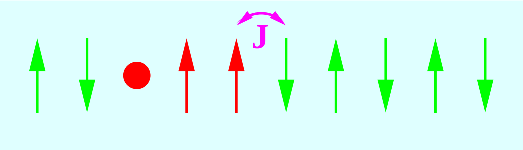

Luttinger liquids

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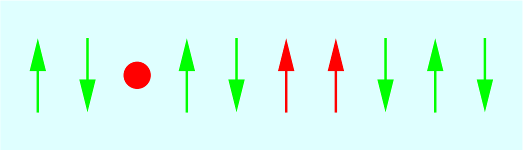

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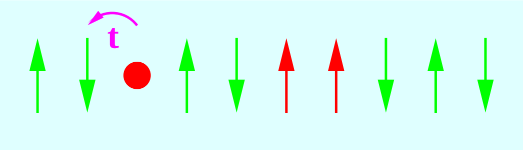

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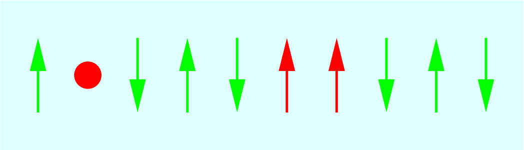

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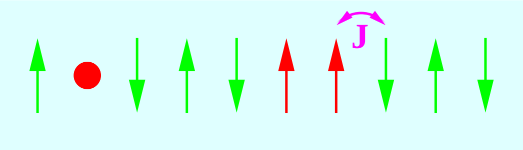

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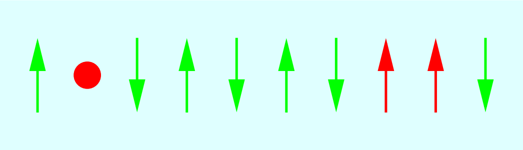

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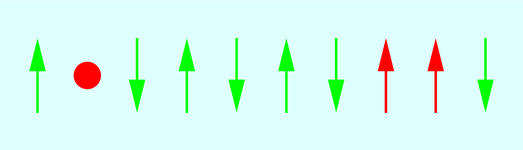
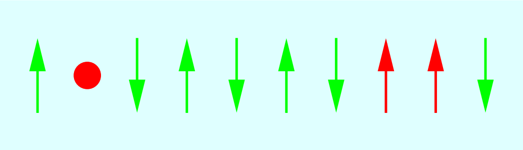


Charge velocity $\sim t$
Domain wall velocity $\sim J$

New elementary excitations

Hole in a quantum antiferromagnet



Charge velocity $\sim t$
Domain wall velocity $\sim J$ $\longrightarrow$ **Charge-spin separation**

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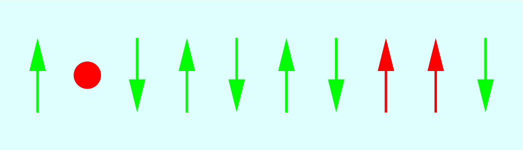


Charge velocity $\sim t$
Domain wall velocity $\sim J$ $\longrightarrow$ **Charge-spin separation**

Holon: $Q = -e$, $S = 0$ **Spinon:** $Q = 0$, $S = \frac{1}{2}$

BCS-BEC crossover

BCS-BEC crossover

From weakly attractive to tightly bound pairs

BCS-BEC crossover

From weakly attractive to tightly bound pairs

BCS limit: Fermi liquid

BCS-BEC crossover

From weakly attractive to tightly bound pairs

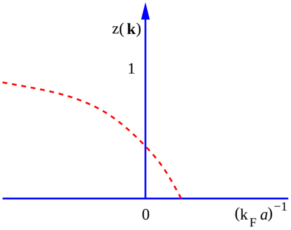
BCS limit: Fermi liquid

BEC limit: no independent fermions

From weakly attractive to tightly bound pairs

BCS limit: Fermi liquid

BEC limit: no independent fermions

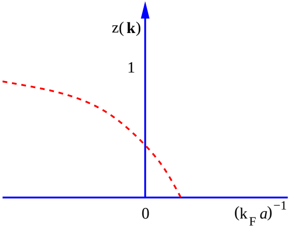


BCS-BEC crossover

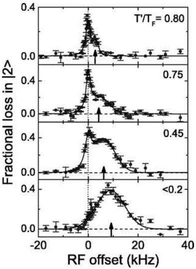
From weakly attractive to tightly bound pairs

BCS limit: Fermi liquid

BEC limit: no independent fermions



Pseudogap at BCS-BEC crossover



Coexistence of single particles and preformed pairs

C. Chin *et al*, Science **305**, 1128 (2004)

Gauge fields

Gauge fields

Coulomb interaction

From Gauss-law: $v(\mathbf{q}) = \frac{4\pi e}{\mathbf{q}^2}$

Gauge fields

Coulomb interaction

From Gauss-law: $v(\mathbf{q}) = \frac{4\pi e}{\mathbf{q}^2} \rightarrow$ **infrared divergent**

Gauge fields

Coulomb interaction

From Gauss-law: $v(\mathbf{q}) = \frac{4\pi e}{\mathbf{q}^2} \rightarrow$ **infrared divergent**

Screening: $v(\mathbf{q}) = \frac{4\pi e}{\mathbf{q}^2 + \kappa^2}$, κ^{-1}: **Thomas-Fermi screening length**

Coulomb interaction

From Gauss-law: $v(\mathbf{q}) = \frac{4\pi e}{\mathbf{q}^2} \rightarrow$ **infrared divergent**

Screening: $v(\mathbf{q}) = \frac{4\pi e}{\mathbf{q}^2 + \kappa^2}$, κ^{-1}: **Thomas-Fermi screening length**

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$$z \xrightarrow{T \to 0} 0$$

Gauge fields

Emergent gauge fields

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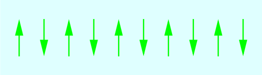
Quantum antiferromagnets



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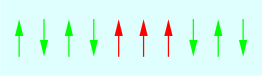

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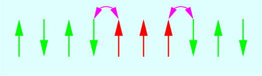

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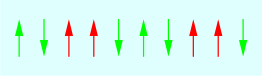

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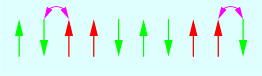

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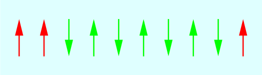

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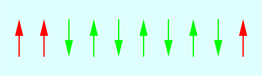

$\longrightarrow$ Deconfined spinons in 1D

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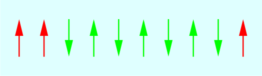

