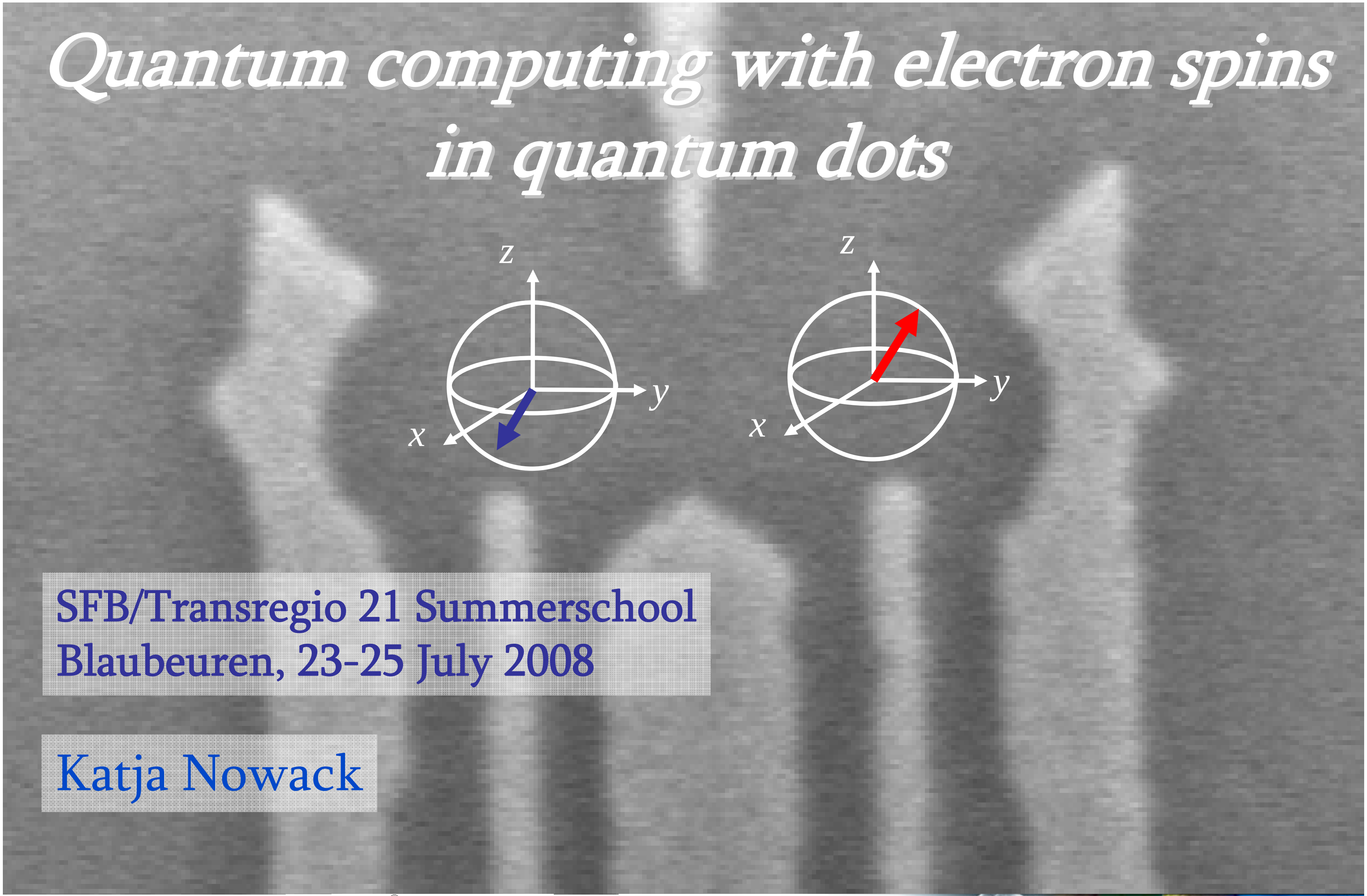
Quantum computing with electron spins in quantum dots



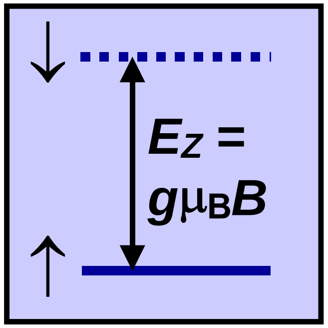
SFB/Transregio 21 Summerschool
Blaubeuren, 23-25 July 2008

Katja Nowack

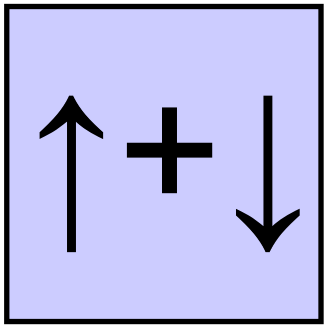
NWO FOM TUDelft Delft University of Technology Kavli Institute of Nanoscience

Electron spin as a quantum bit

natural
2-level system

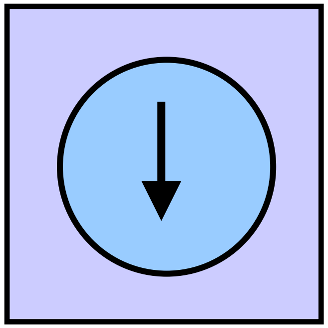


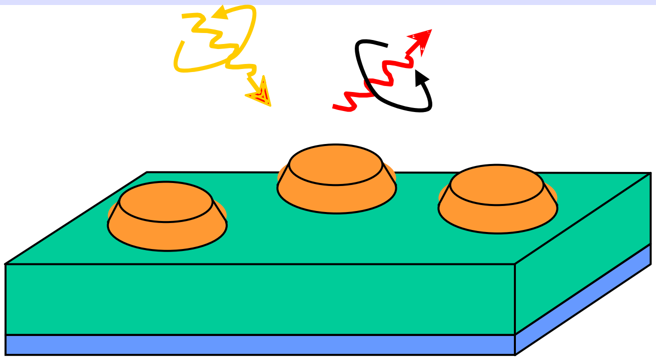
long
coherence time



Charge property:

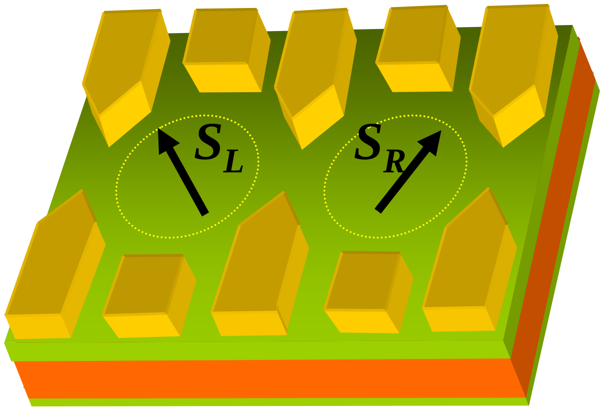
- Confinement
- Read-out





quantum dots (optical)

Imamoglu et al, *PRL* 1999



quantum dots (electrical)

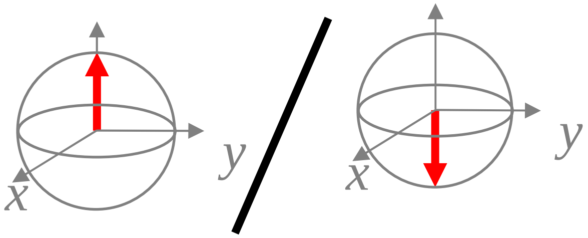
Loss, DiVincenzo, *PRA* ('98)

Spin qubits in quantum dots

Loss & DiVincenzo, PRA 1998

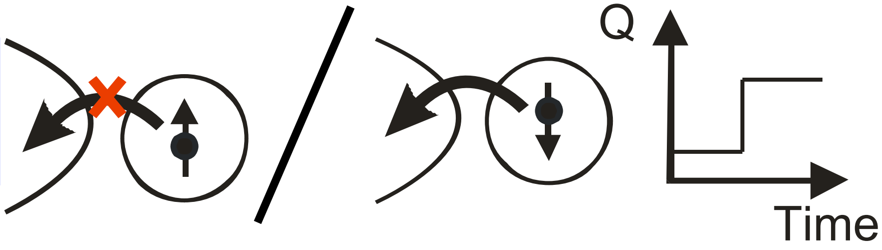
Well-defined qubit states

- *Confine single electrons*



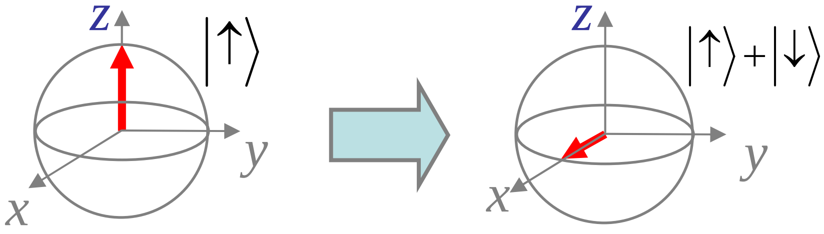
Initialize and read-out

- *Spin to charge conversion*



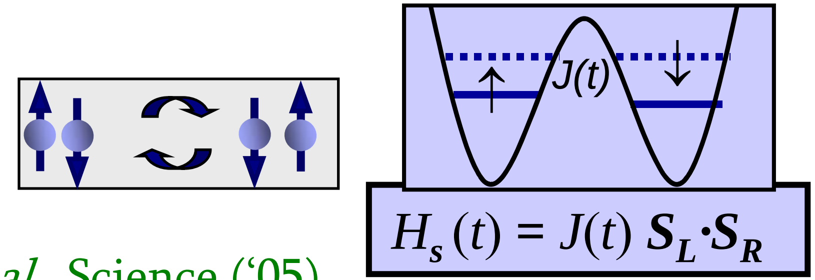
1-qubit gate

- *Electron spin resonance*



2-qubit gate

- *Exchange interaction*



$$H_s(t) = J(t)\, \boldsymbol{S_L} \cdot \boldsymbol{S_R}$$

Petta *et al.*, Science ('05)

Spin qubits in quantum dots

Loss & DiVincenzo, PRA 1998

Well-defined qubit states

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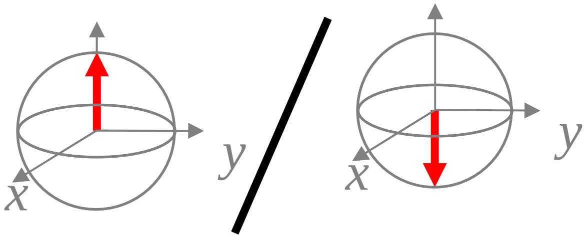

Initialize and read-out

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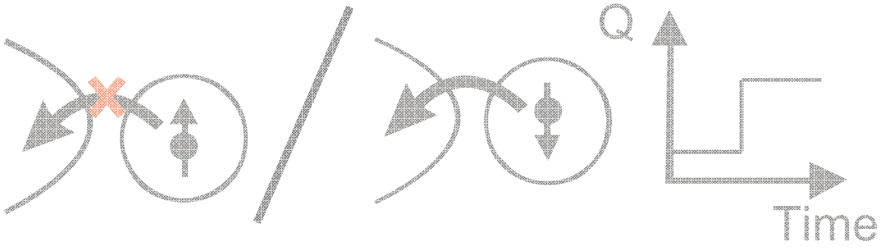

1-qubit gate

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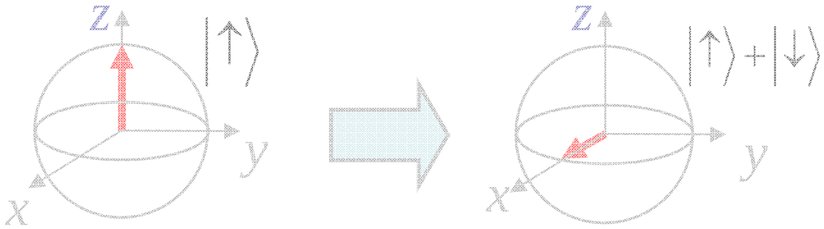
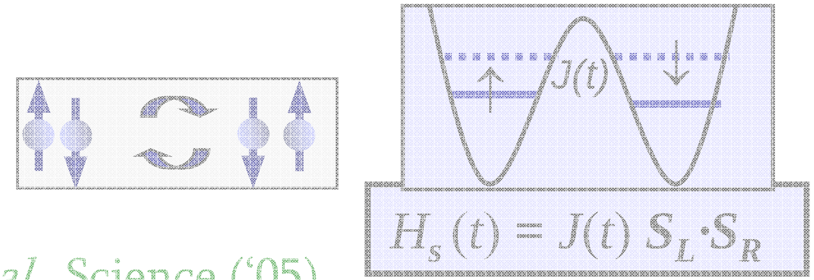


2-qubit gate

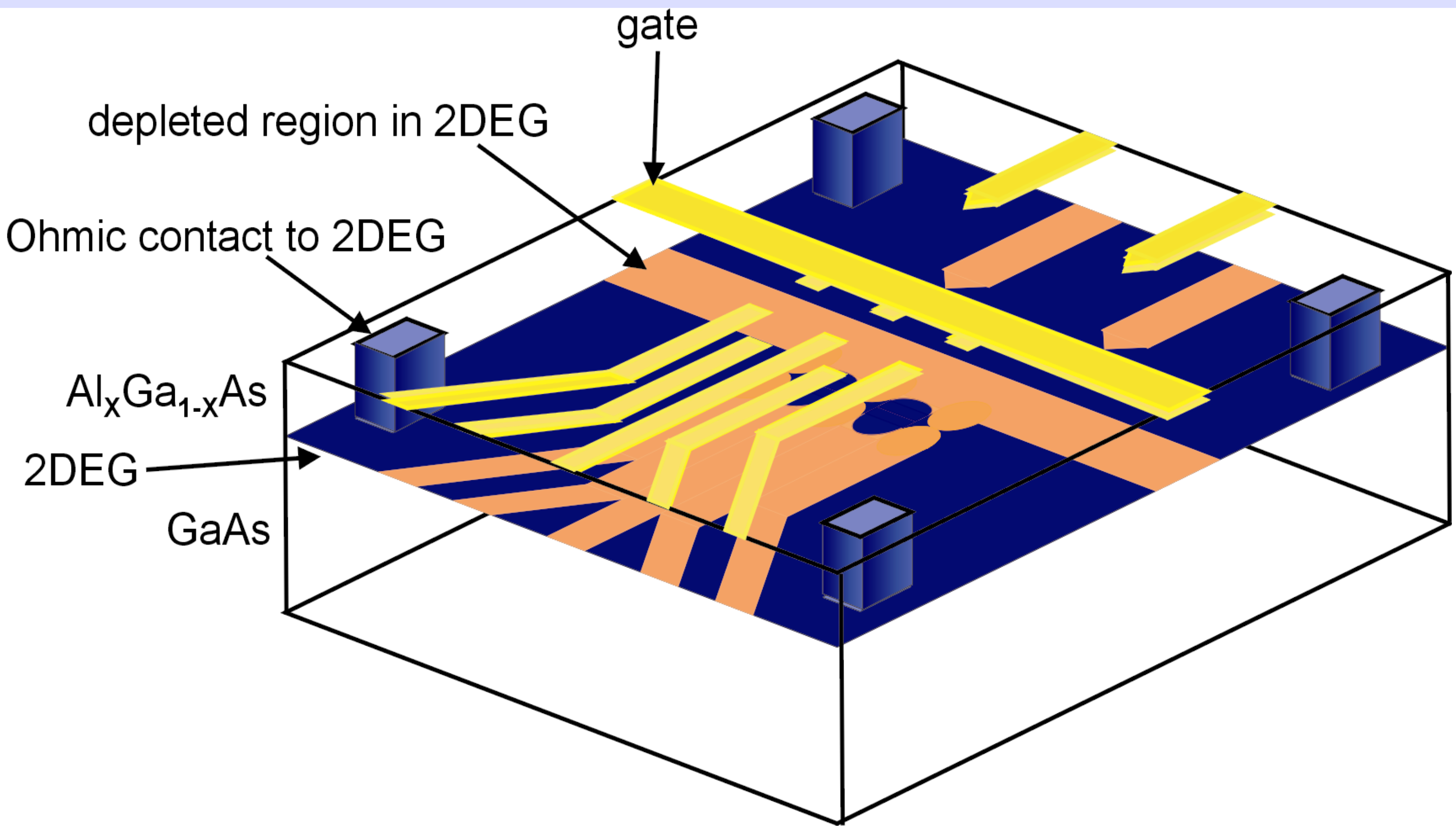
- *Exchange interaction*



$$H_s(t) = J(t)\, S_L \cdot S_R$$

Petta *et al.*, Science ('05)

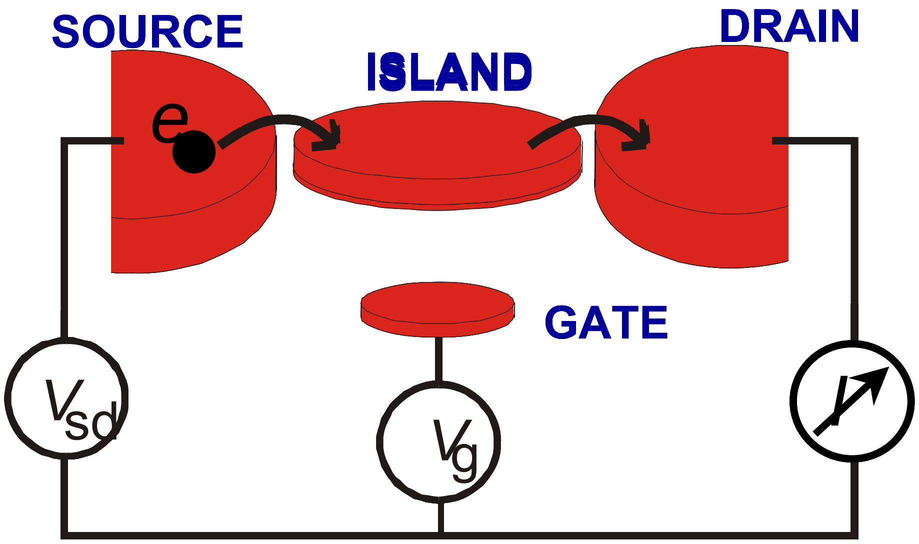
Electrostatically defined quantum dots



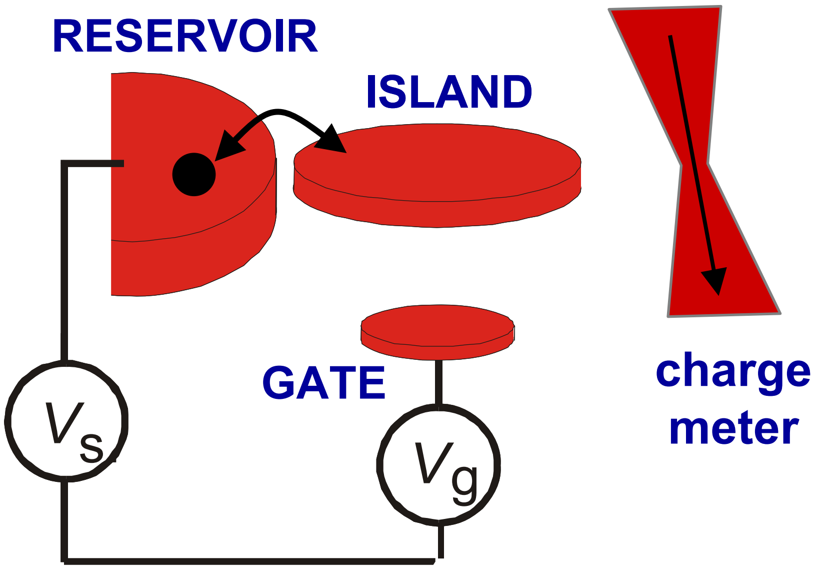
- Electrically measured (contact to 2DEG)
- Electrically controlled number of electrons
- Electrically controlled tunnel barriers