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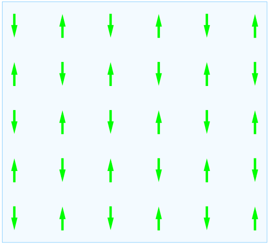

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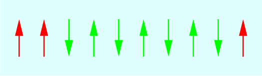

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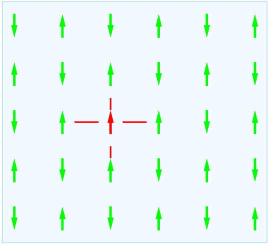

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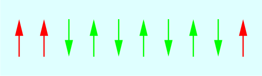

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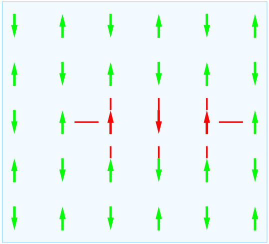

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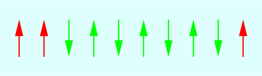

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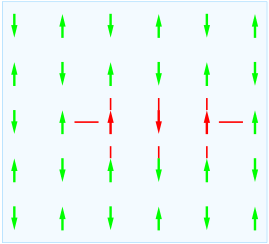

$\longrightarrow$ **Linear confinement of spinons in 2D**

Critical fluctuations

Quantum critical point

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Quantum critical point

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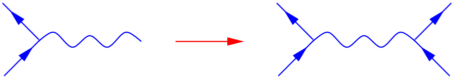

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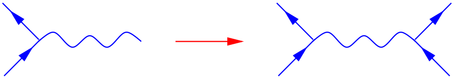

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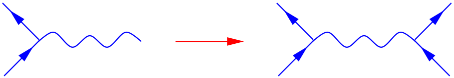

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- Large critical region around a quantum critical point

Outline

Heavy fermions

Strongly interacting Fermi liquid

- **Compounds with d- and f-electrons**

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- **Large specific heat and Pauli susceptibility**

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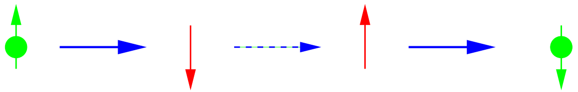
  $\hookrightarrow$ Kondo-lattice

Kondo screening



Kondo lattice

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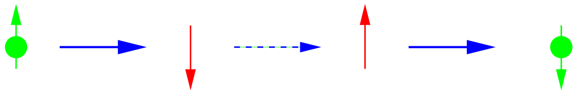

- **High temperature: impurity scattering**

Kondo lattice

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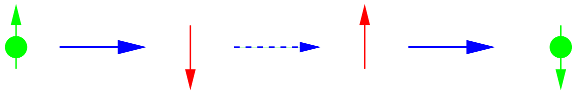

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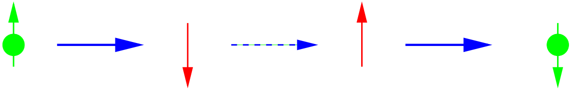
Kondo lattice

Kondo screening



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Kondo lattice

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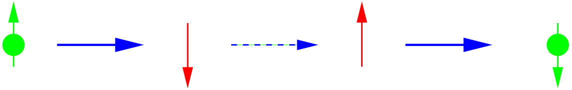

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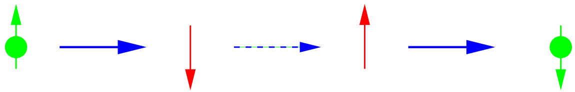

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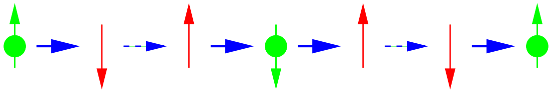
- **Heavy Fermi liquid formed when Kondo screening clouds overlap** $\longrightarrow$ $T_{\rm coh}$ **coherence temperature**

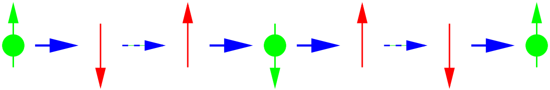
Kondo lattice

RKKY interaction



Kondo lattice

RKKY interaction



↪ **Exchange interaction among localized moments mediated by conduction electrons**

Kondo lattice

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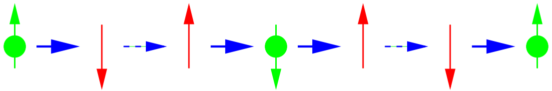

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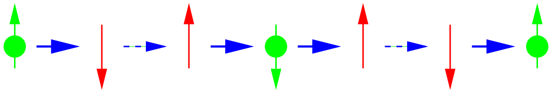
Kondo screening vs. RKKY interaction

Kondo lattice

RKKY interaction



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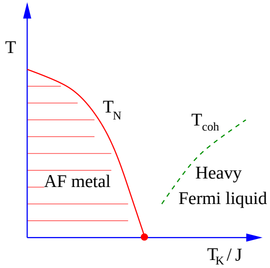

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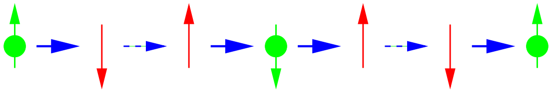

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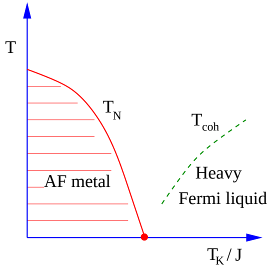
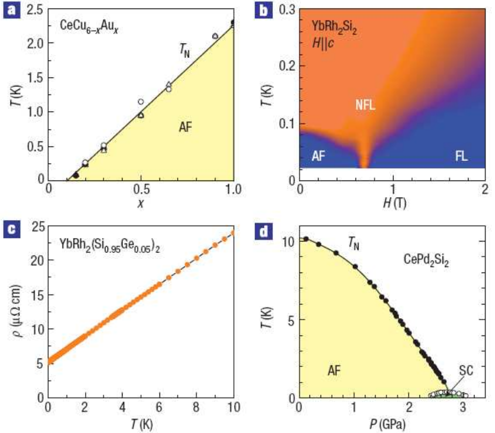