Electrical measurement of quantum dots

Quantum dot = island with a discrete number of electrons
occupying discrete set of orbitals



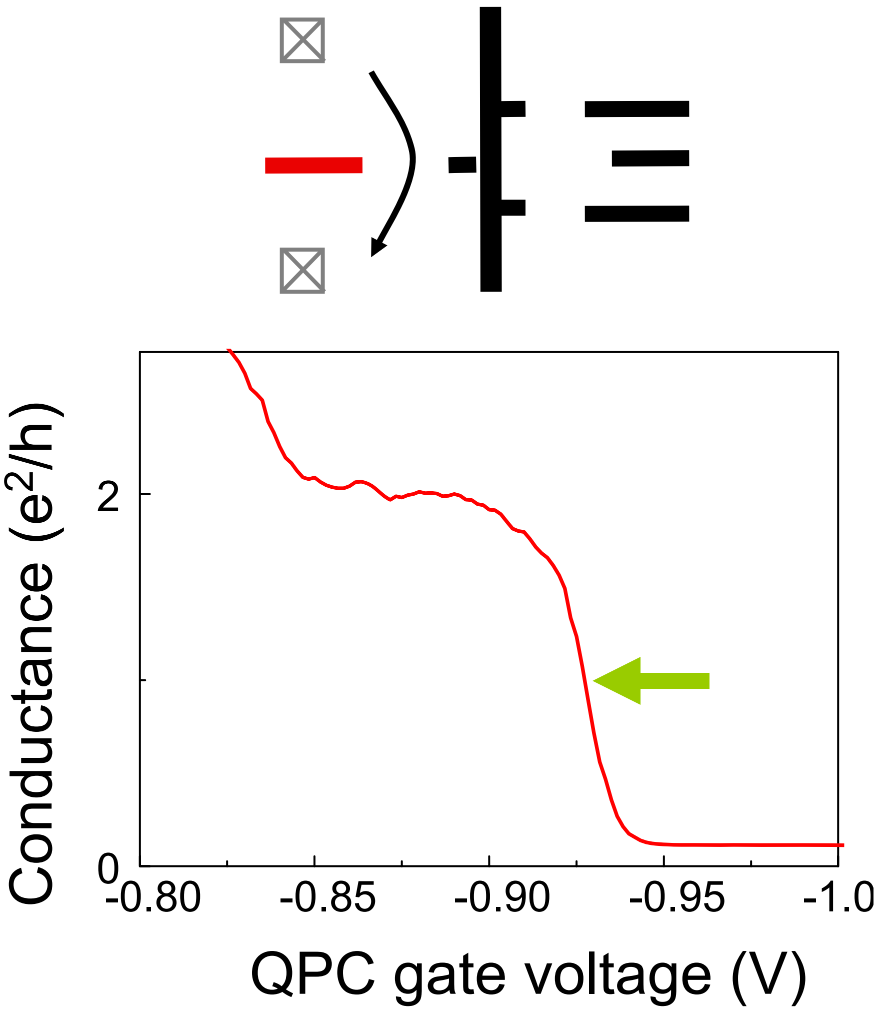
Electron transport
through dot

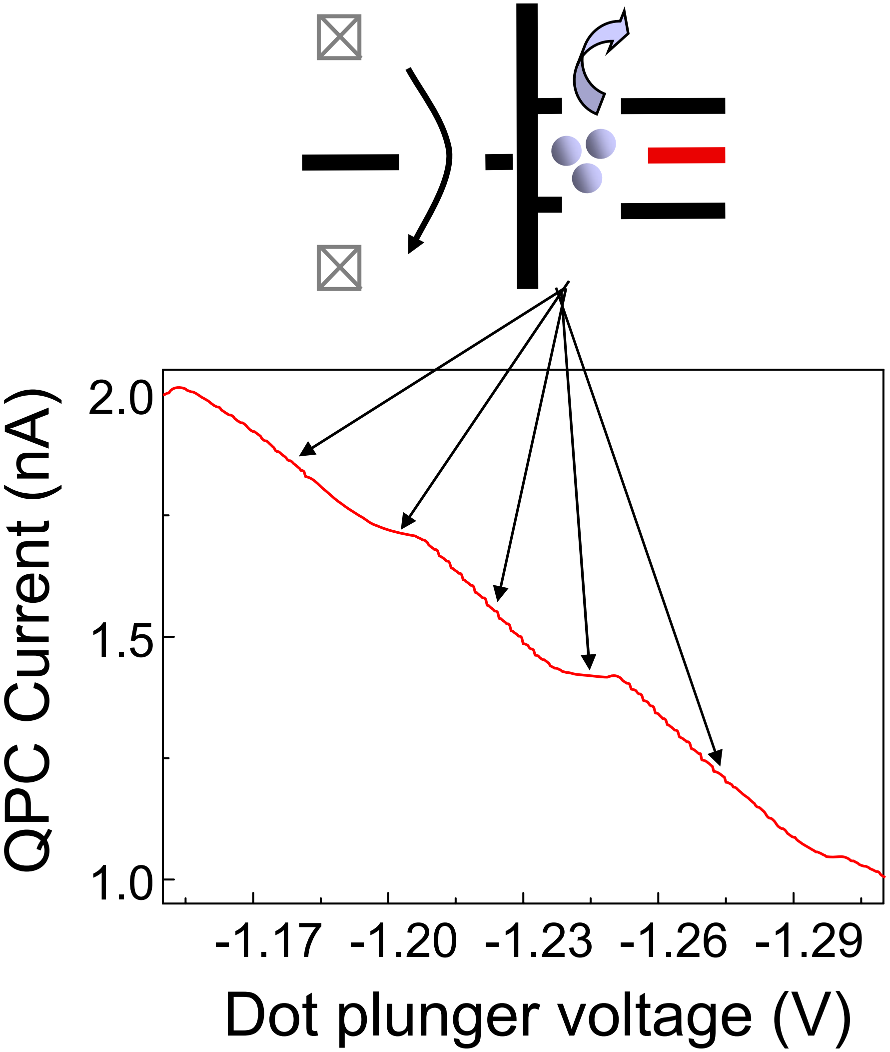


Electron transport
through QPC

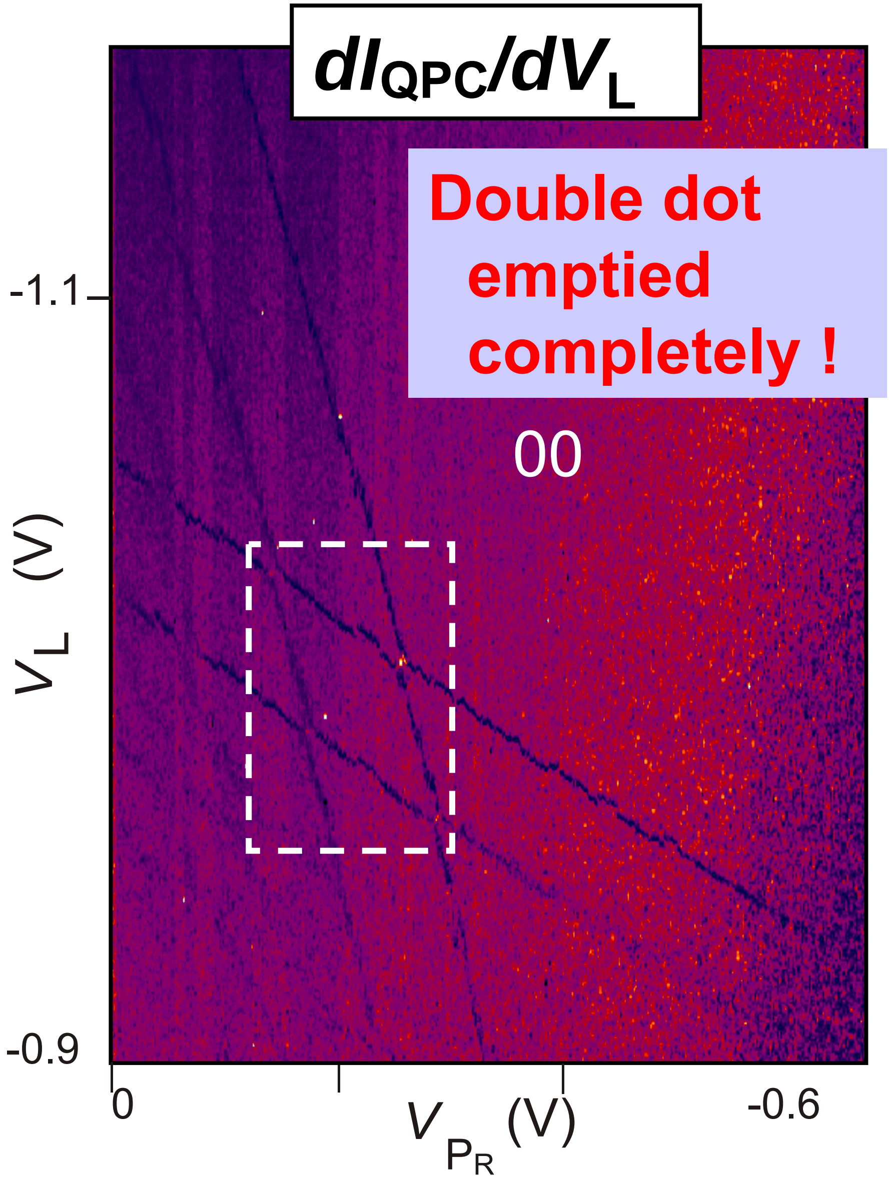
A quantum point contact (QPC) as a charge detector

Field *et al*, PRL 1993

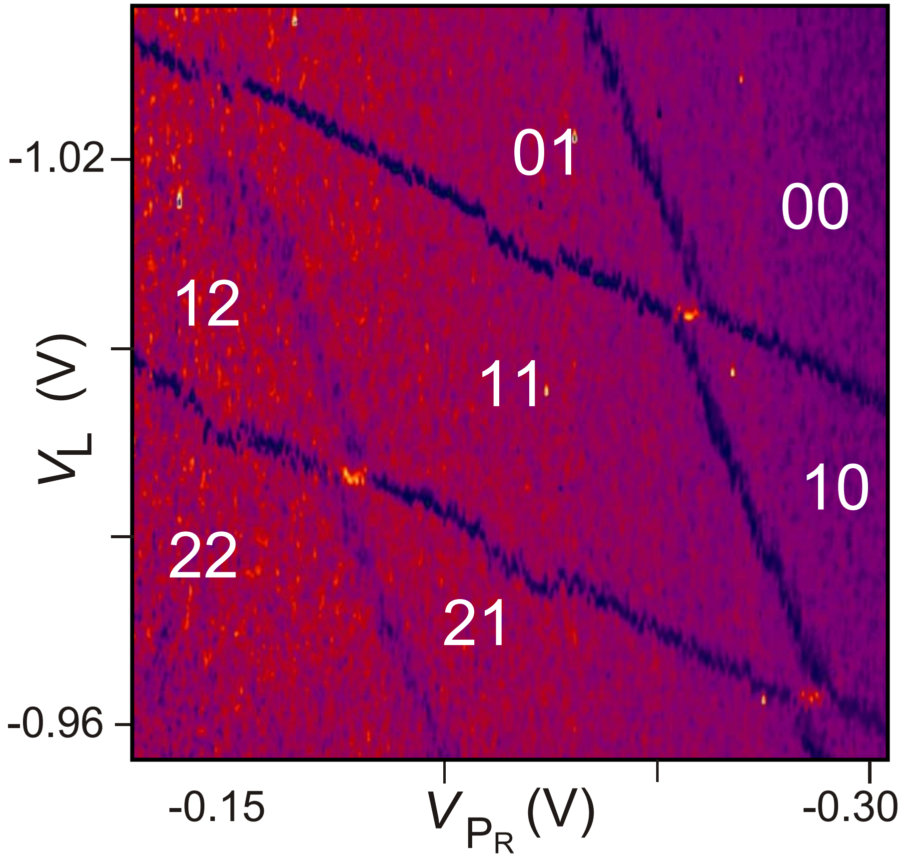




QPC – find the last electron

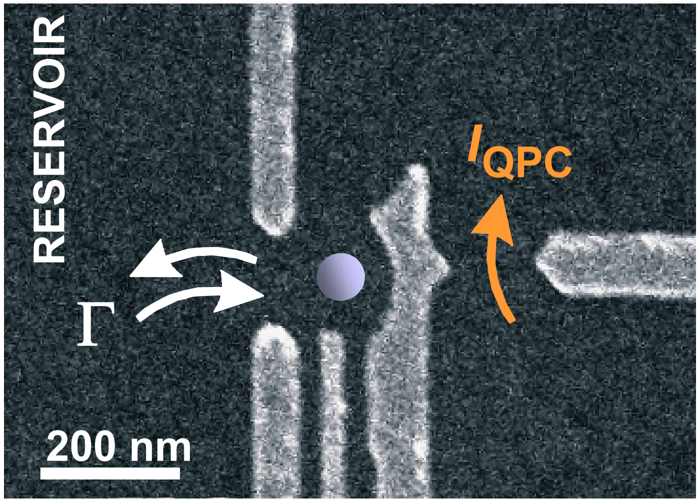


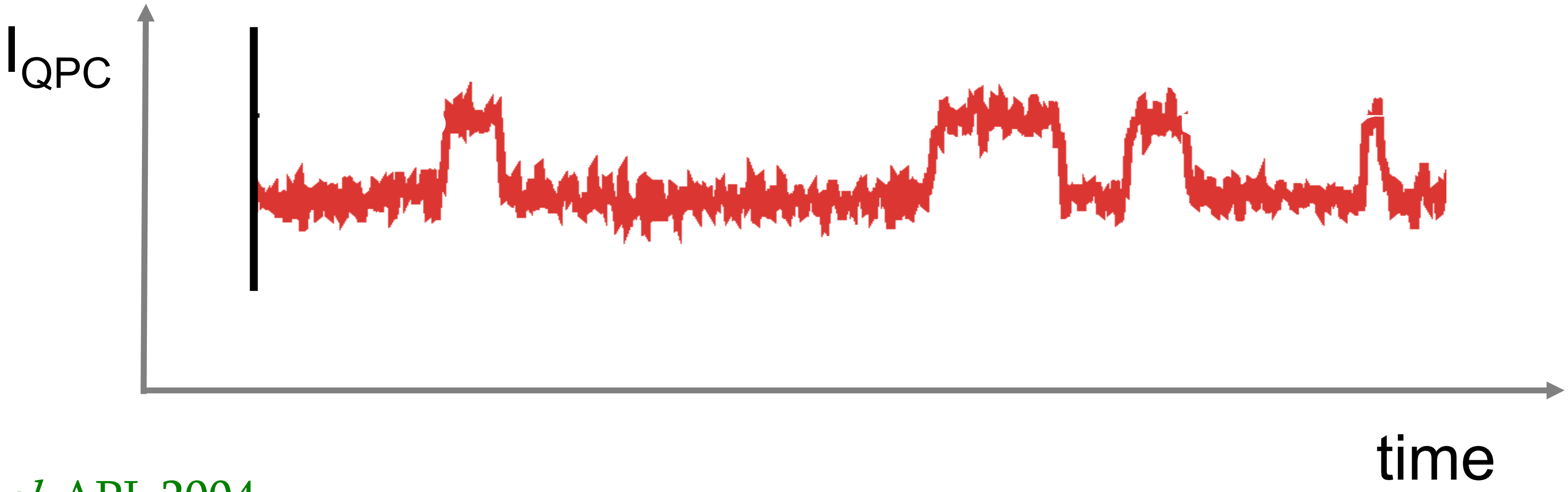
Double dot emptied completely !