Quantum critical point tunable by interactions

Quantum critical points, non-Fermi liquids, and superconductivity

P. Gegenwart, Q. Si, and F. Steglich, Nature Physics **4**, 186 (2008)



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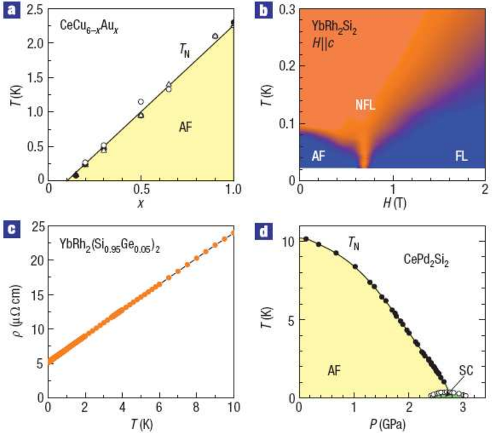

a **QCP by doping**

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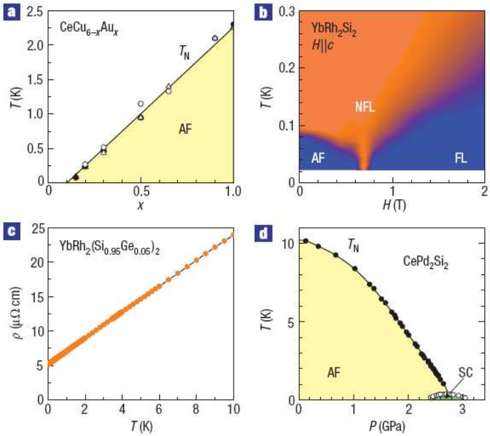

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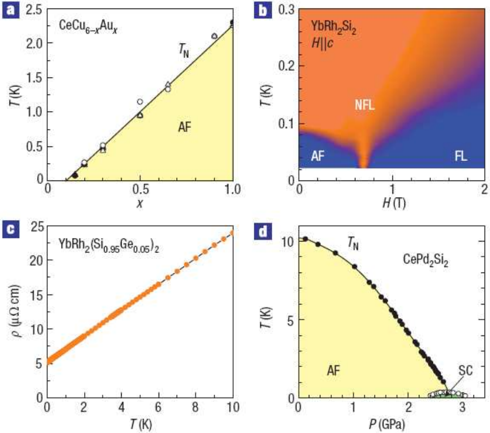

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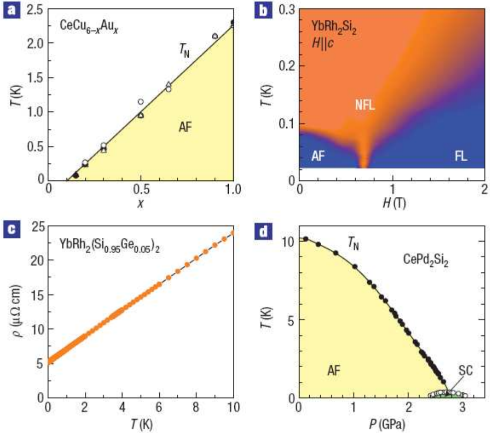
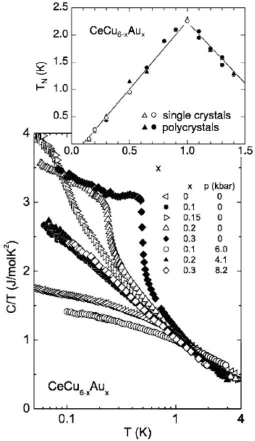


- a QCP by doping
- b QCP by magnetic field
- c Non-Fermi liquid
- d Superconductivity at the QCP

Non-Fermi liquid behavior

Non-Fermi liquid behavior

Specific heat close to a quantum critical point



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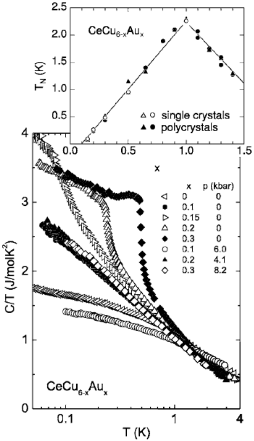

- $\frac{c_V}{T} \sim \ln \frac{T_0}{T}$ **over two decades in temperature**

Non-Fermi liquid behavior

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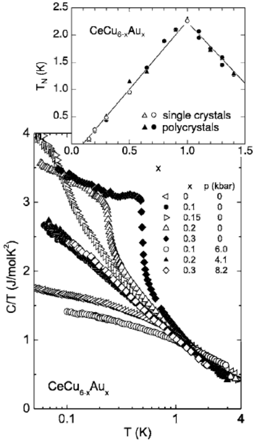


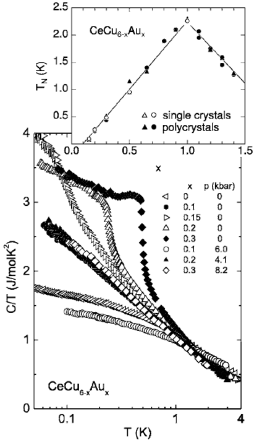
- $\frac{c_V}{T} \sim \ln \frac{T_0}{T}$ over two decades in temperature

  $\hookrightarrow$ Additional entropy close to the QCP

Quantum critical points in heavy fermion systems

Non-Fermi liquid behavior

Specific heat close to a quantum critical point



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  $\hookrightarrow$ Additional entropy close to the QCP

- Consistent with additional scattering channels for resistivity

Superconductivity

Superconductivity

Example: $CeCu_2(Si_{1-x}\ Ge_x)$ and $CeCu_2Si_2$

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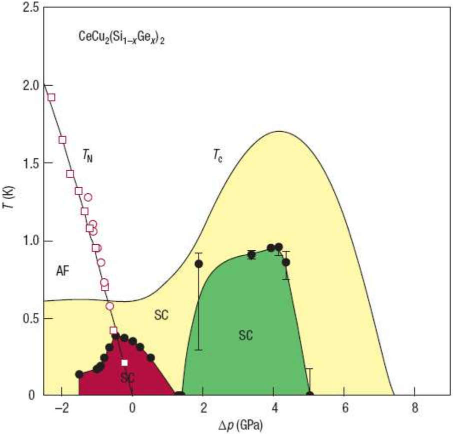

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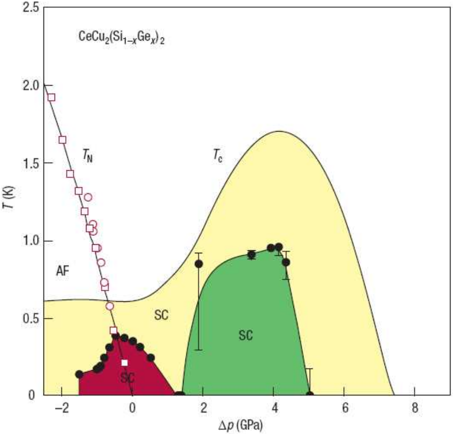

- **Superconducting dome around the QCP: general feature**

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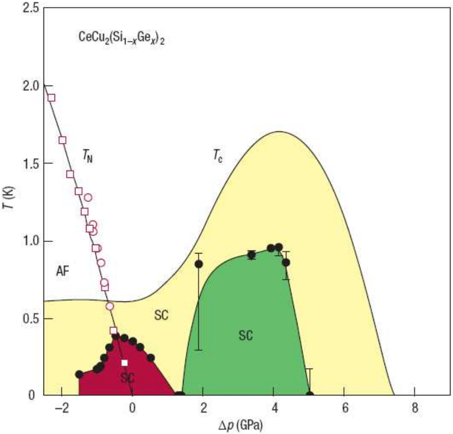


- Superconducting dome around the QCP: general feature
- Unconventional order parameter for superconductivity

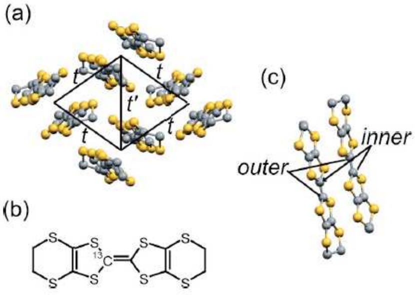
Organic superconductors

Strongly correlated electrons on geometrically frustrated lattices

Organic superconductors

Strongly correlated electrons on geometrically frustrated lattices

κ-(BEDT-TTF)$_2$Cu$_2$X



a **Dimer sheet in κ-(BEDT-TTF)$_2$Cu$_2$(CN)$_3$ forming a triangular lattice**

Organic superconductors

Strongly correlated electrons on geometrically frustrated lattices

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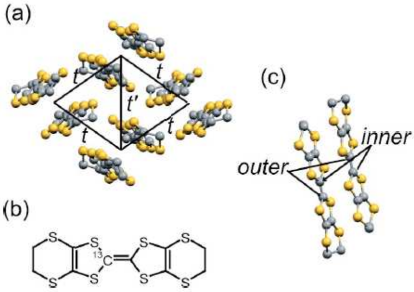


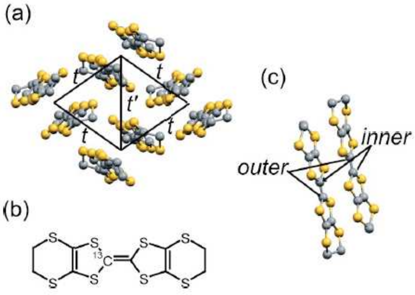
a **Dimer sheet in κ-(BEDT-TTF)$_2$Cu$_2$(CN)$_3$ forming a triangular lattice**

b **Single BEDT-TTF molecule**

Organic superconductors

Strongly correlated electrons on geometrically frustrated lattices

κ-**(BEDT-TTF)$_2$Cu$_2$X**



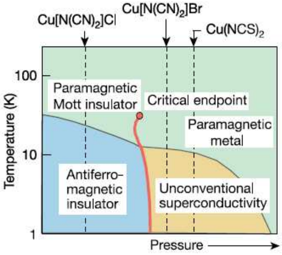
a **Dimer sheet in κ-(BEDT-TTF)$_2$Cu$_2$(CN)$_3$ forming a triangular lattice**

b **Single BEDT-TTF molecule**

c **Dimer at the sites of the triangular lattice**

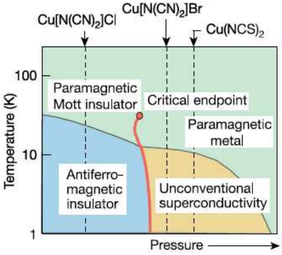
Organic superconductors

Mott-insulators and superconductivity



Organic superconductors

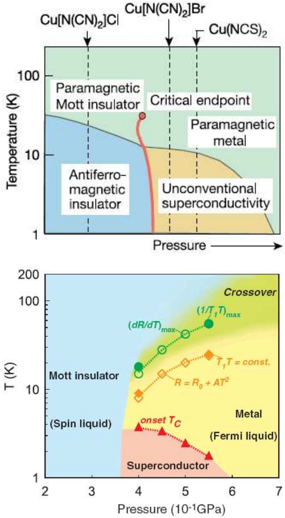
Mott-insulators and superconductivity



Hubbard model at half-filling on a triangular lattice vs. U/t

F. Kagawa, K. Miyagawa, and K. Kanoda, Nature **436**, 534 (2005)

Mott-insulators and superconductivity



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Organic superconductors

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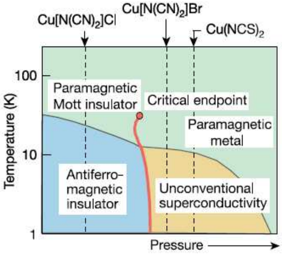
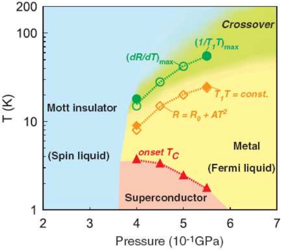




Hubbard model at half-filling on a triangular lattice vs. U/t

F. Kagawa, K. Miyagawa, and K. Kanoda,

Nature **436**, 534 (2005)

Spin-liquid phase up to lowest temperatures

Y. Kurosaki,Y. Shimizu, K. Miyagawa,

K. Kanoda, and G. Saito,

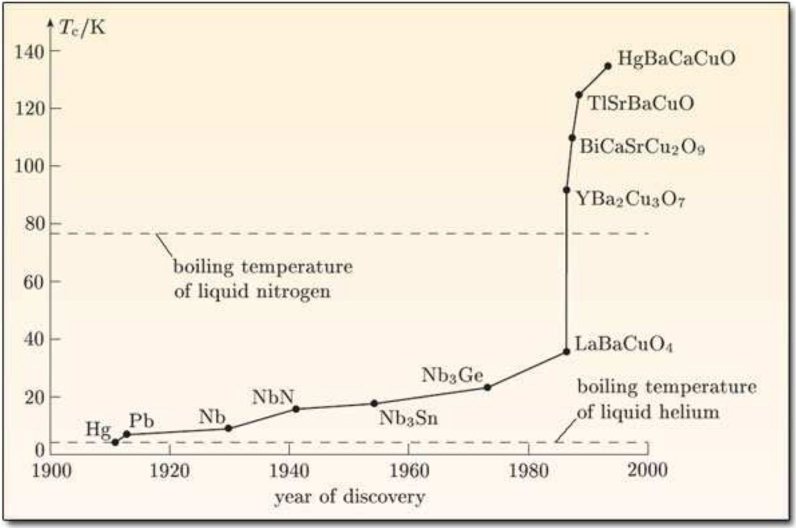
Phys. Rev. Lett. **95**, 177001 (2005)