QPC detects *all* charge transitions, also between dots

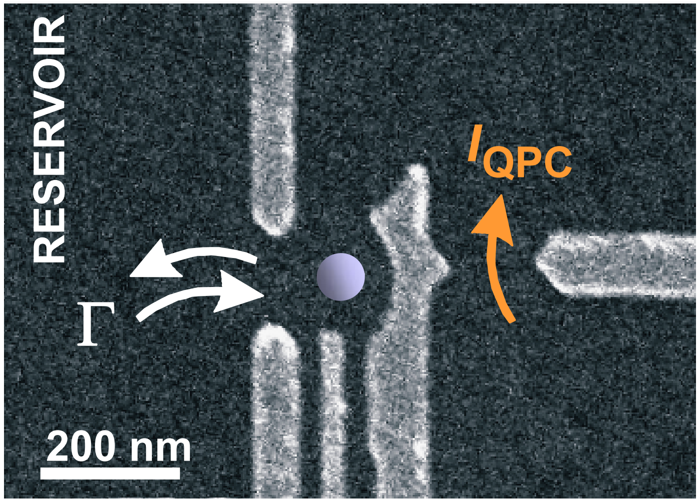
Real-time charge detection





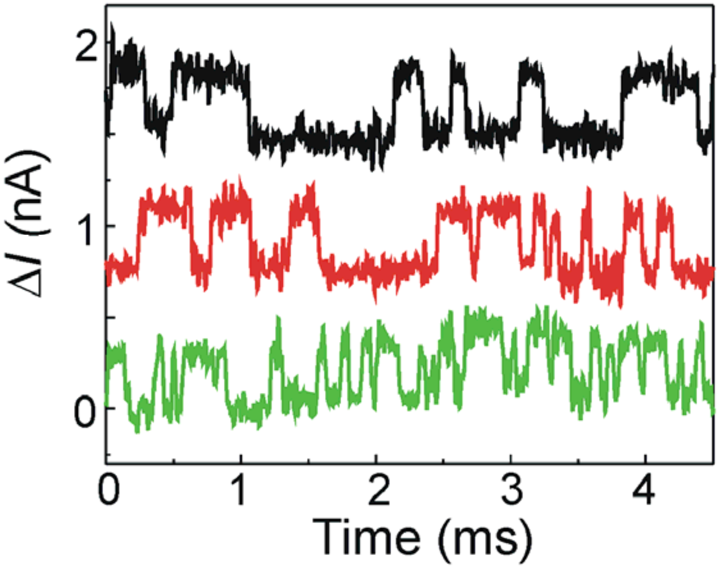
Vandersypen *et al*, APL 2004

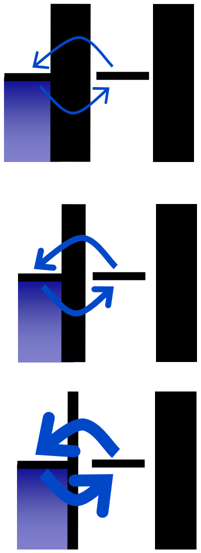
Real-time charge detection



- V_{SD} ~ 1 mV
- I_{QPC} ~ 30 nA
- ΔI_{QPC} ~ 0.3 nA
- Shortest steps ~ 8 μs
- With cryogenic preamplifier (HEMT) shortest steps ~300ns

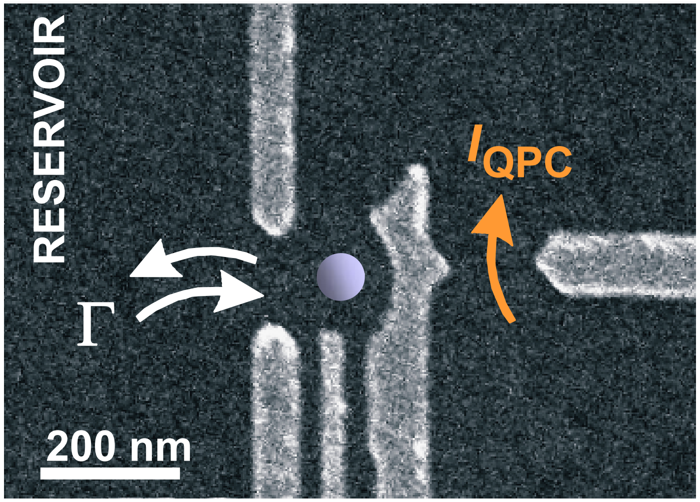
Vandersypen et al, *APL* ('04);
fast detection: Vink et al., *APL* ('07)





Change tunnel rate Γ by tuning thickness of tunnel barrier

Real-time charge detection



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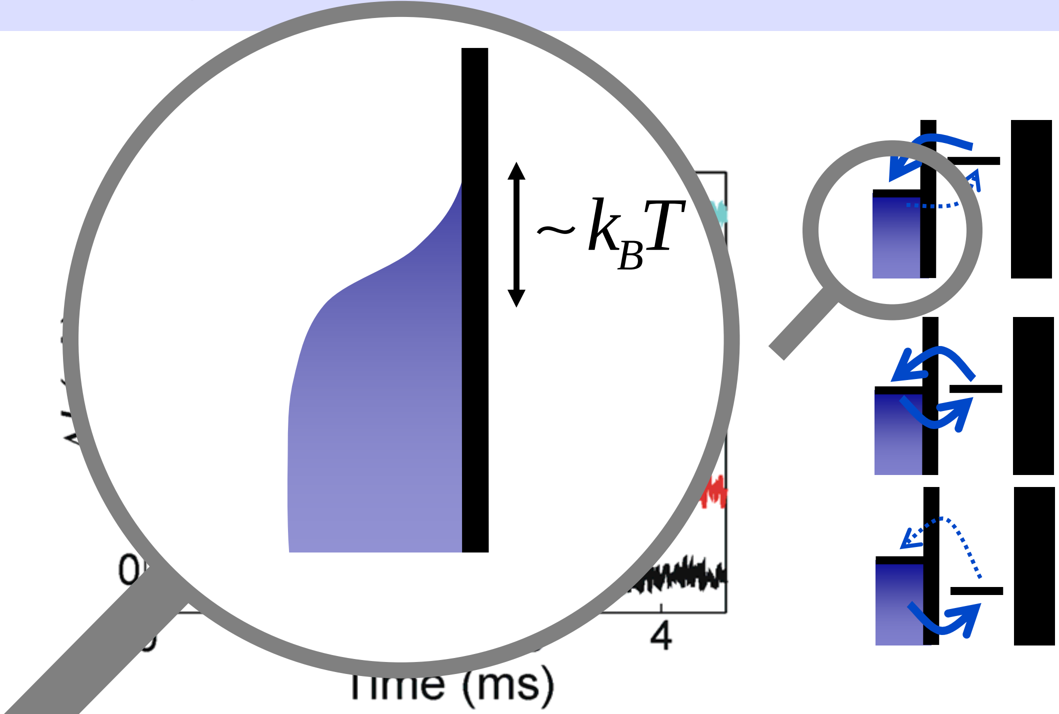
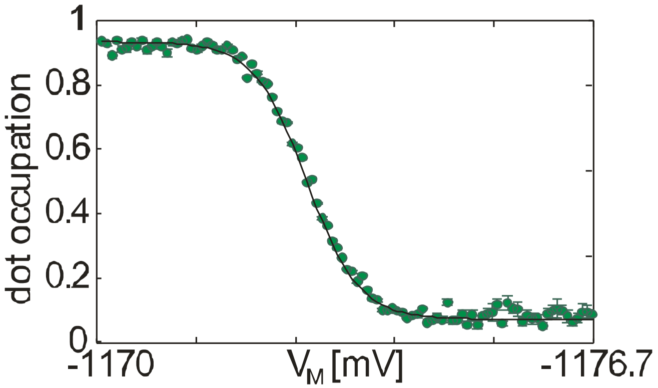


Change tunnel rate Γ by scanning through fermi distribution of the reservoir



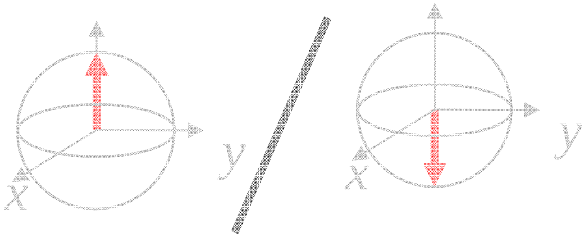
Vink et al., *APL* ('07)

Spin qubits in quantum dots

Loss & DiVincenzo, PRA 1998

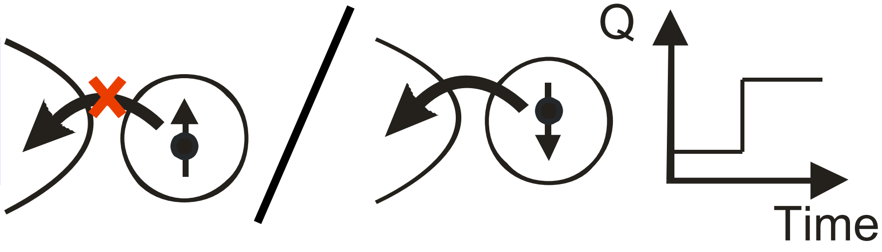
Well-defined qubit states

- *Confine single electrons*



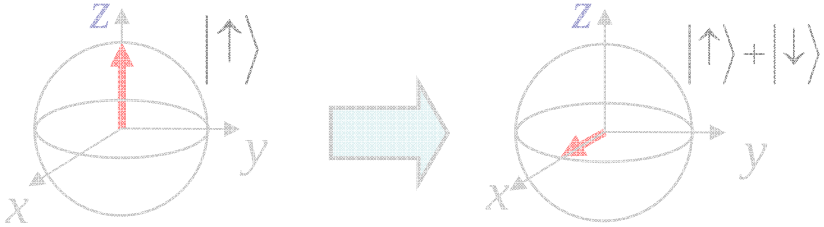
Initialize and read-out

- *Spin to charge conversion*



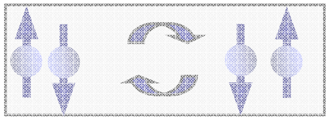
1-qubit gate

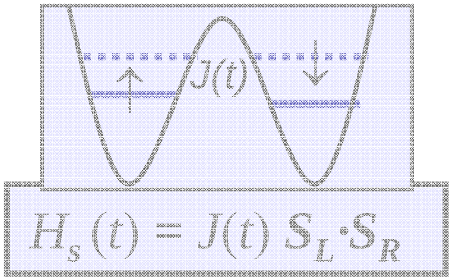
- *Electron spin resonance*



2-qubit gate

- *Exchange interaction*

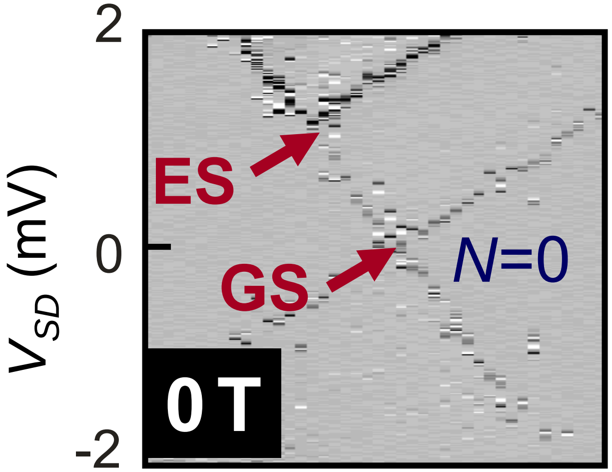


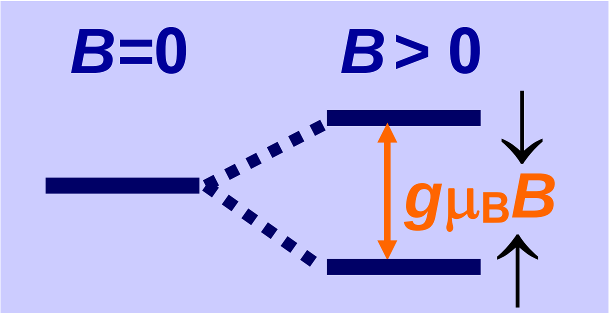


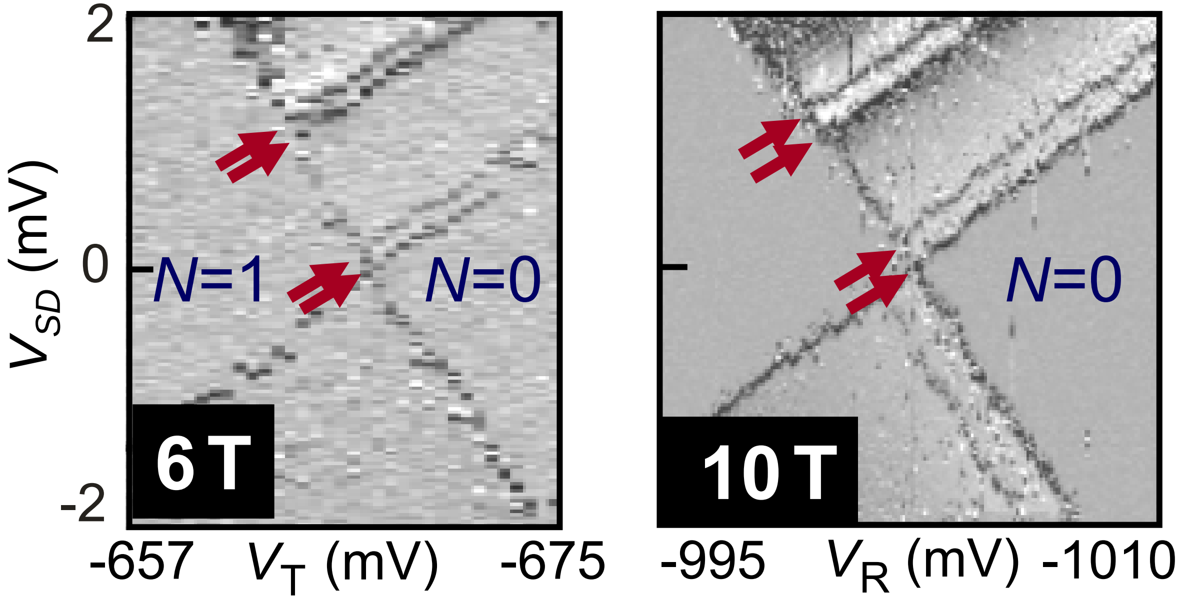
Petta *et al.*, Science ('05)

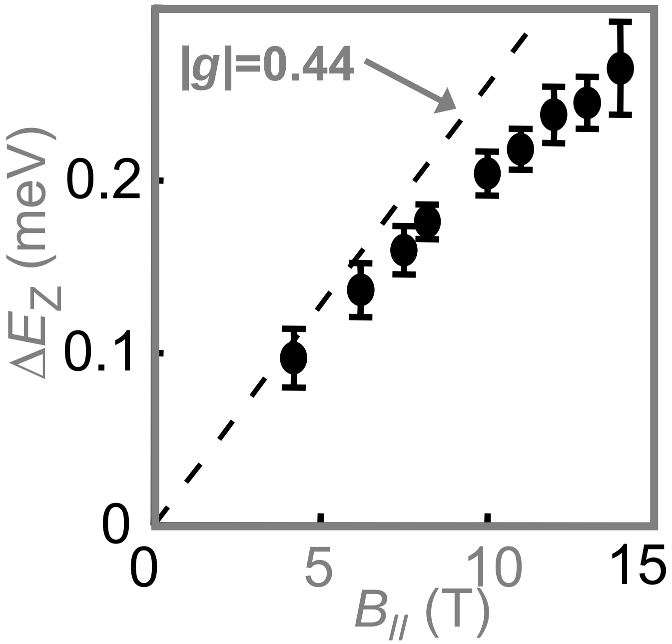
Single electron Zeeman splitting in $B_{//}$

Hanson et al, PRL 91, 196802 (2003)
Also: Potok et al, PRL 91, 016802 (2003)



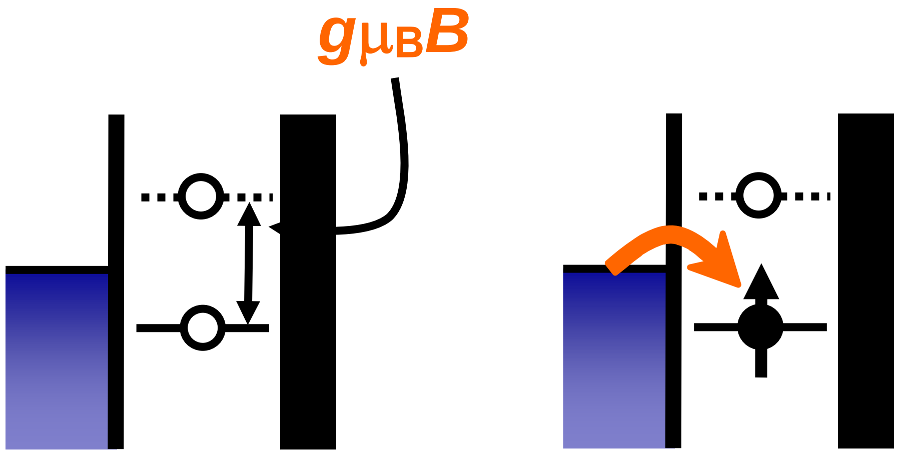




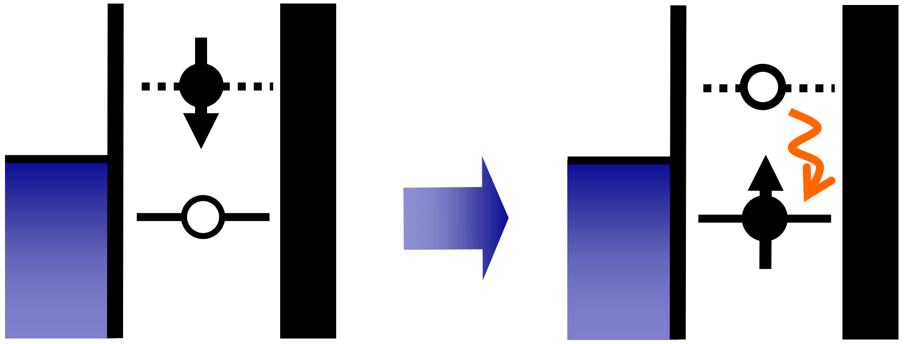


Initialization of a single electron spin

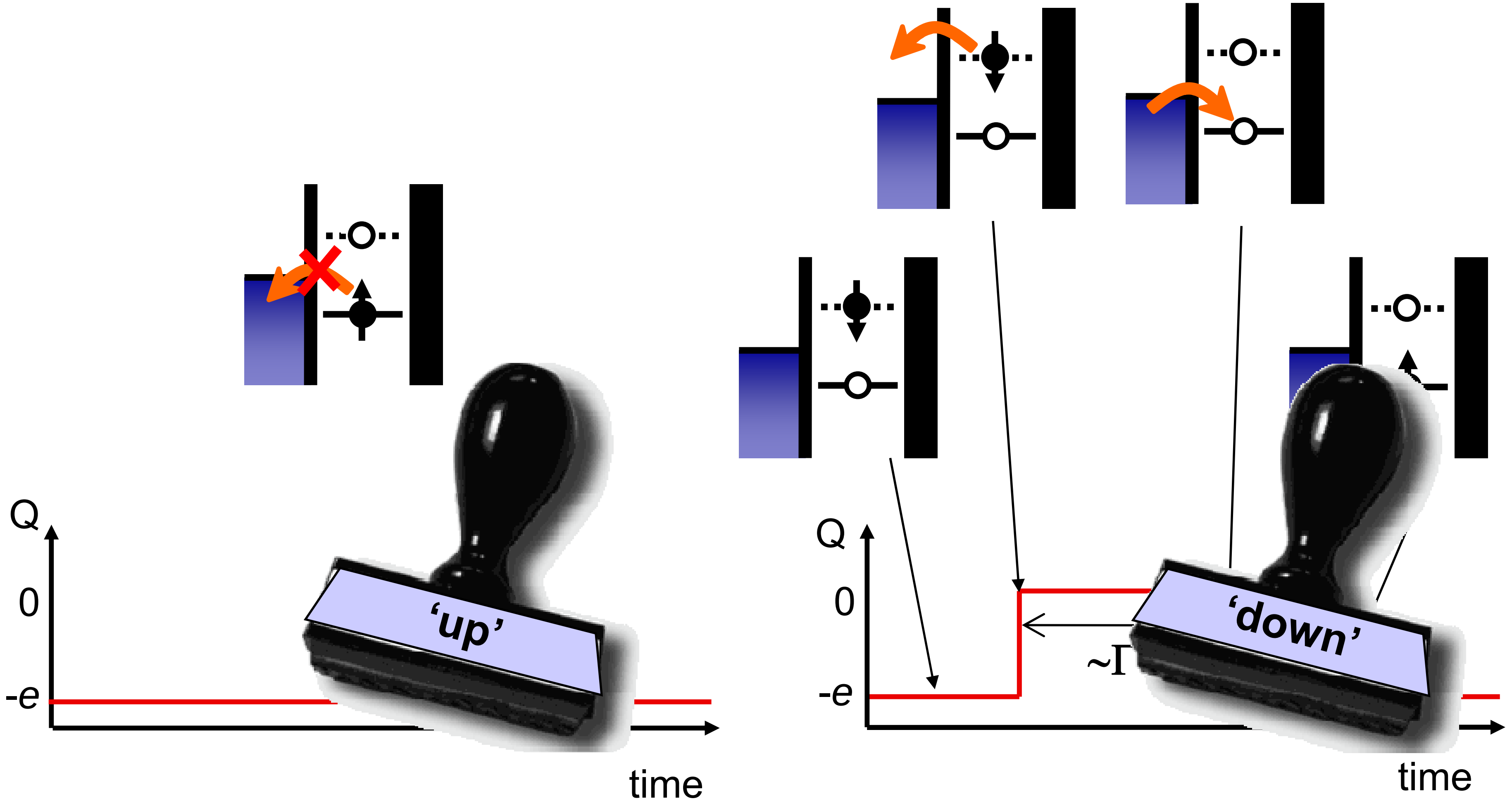
Method 1:
spin-selective
tunneling



Method 2:
relaxation to
ground state



Energy-selective readout



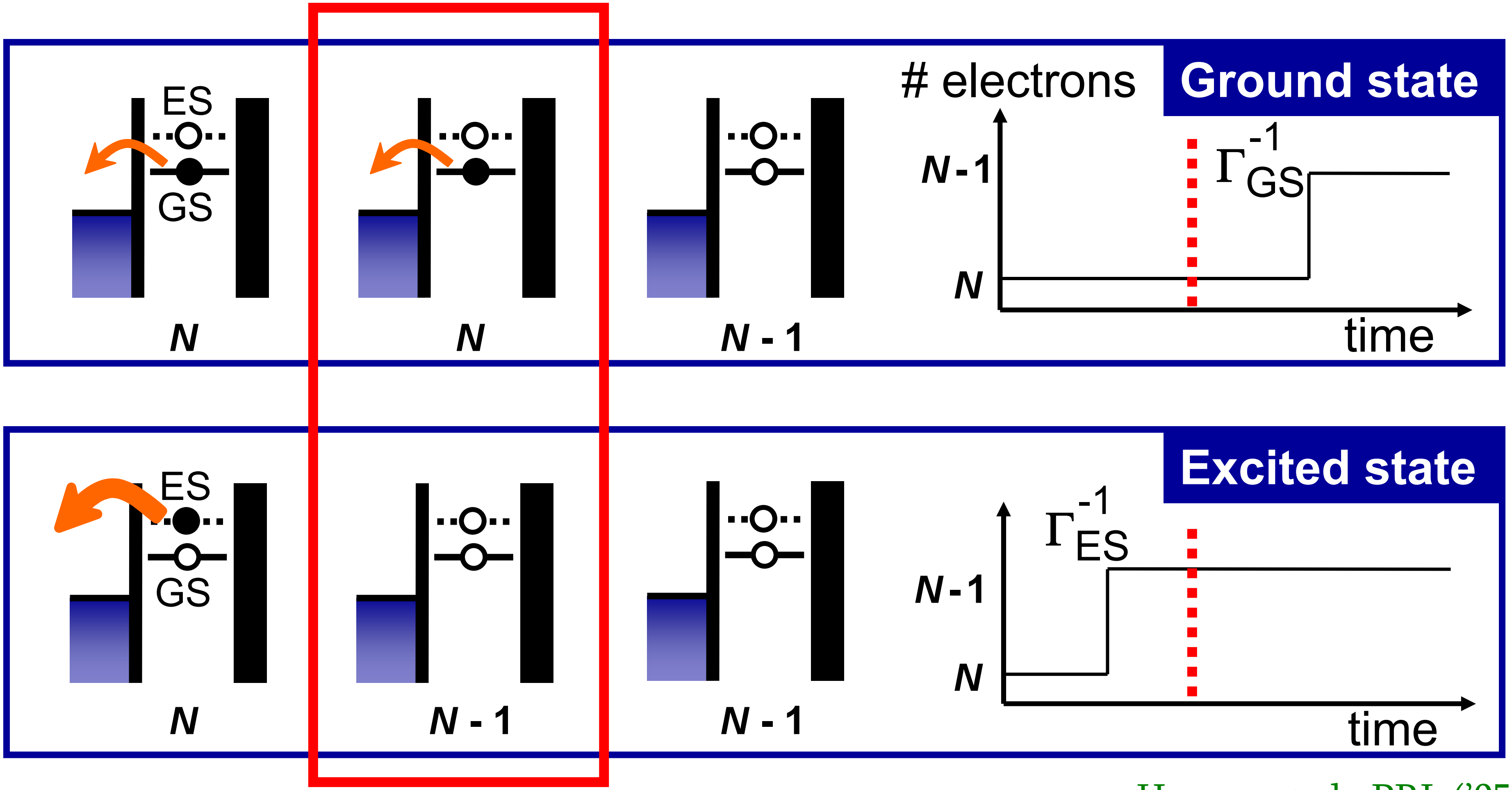
Limitations:

- Requires $\Delta E >> k_B T$
- Sensitive to background charge fluctuations ("switching")
- Sensitive to HF noise (photon-assisted tunneling)

Elzermann et al., *Nature* ('04)

Tunnel-rate-selective readout

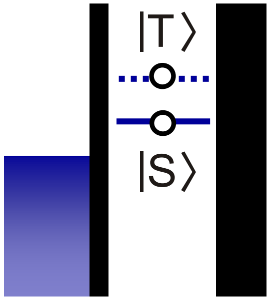
- Needed: different tunnel rates for ES and GS; e.g. $\Gamma_{ES} >> \Gamma_{GS}$
- Do not measure *whether* an electron tunnels, but *when* it tunnels

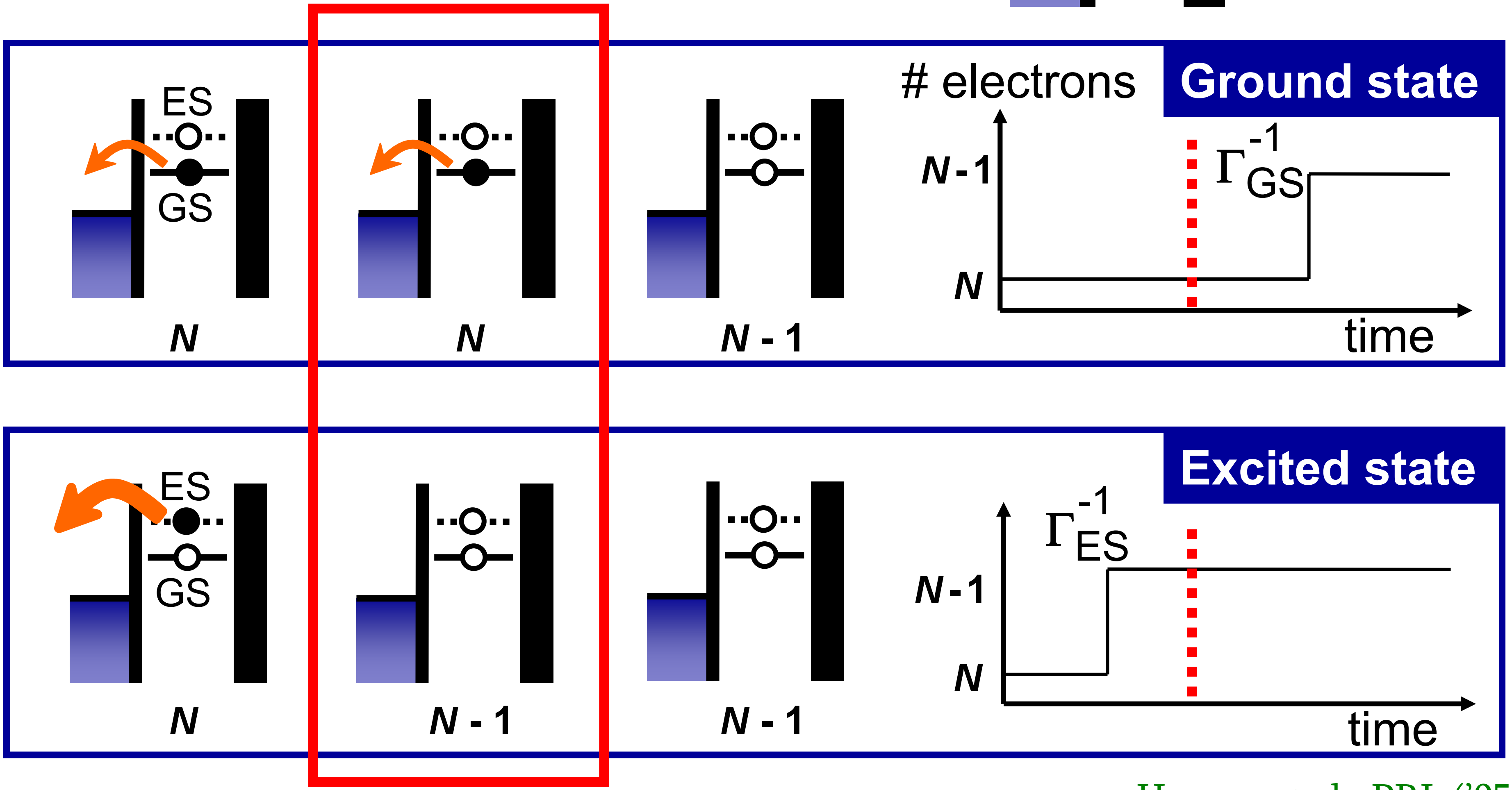


Hanson et al., PRL ('05)

Tunnel-rate-selective readout

- e.g. singlet and triplet in single dot $\Gamma_T >> \Gamma_S$

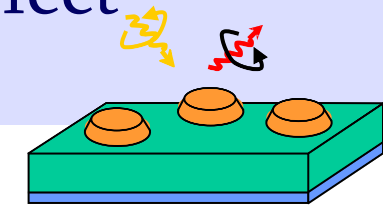


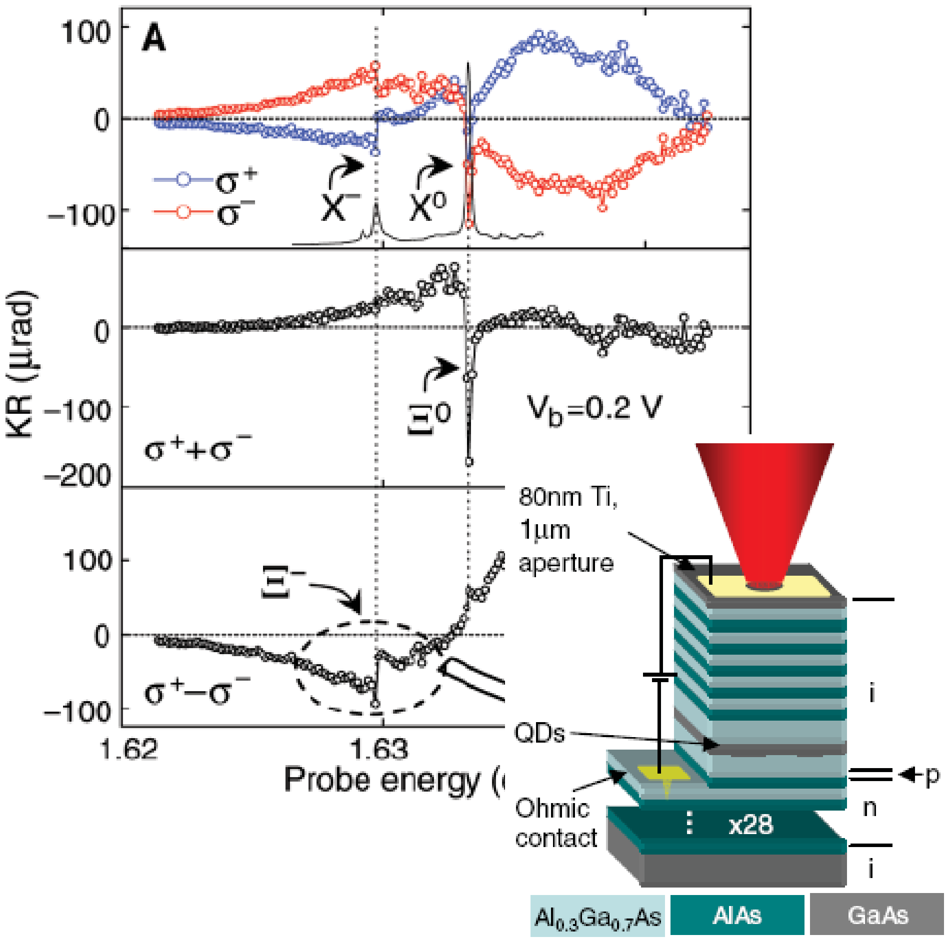


Hanson et al., PRL ('05)

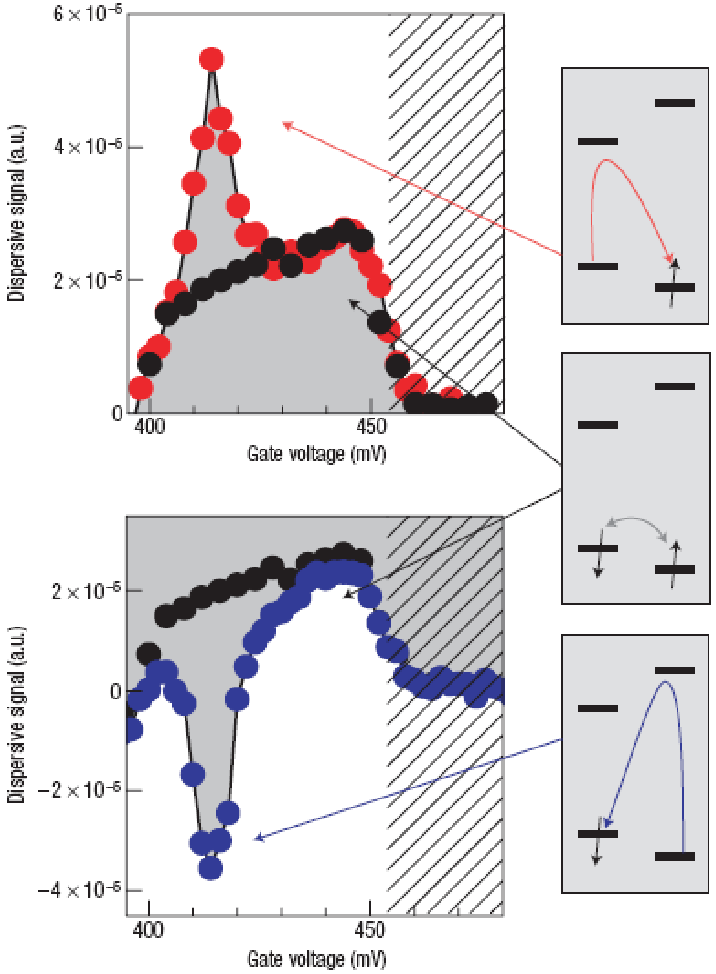
Single-spin detection via Kerr/Faraday effect

(*not yet single-shot read-out*)





Berezowsky, Mikkelsen, Gywat, Stoltz, Coldren, Awschalom, Science 2006



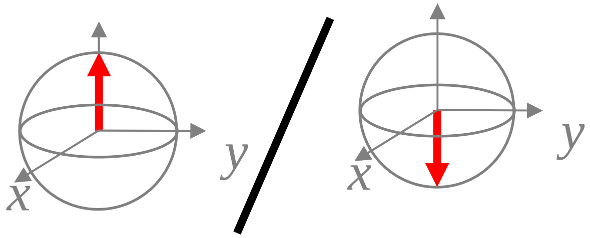
Atature, Dreiser, Badolato, Imamoglu, Nat. Phys. 2007

Spin qubits in quantum dots

Loss & DiVincenzo, PRA 1998

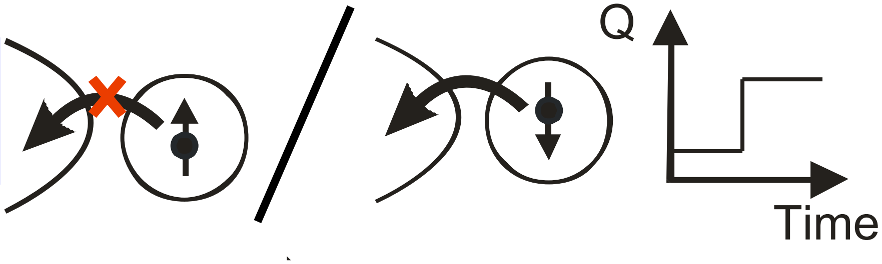
Well-defined qubit states

- *Confine single electrons*



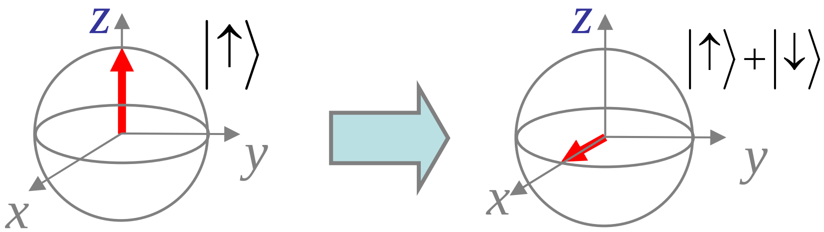
Initialize and read-out

- *Spin to charge conversion*



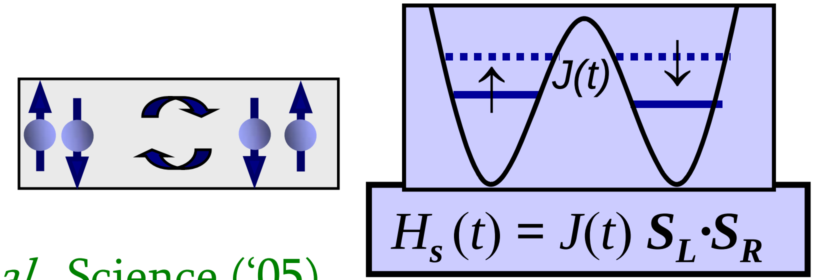
1-qubit gate

- *Electron spin resonance*



2-qubit gate

- *Exchange interaction*



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Petta *et al.*, Science ('05)