High temperature superconductors

Critical temperature

High temperature superconductors

Critical temperature

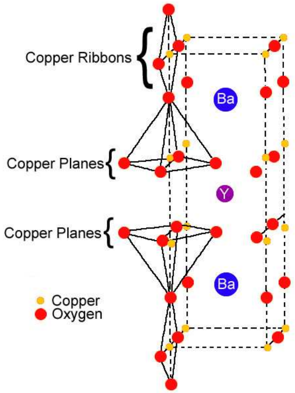


High temperature superconductors

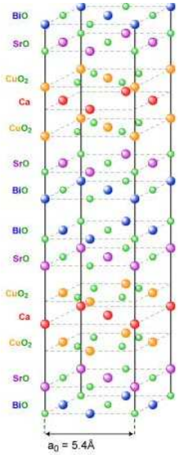
High temperature superconductors

Structure

$YBa_2Cu_3O_7$ - $T_c \simeq 95$ K



$Bi_2Sr_2CaCu_2O_8$ - $T_c \simeq 110$ K



High temperature superconductors

Structure

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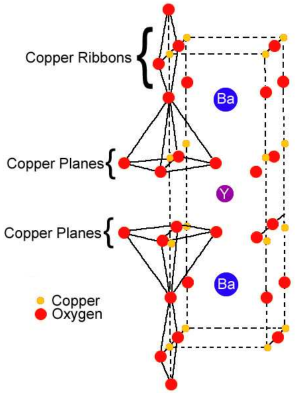
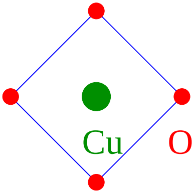
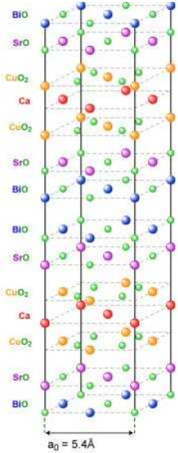
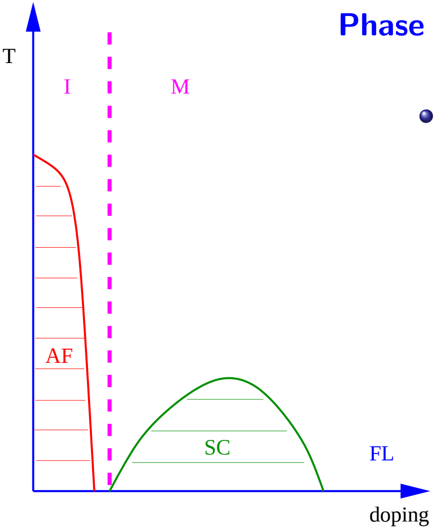






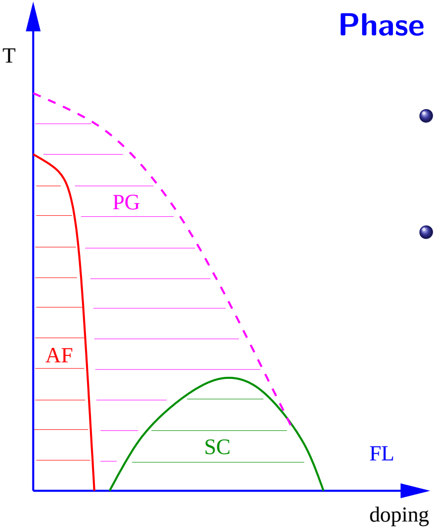
Phase diagram I

High temperature superconductors



Phase diagram I

- Antiferromagnetic Mott-insulator without doping

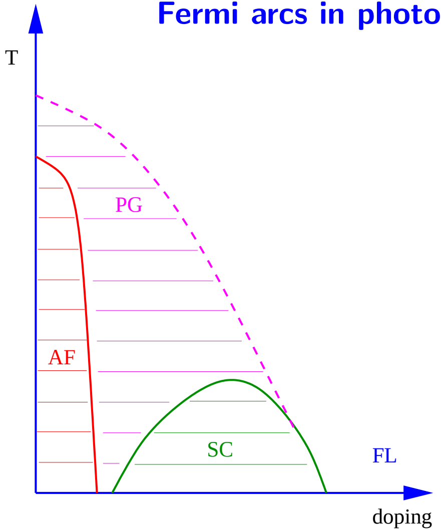


Phase diagram I

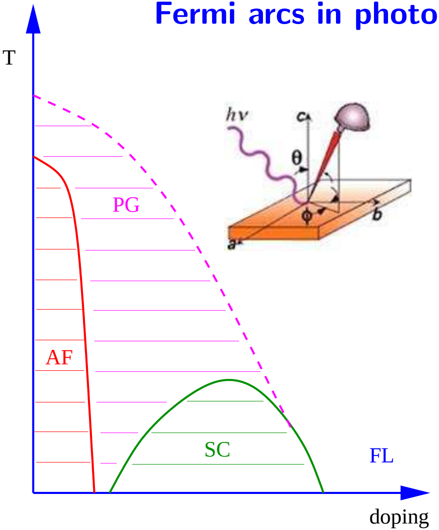
- **Antiferromagnetic Mott-insulator without doping**
- **Decrease of c_V and χ**

Pseudogap region in high T_c superconductors

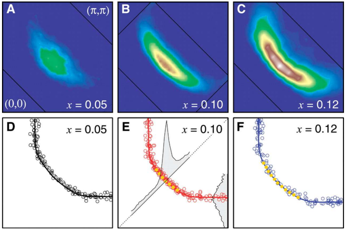
Fermi arcs in photoemission spectroscopy



Fermi arcs in photoemission spectroscopy



Fermi arcs

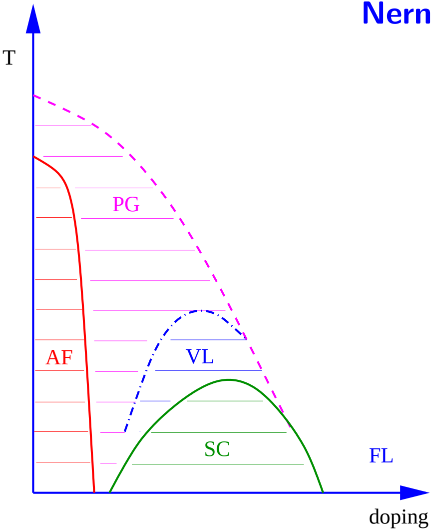


A. Damascelli, Z. Hussain, and Z.-X. Shen

Rev. Mod. Phys. **75**, 473 (2003)

Pseudogap region in high T_c superconductors

Nernst effect



Pseudogap region in high T_c superconductors

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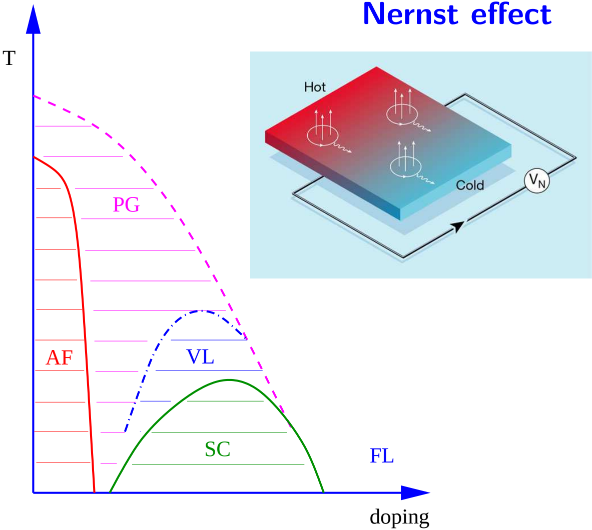
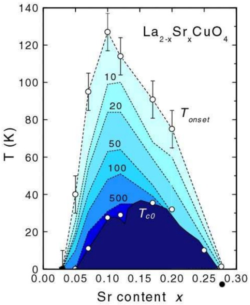
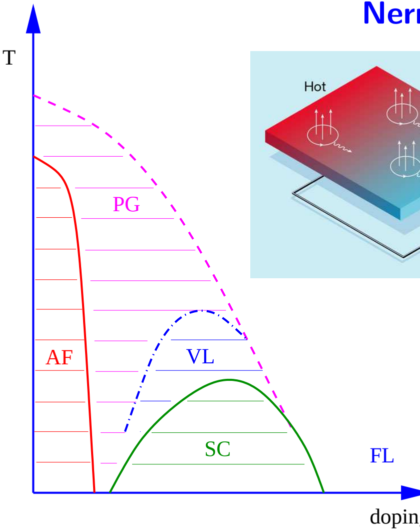




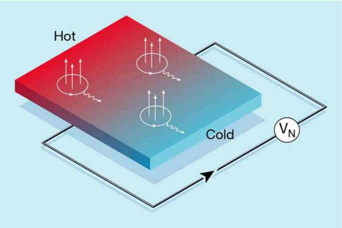
Z. Xu *et al*,

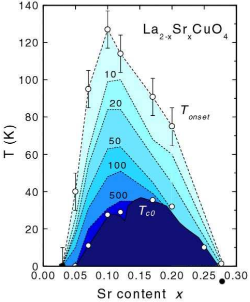
Nature **406**, 486 (2000)

Pseudogap region in high T_c superconductors



Nernst effect





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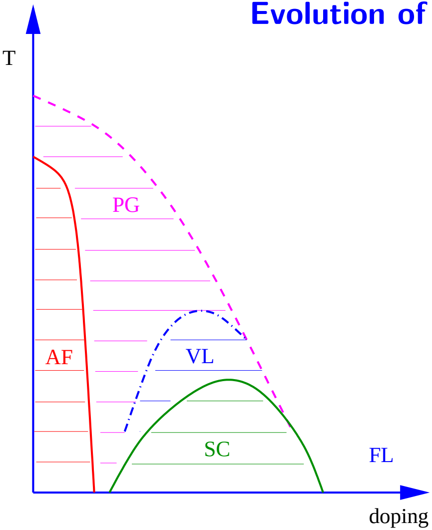
preformed pairs

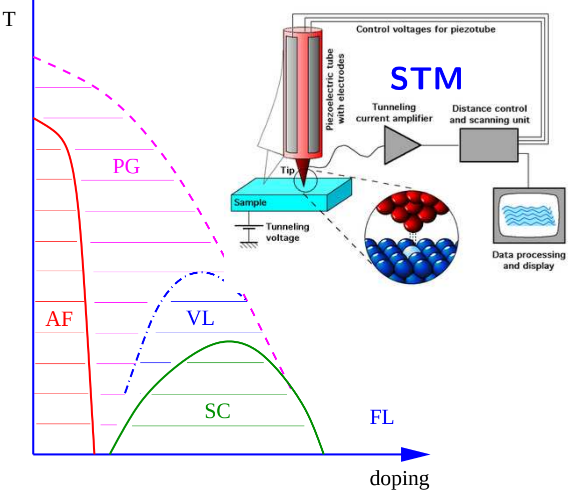
Pseudogap region in high T_c superconductors

Evolution of gaps vs. doping



Pseudogap region in high T_c superconductors

Evolution of gaps vs. doping



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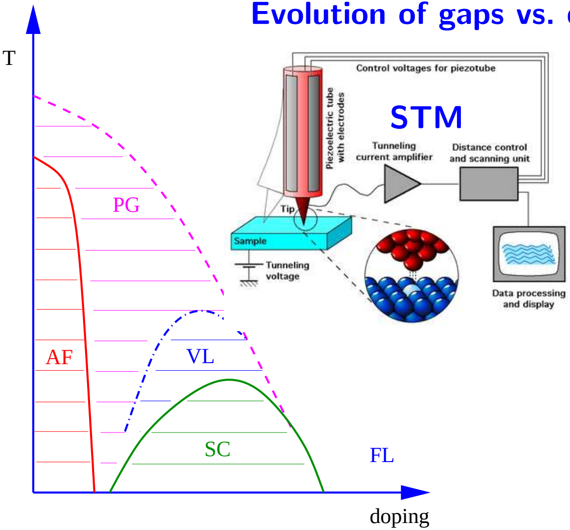
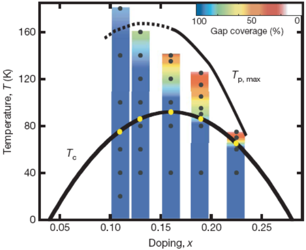
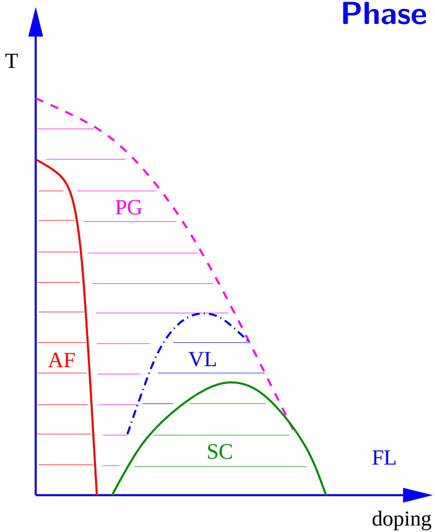