Spin qubits in quantum dots

Loss & DiVincenzo, PRA 1998

Well-defined qubit states

- *Confine single electrons*

Initialize and read-out

- *Spin to charge conversion*

1-qubit gate

- *Electron spin resonance*

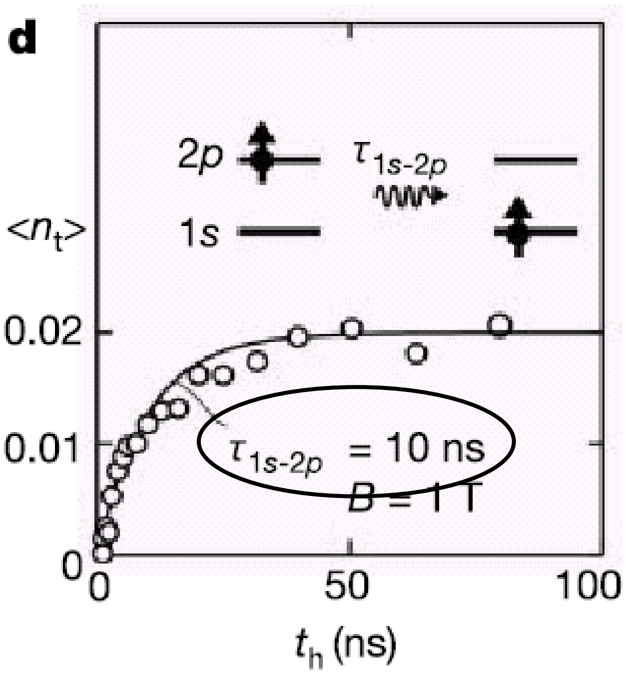
2-qubit gate

- *Exchange interaction*

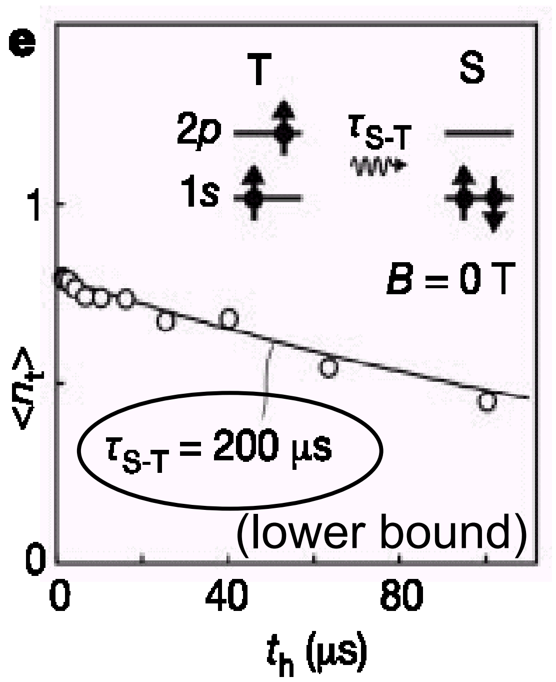
Study energy relaxation (T_1)

Spin relaxation in quantum dots – *much* slower than orbital relaxation

orbital

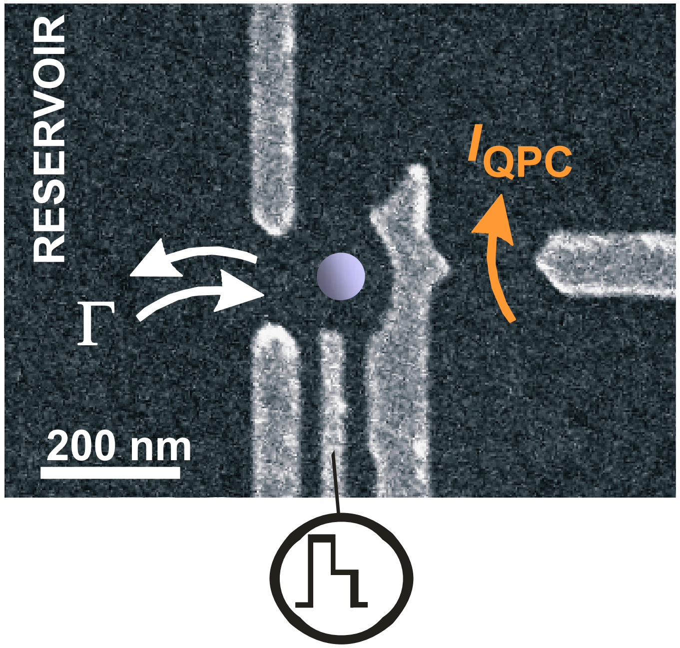


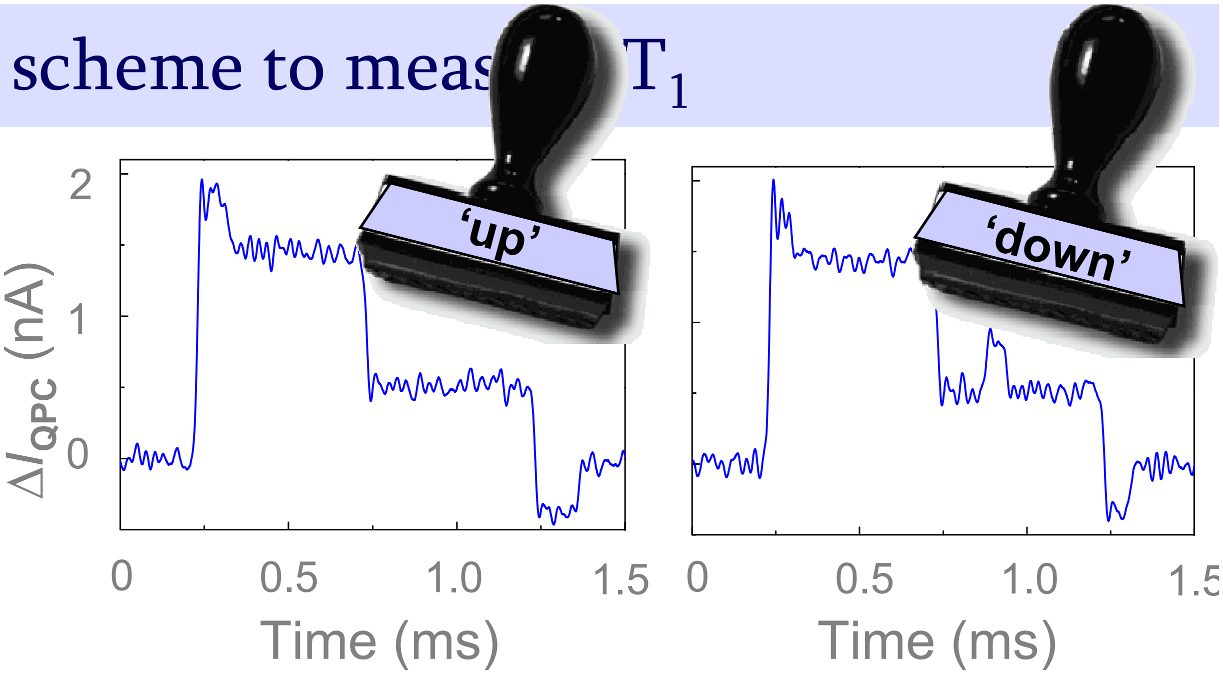
spin
(two-electron)

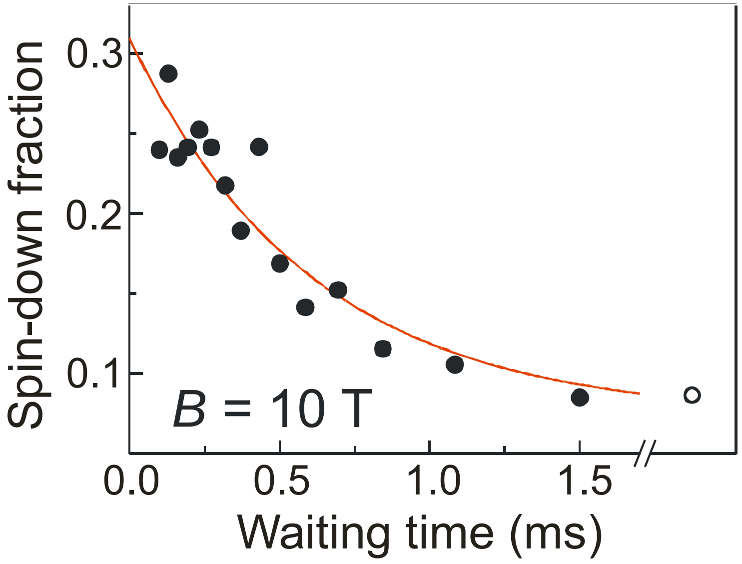


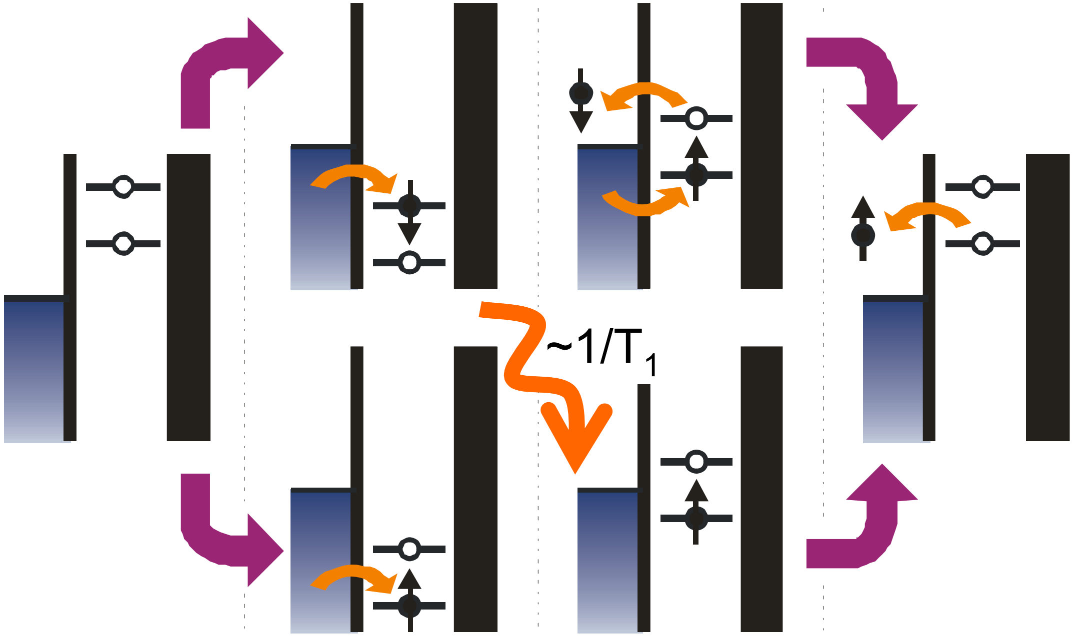
Fujisawa, Austing, Tokura, Hirayama, Tarucha,
Nature ('02)

Pulse scheme to meas T$_1$





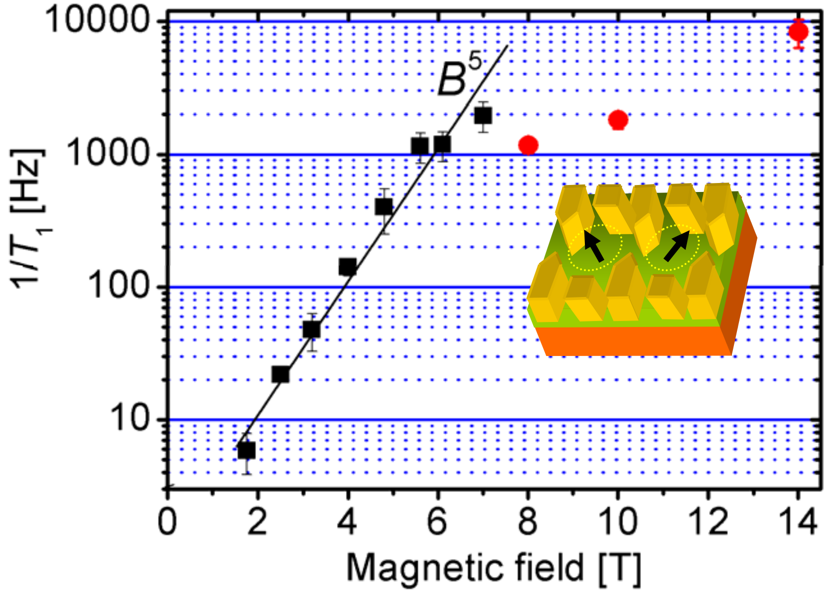




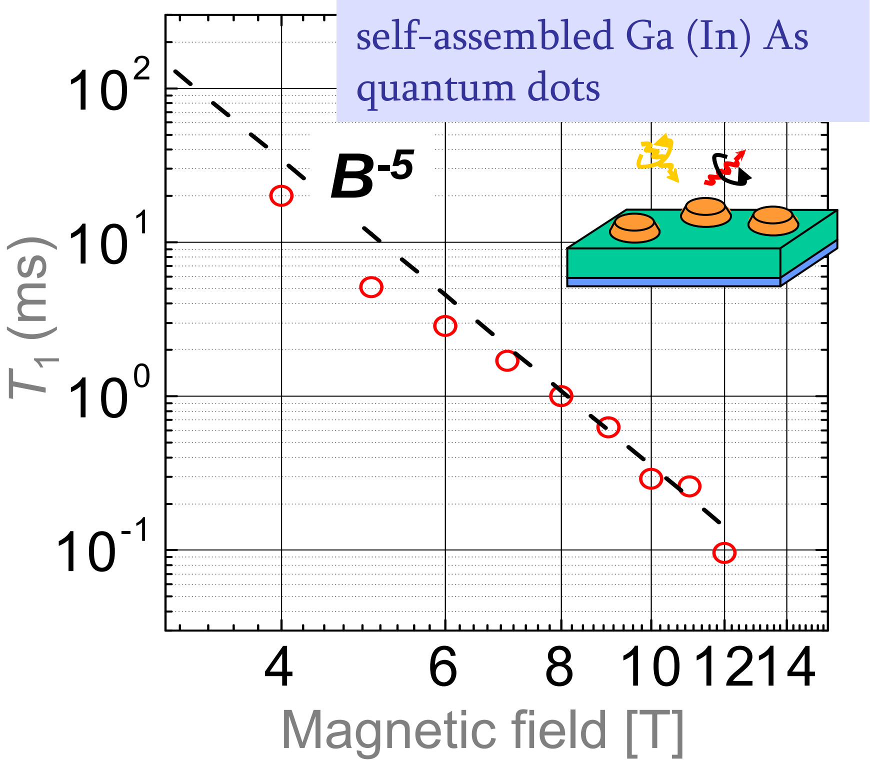
Elzermann et al.,*Nature* (‘04)

Single spin relaxation in quantum dots

timescale: 100 μs up to > 1 s (!)
mechanism: electric field fluctuations (from phonons)
via spin-orbit coupling (Rashba, Dresselhaus)



● Elzerman *et al.*, *Nature* 2004
■ Amasha *et al.*, cond-mat/0607110


self-assembled Ga (In) As quantum dots

Kroutvar *et al.*, *Nature* 2004

Theory: Khaetskii & Nazarov, *PRB* 2000, 2001

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Loss & DiVincenzo, PRA 1998

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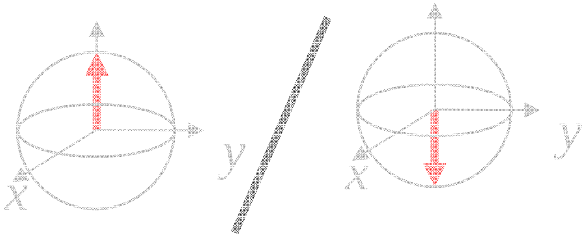
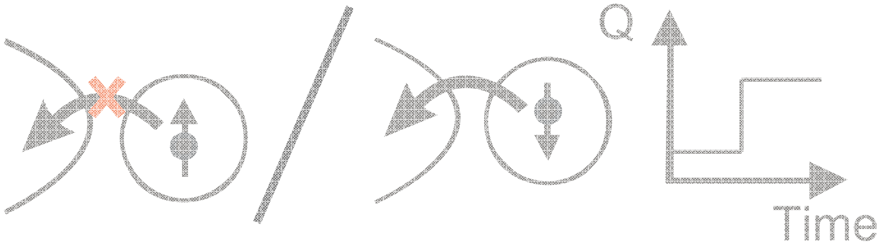