K.K. Gomes *et al*,

Nature **447**, 569 (2007)

Phase diagram I



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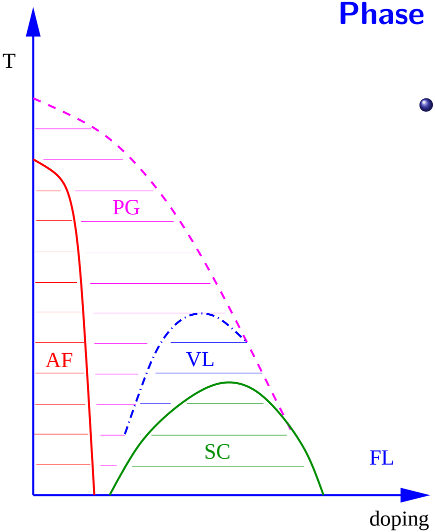
Phase diagram I



- **Nernst effect $\longrightarrow$ preformed pairs**

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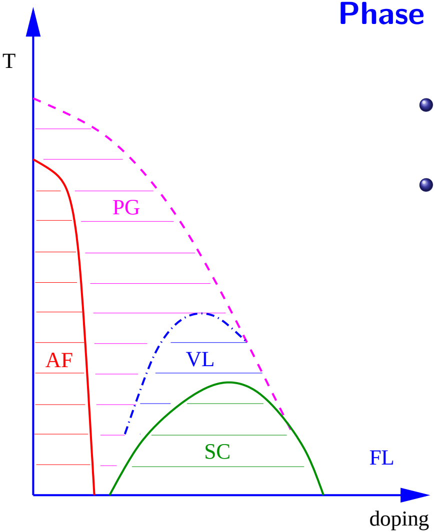
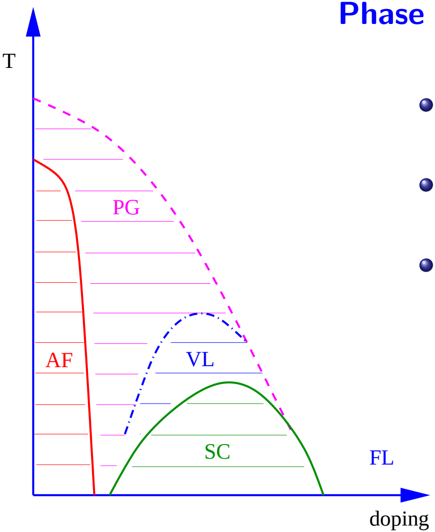


Phase diagram I

- Nernst effect $\longrightarrow$ preformed pairs
- The pseudogap is due to the formation of pairs above T_c



Phase diagram I

- Nernst effect ⟶ preformed pairs
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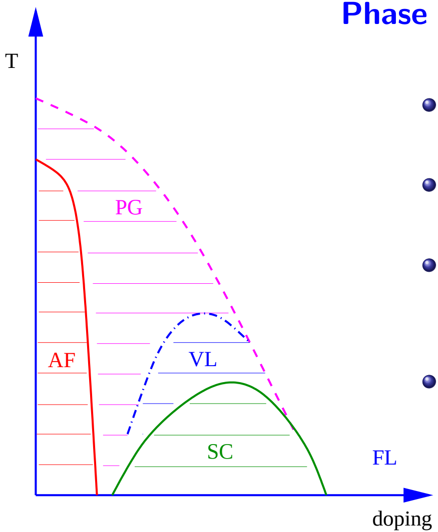
Pseudogap region in high T$_c$ superconductors



Phase diagram I

- Nernst effect $\longrightarrow$ preformed pairs
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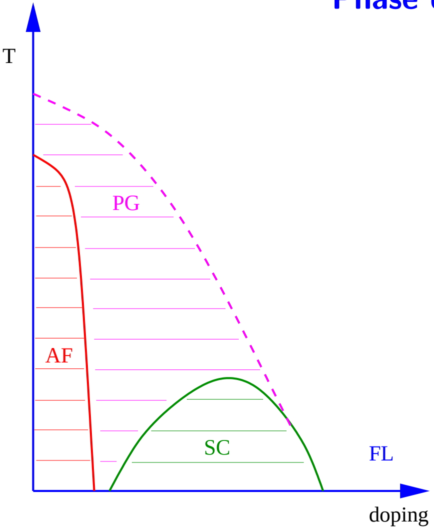
- Consistent with the absence of thermodynamic signals for a phase transition
- STM suggests: pseudogap and superconducting gap have the same origin

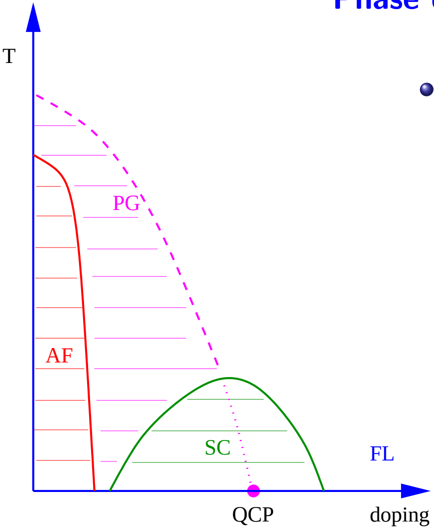
Pseudogap region in high T_c superconductors

Phase diagram II

Phase diagram II

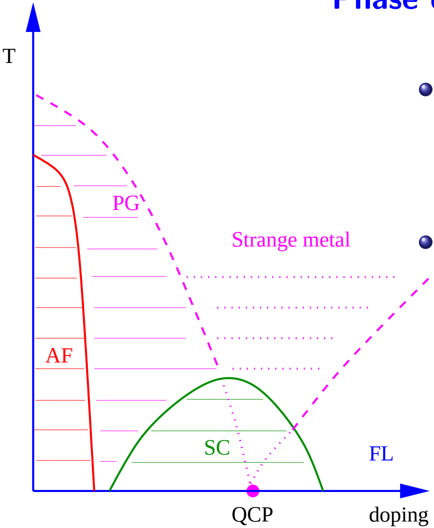


Phase diagram II



- PG-line corresponds to a spontaneous symmetry breaking → QCP hidden by the superconducting dome

Phase diagram II



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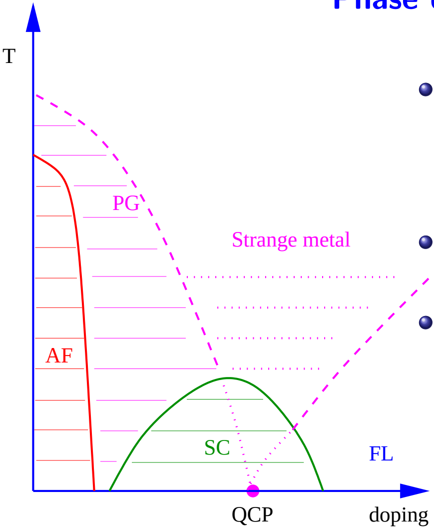

- PG-line corresponds to a spontaneous symmetry breaking → QCP hidden by the superconducting dome
- Quantum critical region → anomalies in the normal state
- Which symmetry breaking?

Symmetry breaking in pseudogap region

Nematic electronic liquid

V. Hinkov *et al.*, Science **319**, 597 (2008).

Symmetry breaking in pseudogap region

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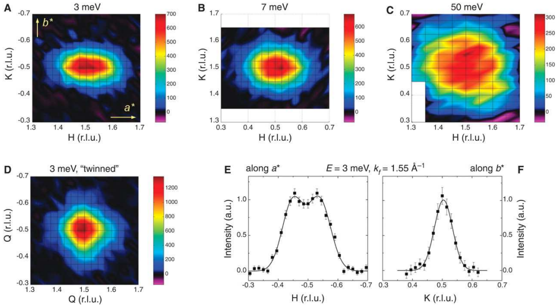
V. Hinkov *et al.*, Science **319**, 597 (2008).

Nematic order



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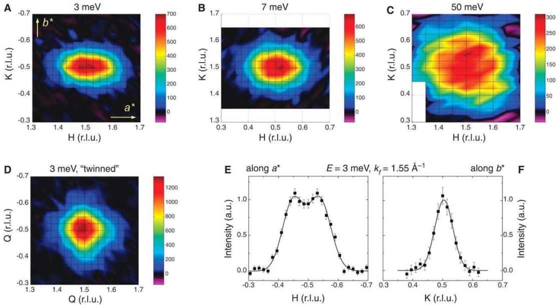
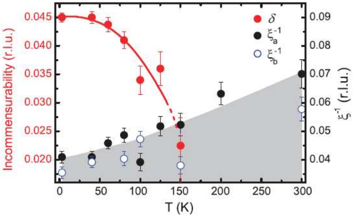
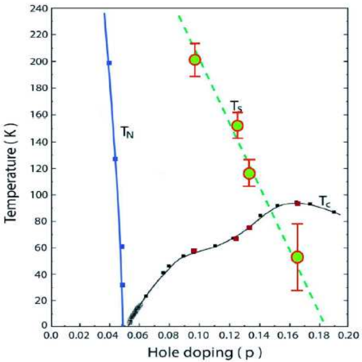


Incommensurability vs. T



Time reversal symmetry breaking I

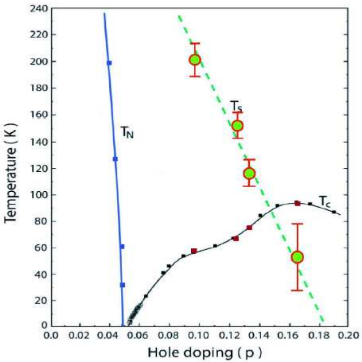
J. Xia *et al.*, Phys. Rev. Lett. **100**, 127002 (2008).



- **Polar Kerr-effect → parity or time reversal symmetry breaking**

Time reversal symmetry breaking I

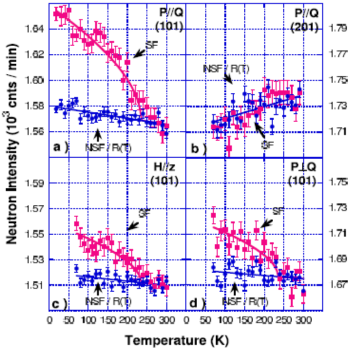
J. Xia *et al.*, Phys. Rev. Lett. **100**, 127002 (2008).



- **Polar Kerr-effect → parity or time reversal symmetry breaking**
- **Non-zero signal at temperatures around the pseudogap T^***

Time reversal symmetry breaking II

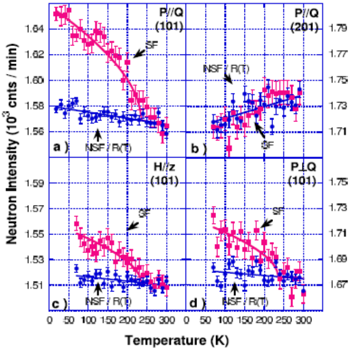
H.A. Mook *et al.*, arXiv:0802.3620



- **Neutron scattering with polarized neutrons** $\hookrightarrow$ **magnetic ordering**

Time reversal symmetry breaking II

H.A. Mook *et al.*, arXiv:0802.3620



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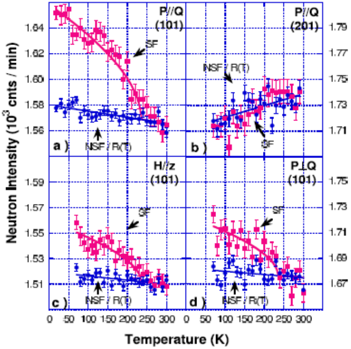

- **Neutron scattering with polarized neutrons**
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- **Non-zero signal at temperatures around the pseudogap T^***
- **Possible origin: orbital currents**

Outline

Summary

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- **Fermi liquid is the rule for a fermionic system**

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- Open questions:

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Summary

Further reading:

Nature Physics **4** (2008). Reviews on quantum phse transitions

"Fermi-liquid instabilities at magnetic quantum phase transitions"
H. v. Löhneysen, A. Rosch, M. Vojta, and P. Wölfle
Reviews of Modern Physics **79**, 1015 (2007)