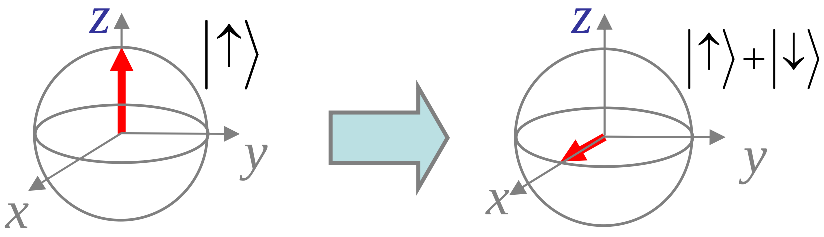
Initialize and read-out

- *Spin to charge conversion*



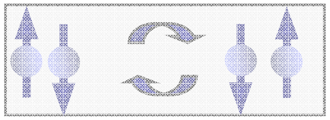
1-qubit gate

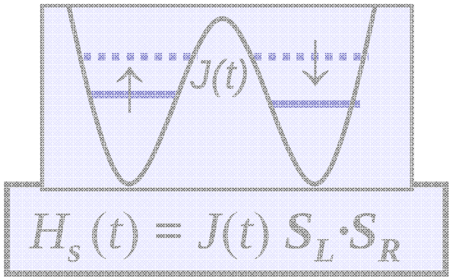
- *Electron spin resonance*



2-qubit gate

- *Exchange interaction*



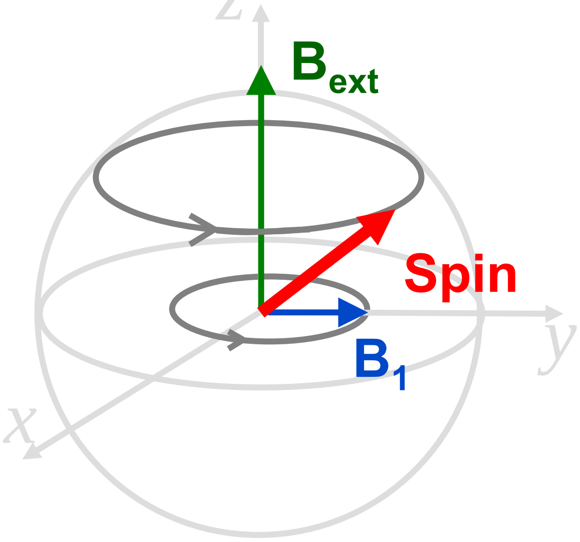


$$H_s(t) = J(t)\, S_L \cdot S_R$$

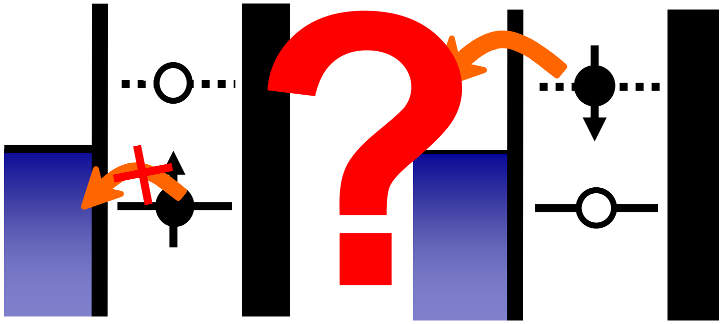
Petta *et al.*, Science ('05)

Therefore we need…

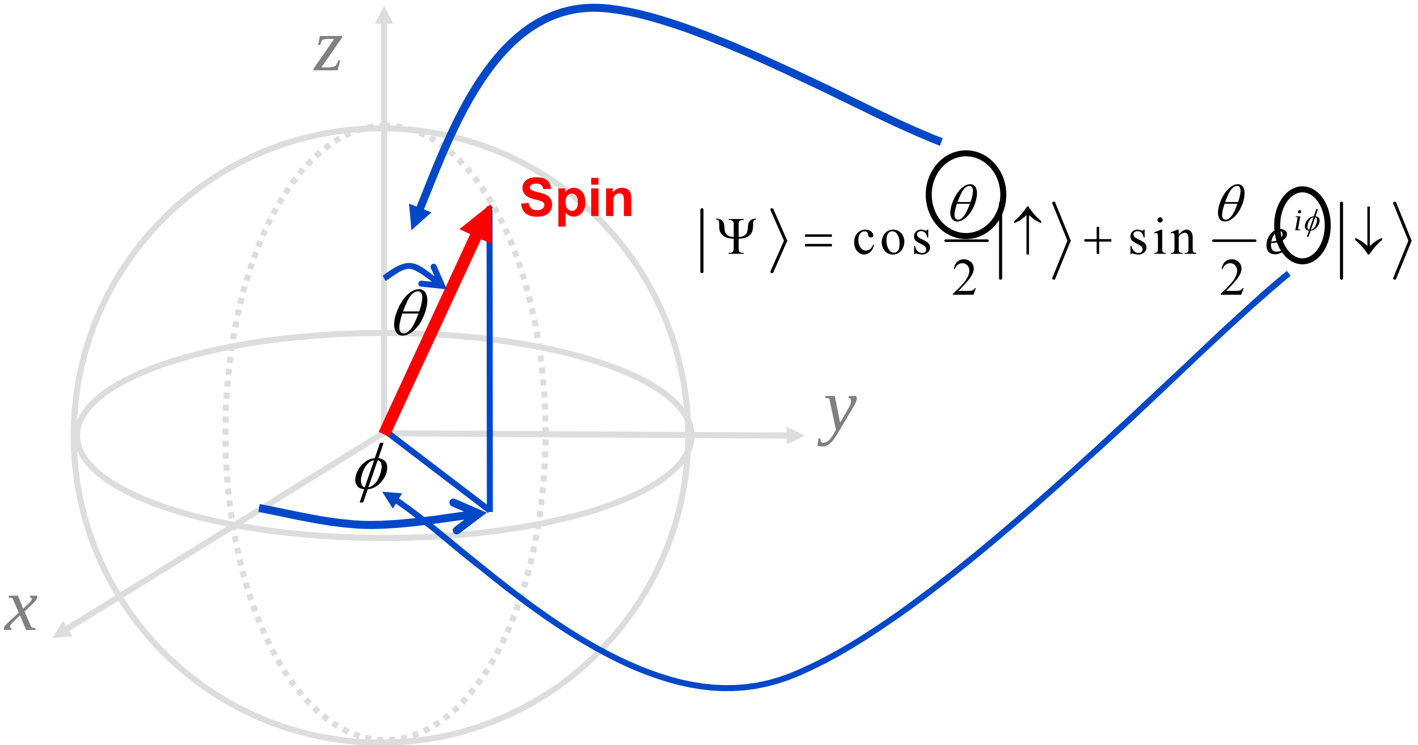
Spin resonance to manipulate…



…and a detector.

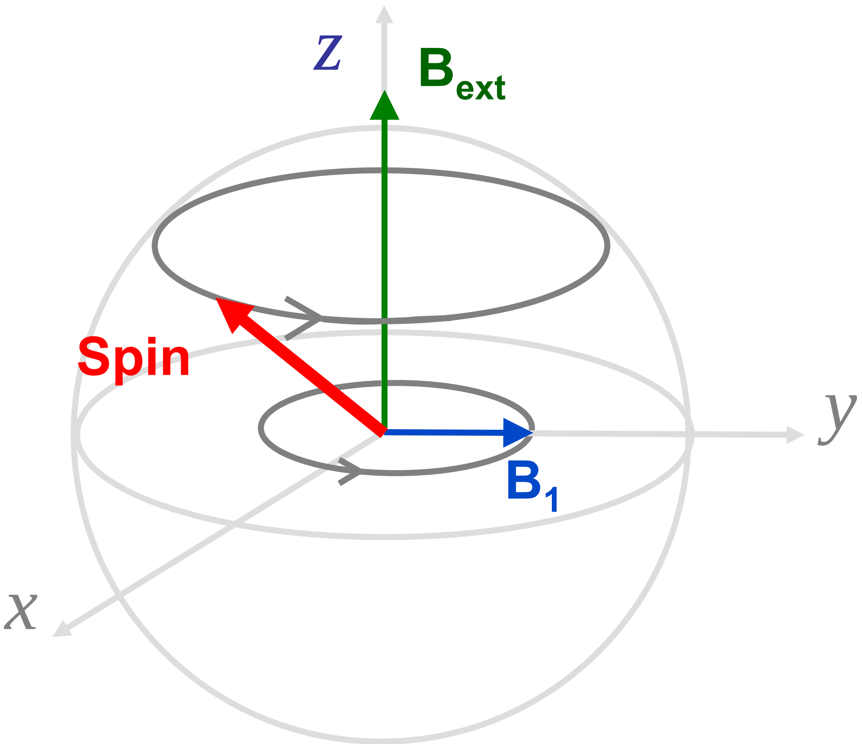


Bloch sphere



$$\left|\Psi\right\rangle = \cos\frac{\theta}{2}\left|\uparrow\right\rangle + \sin\frac{\theta}{2}e^{i\phi}\left|\downarrow\right\rangle$$

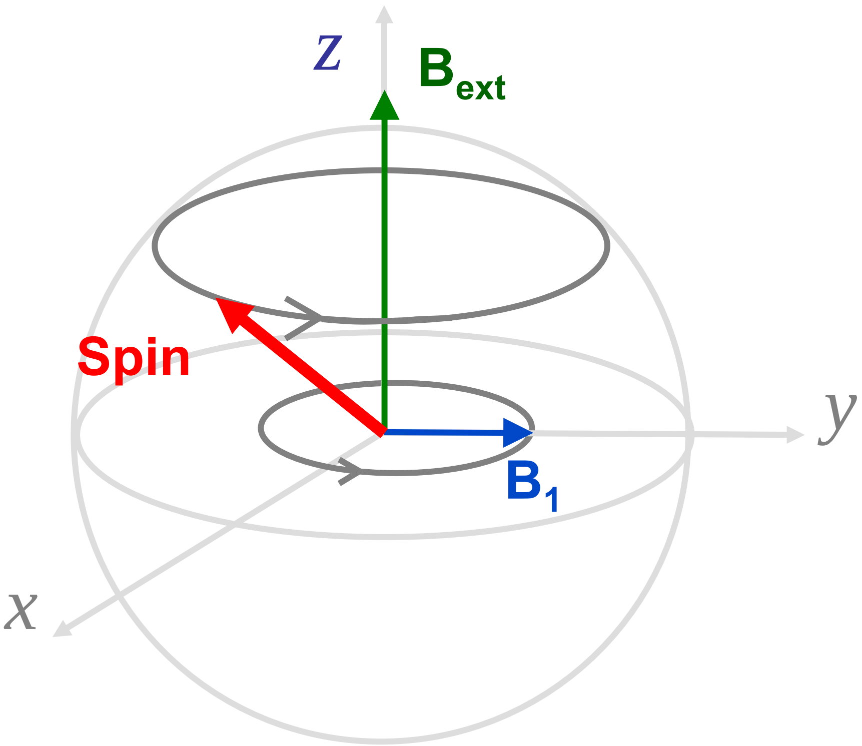
Electron Spin Resonance



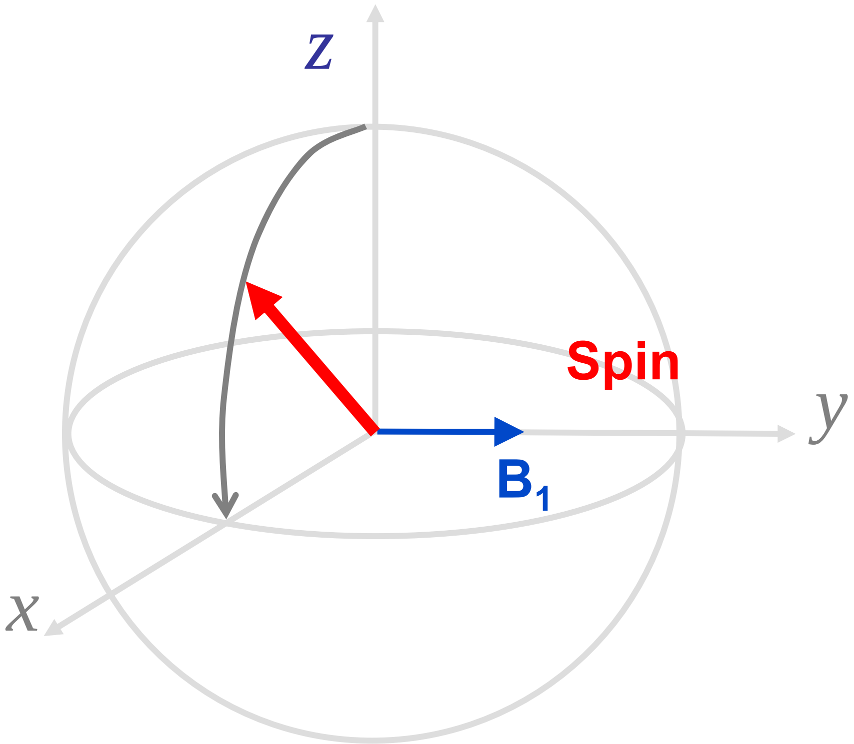
Lab frame

Resonance condition: $g\mu_B B_{ext} = hf_{ac}$

Electron Spin Resonance: Lab frame vs. rotating frame



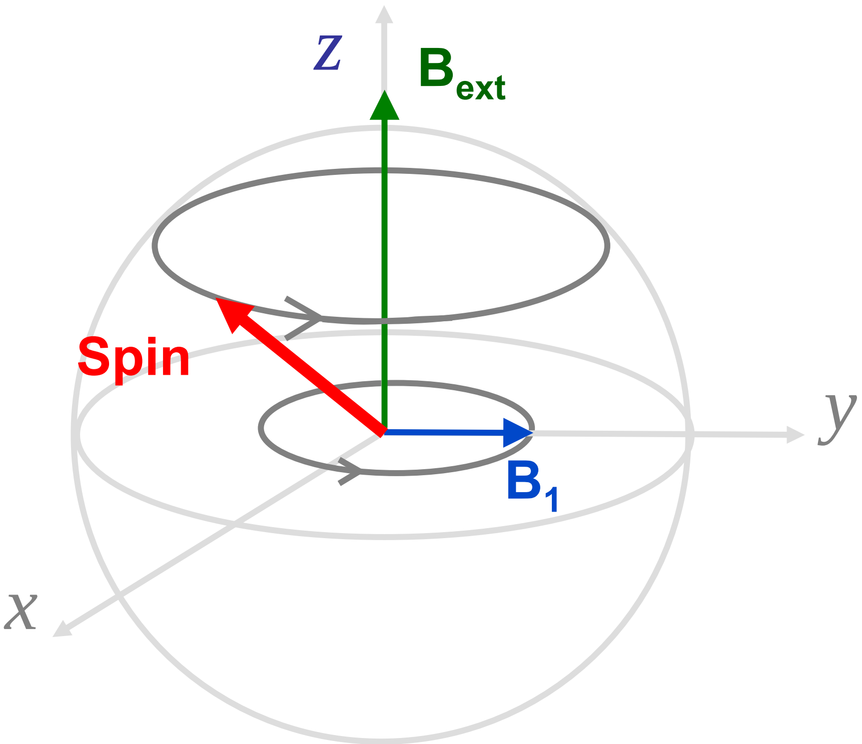
Lab frame



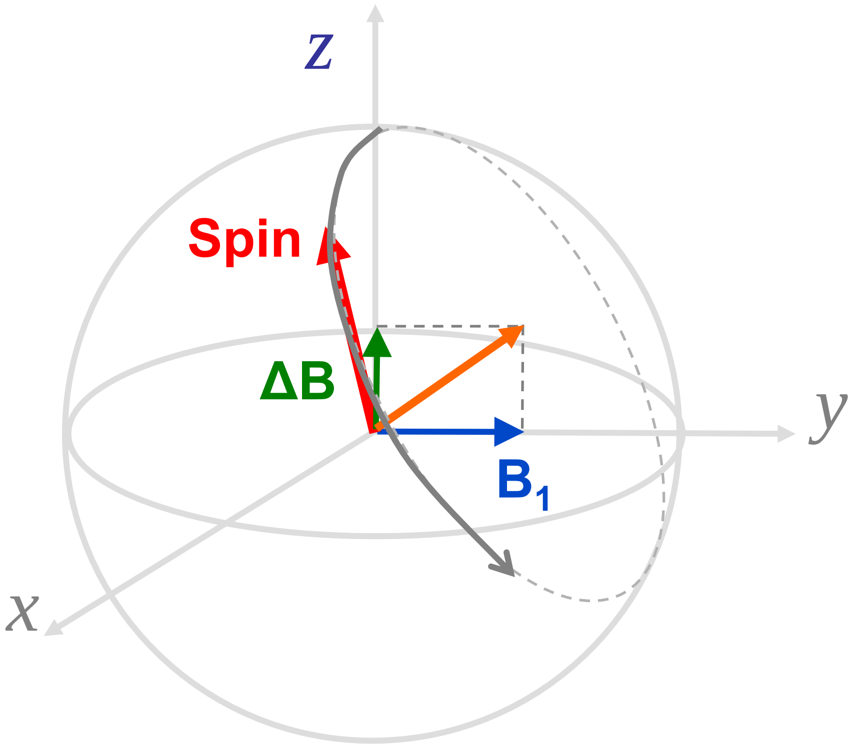
Rotating frame (f_{ac}) exactly on resonance

Resonance condition: $g\mu_B B_{ext} = hf_{ac}$

Electron Spin Resonance: Lab frame vs. rotating frame



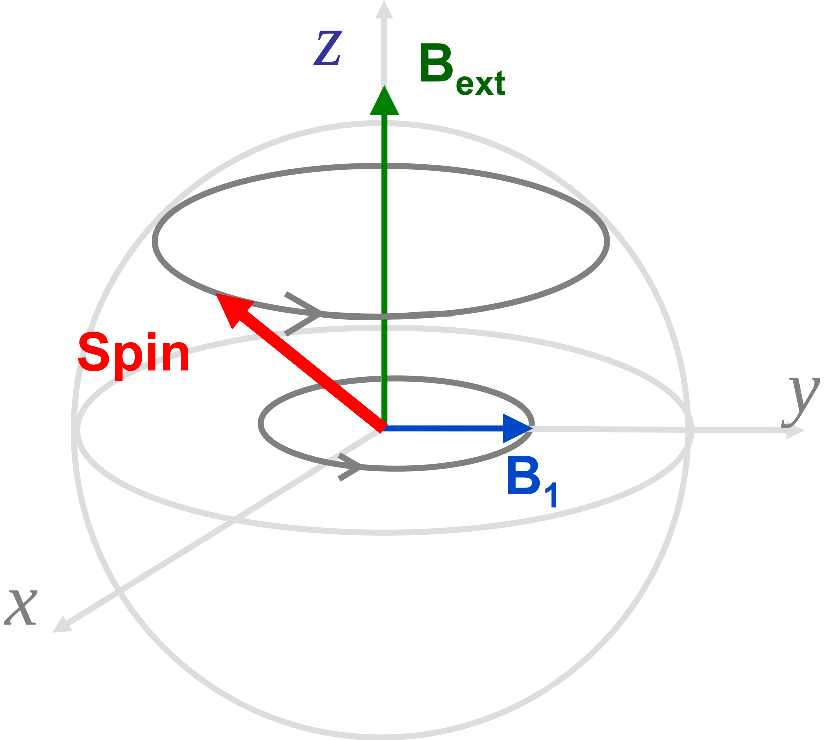
Lab frame



Rotating frame (f_{ac}) slightly off resonance

$$g\mu_B B_{ext} = hf_{ac} + g\mu_B \Delta B$$

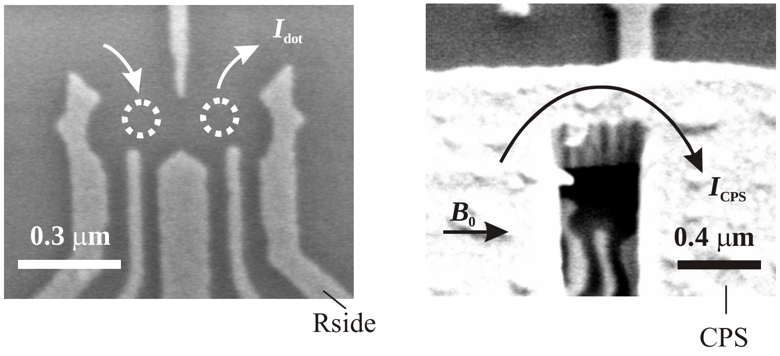
Generating an ac magnetic field



Lab frame

How to generate B_1?

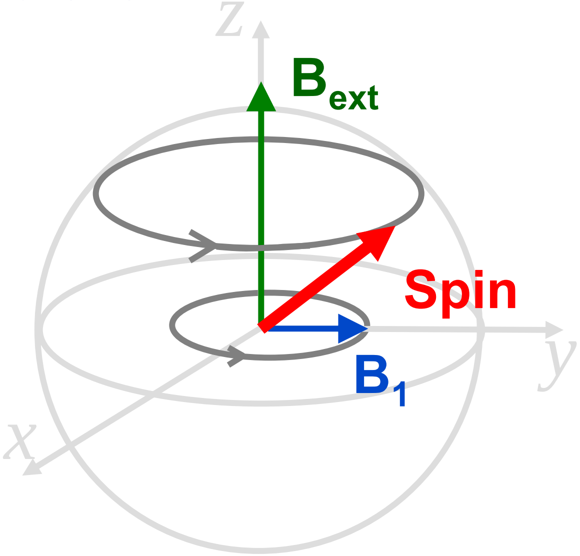
apply oscillating magnetic field B_{AC}



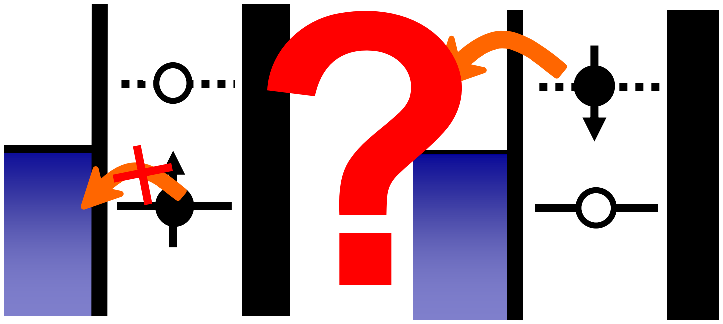
Koppens et al. *Nature* (‘06)

Therefore we need…

Spin resonance to manipulate…



…and a detector.



Detection: can we use energy selective spin read-out??

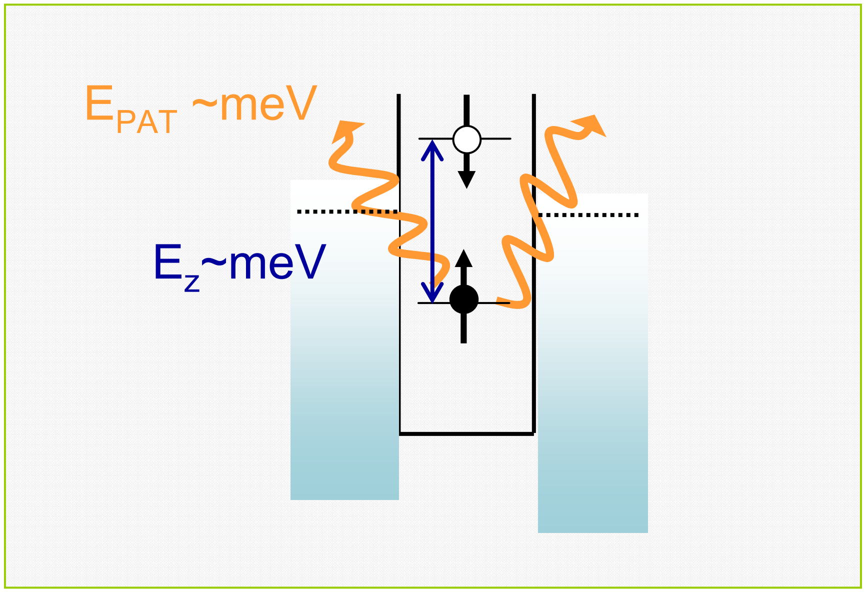
<u>Energy selective read-out</u>

- High Zeeman energy necessary

  → **high frequency** (~30 GHz)

- Sensitive to **electric fields**

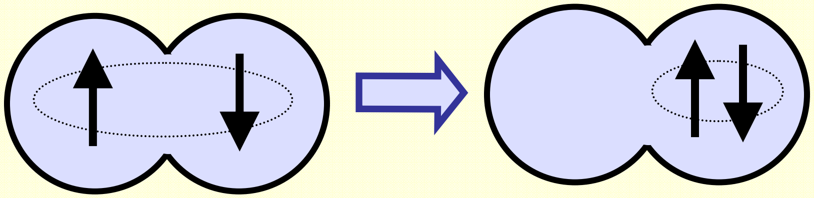
  (Photon Assisted Tunnelling)



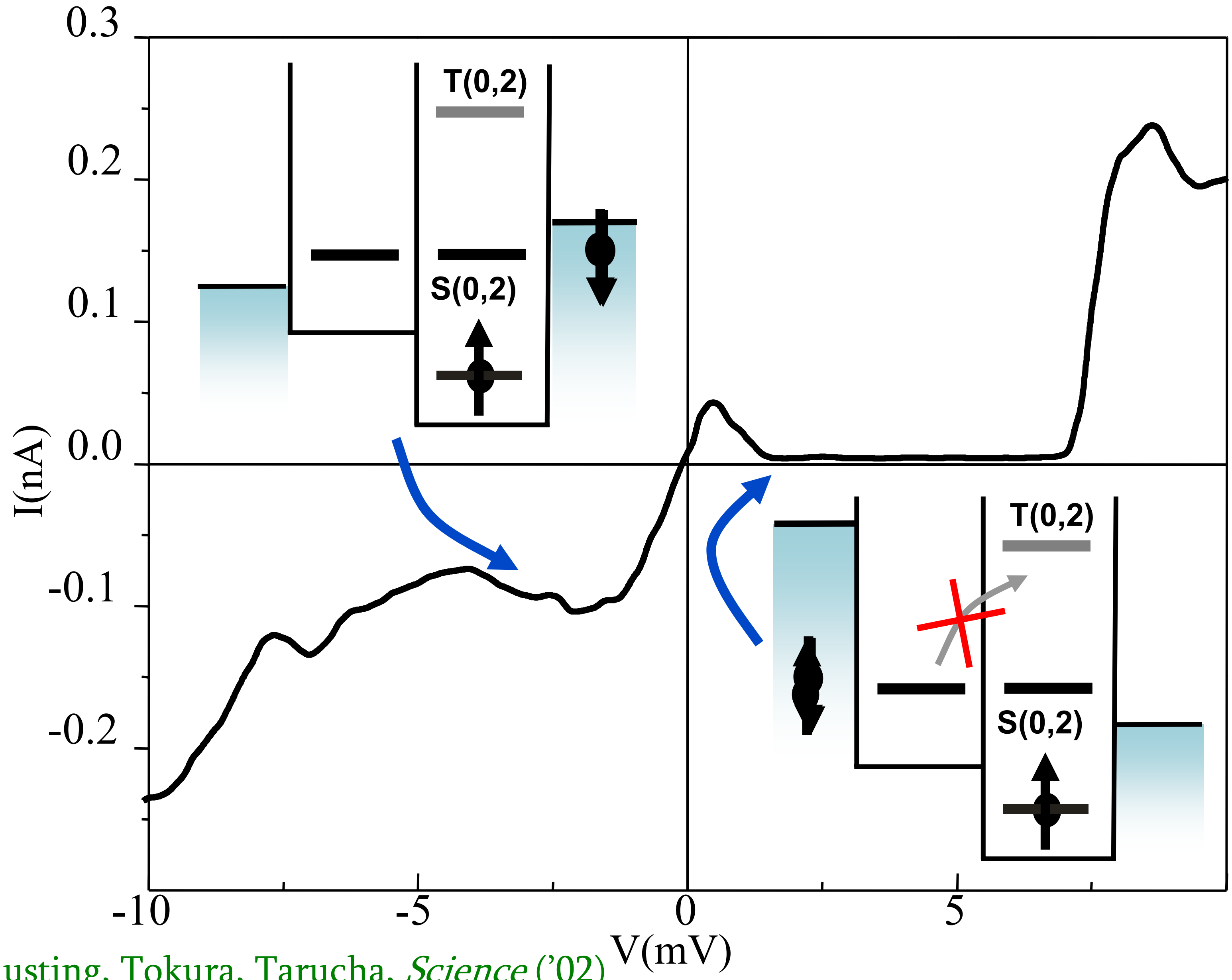
Solution:

Second electron as the detector

- Low frequency (200MHz, 40mT)
- Less sensitive to electric fields

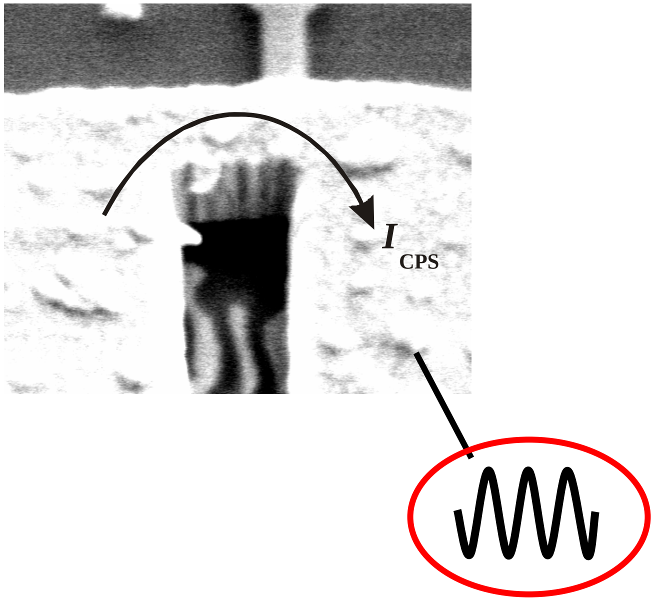


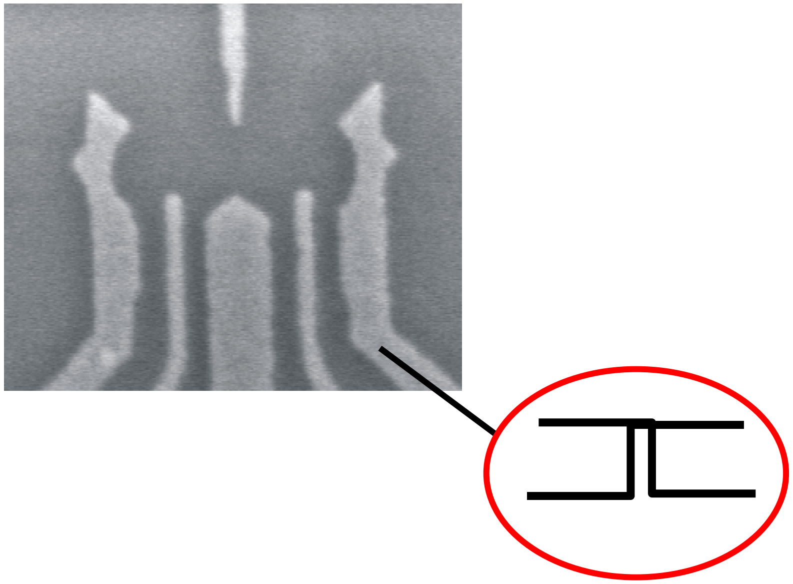
Probe: spin blockade due to Pauli exclusion

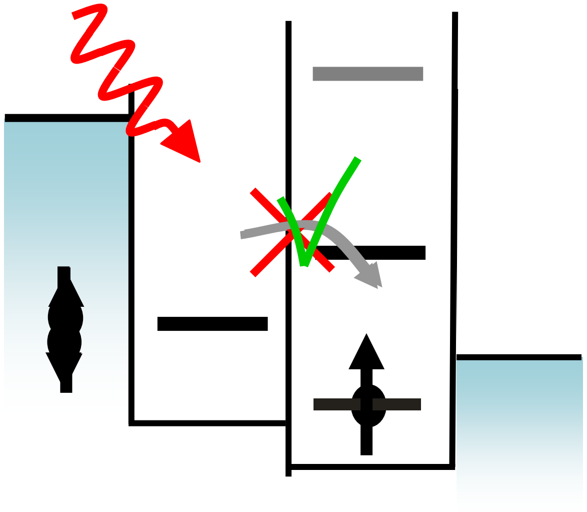


Ono, Austing, Tokura, Tarucha, *Science* ('02)

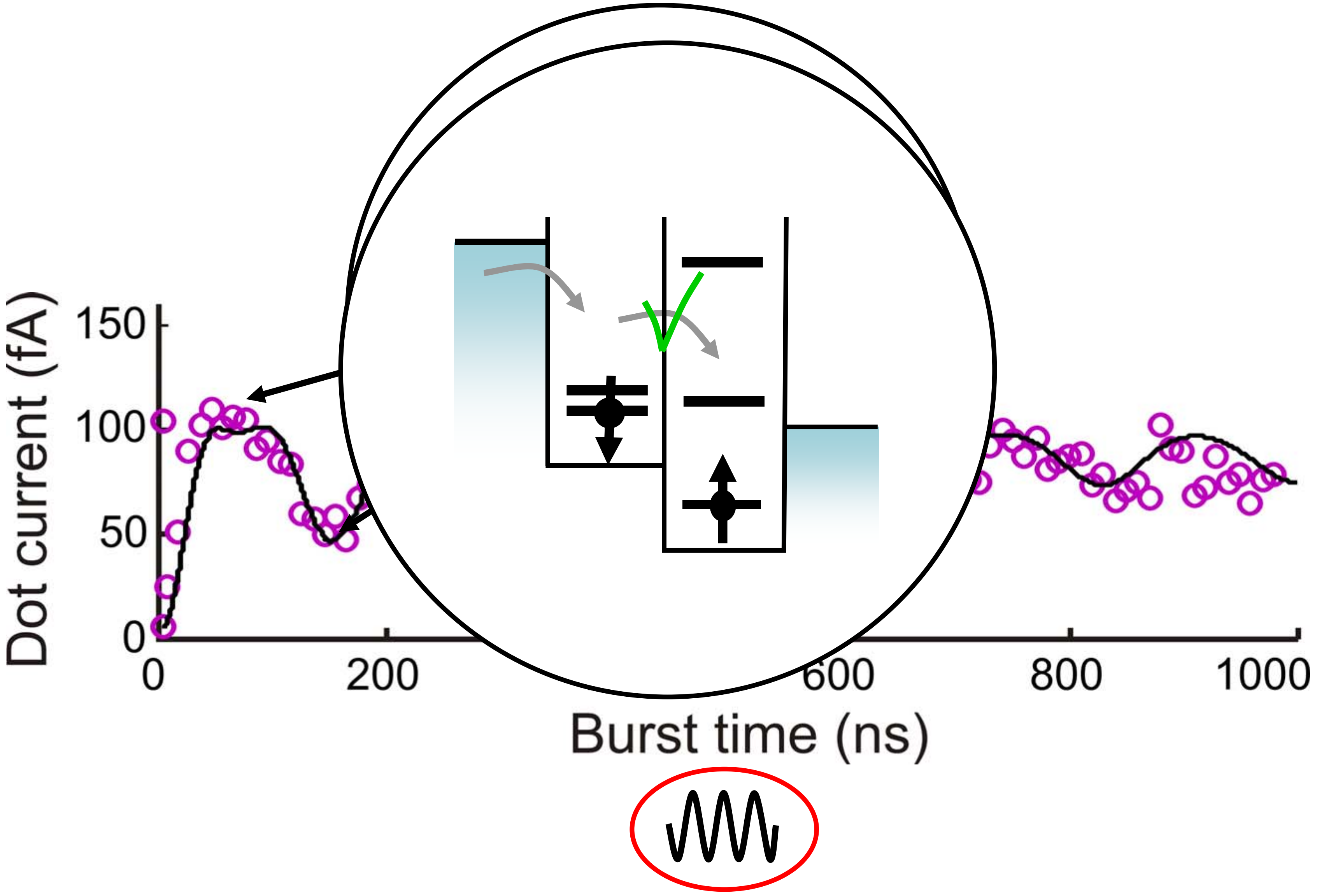
Detection and pulse scheme



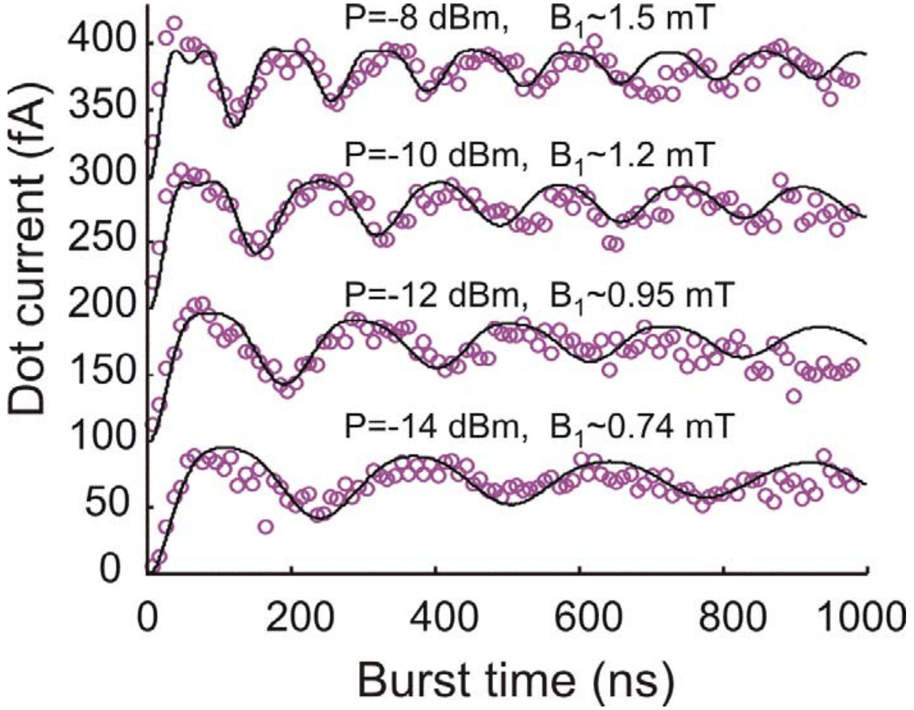


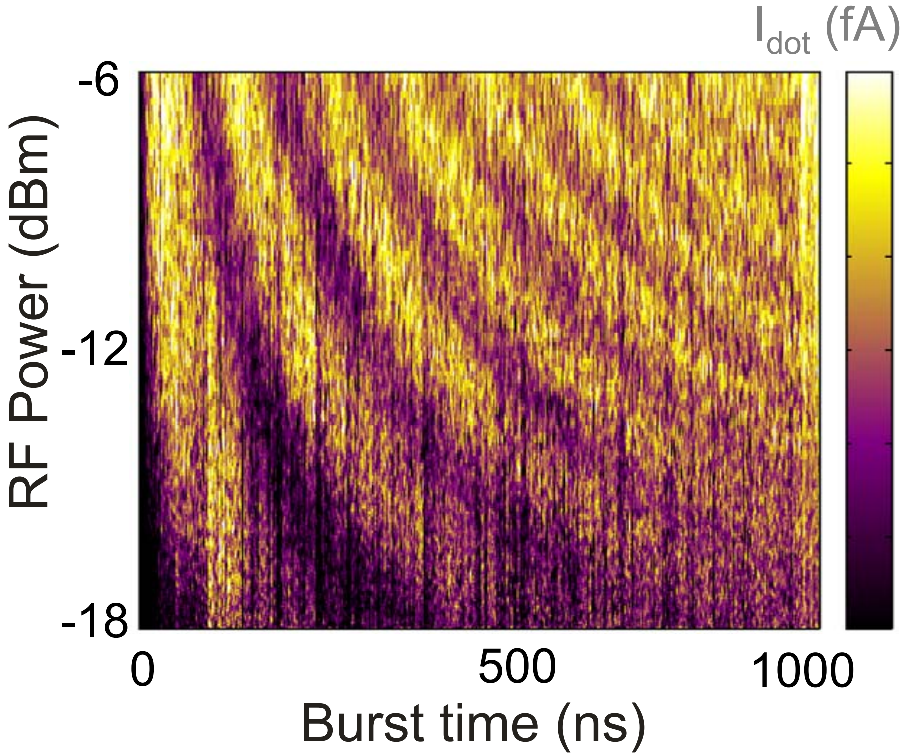


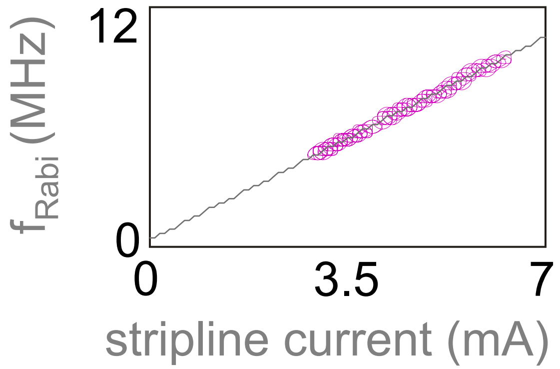
Coherent control of a single electron spin



Coherent control of a single electron spin







up to 8 periods observed
$\pi/2$ rotation in 25ns

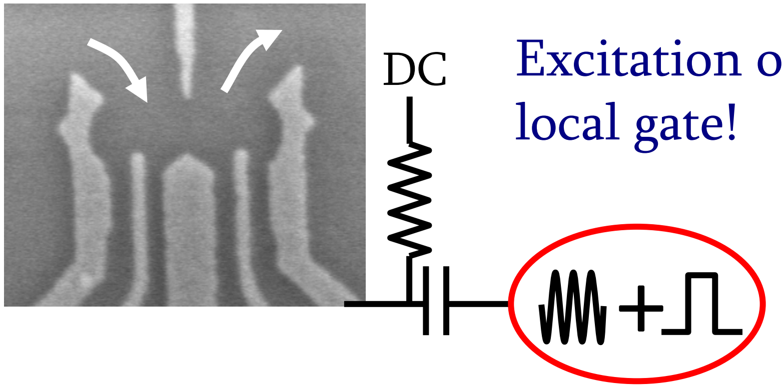
max B_1 ~ 1.9 mT, compare $B_{N,z}$ ~ 1.3 mT

Analysis: Koppens, Klauser, et al, *PRL* 2007

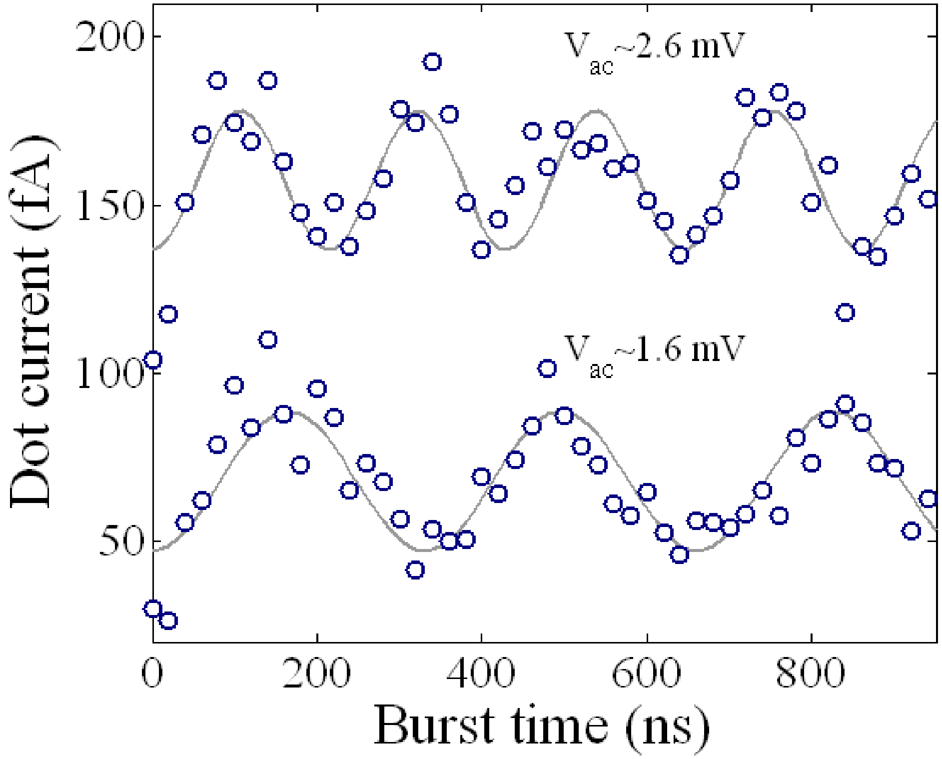
Koppens et al. *Nature* ('06)

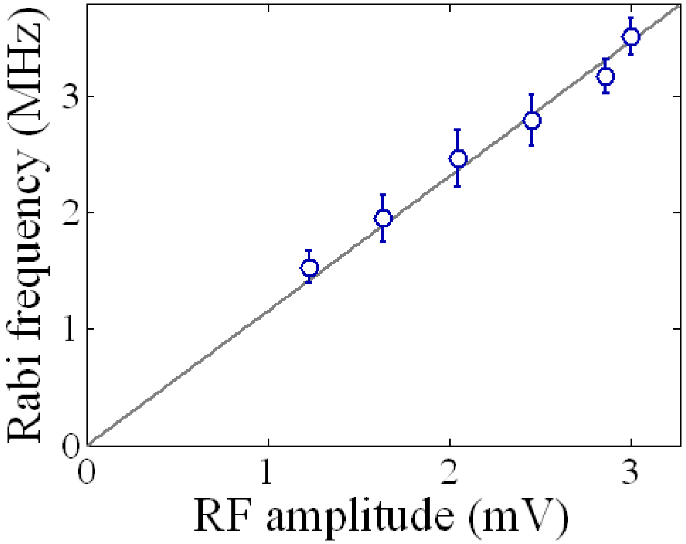
Electrical single-spin control

Easier local addressing
All-electrical control possible



Excitation of a local gate!





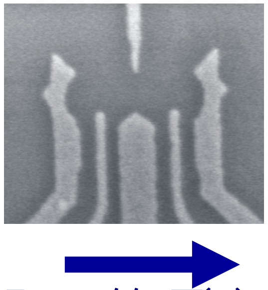
Nowack et al., Science ('07)

Mechanism: spin-orbit interaction

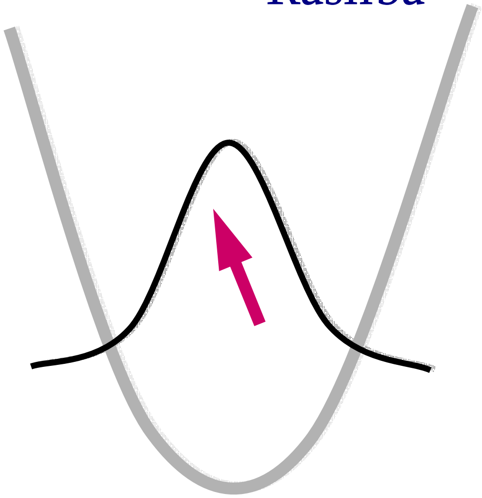
$$H_R = \alpha(\sigma_x p_y - \sigma_y p_x) + \beta(\sigma_x p_x - \sigma_y p_y)$$

Rashba

Dresselhaus



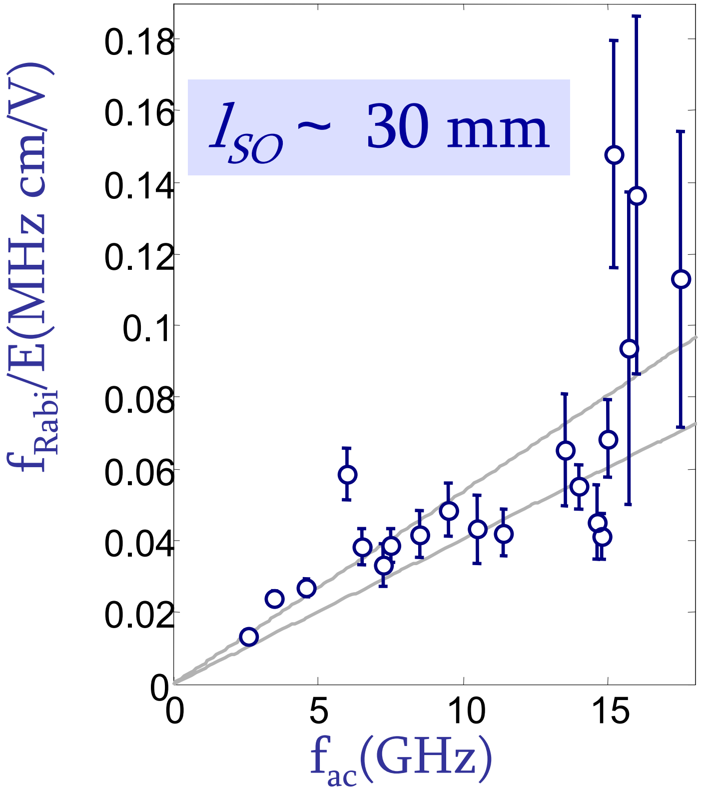
B_{ext} // $E(t)$ // $[1\bar{1}0]$



$$B_{eff}\,,\; f_{Rabi} \sim \frac{1}{l_{SO}}\left|\vec{B}_{ext}\right|\left|\vec{E}\right|$$

$$l_{SO} = \frac{\hbar}{m(\beta - \alpha)}$$

Theory: Golovach et al., *PRB* ('06)



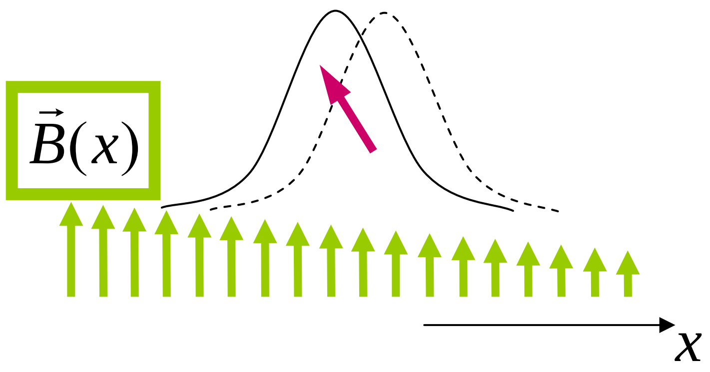
$l_{SO} \sim$ 30 mm

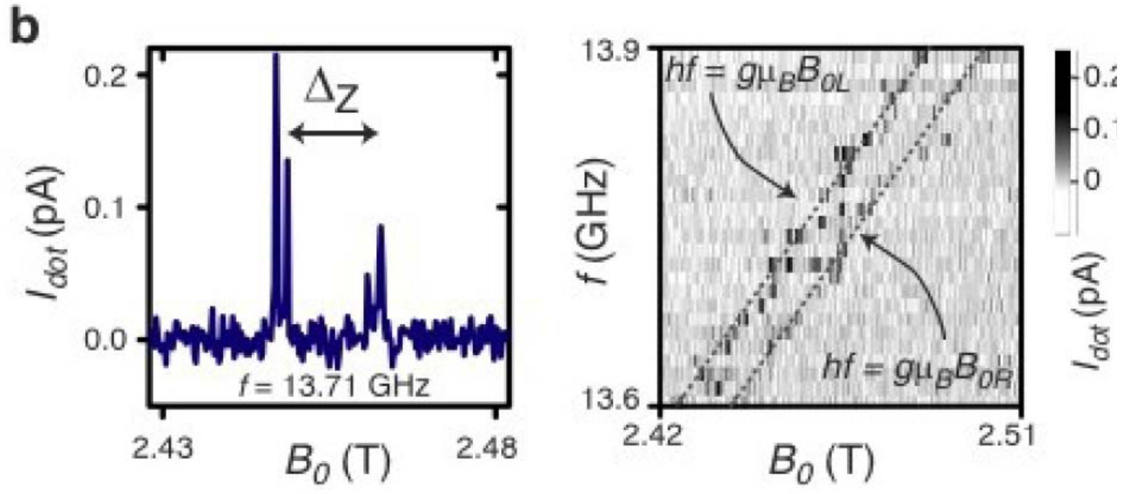
Other mechanisms for electrical driving

Magnetic field gradient

(microfabricated ferromagnet)

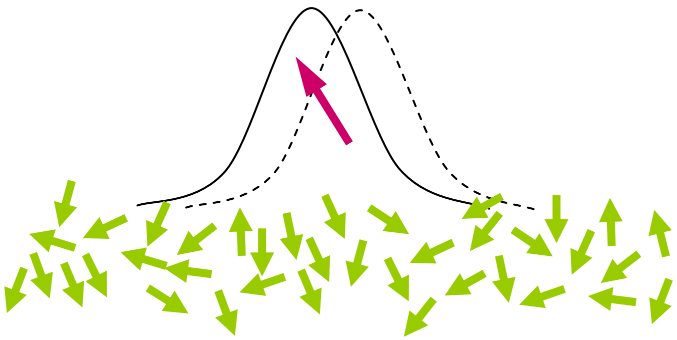
Pioro-Ladriere et al, arXiv:0805.1083

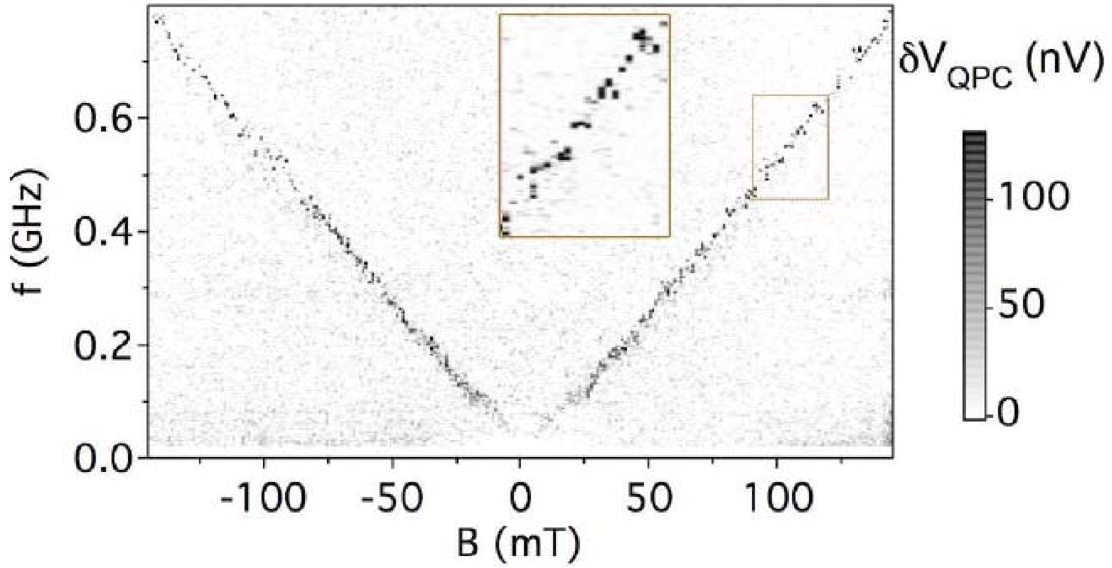




Nuclear field gradient (incoherent only)

Laird et al., *PRL* ('07)



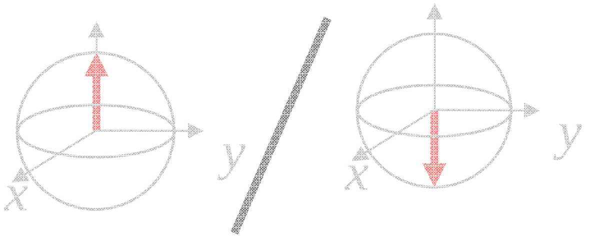


Spin qubits in quantum dots

Loss & DiVincenzo, PRA 1998

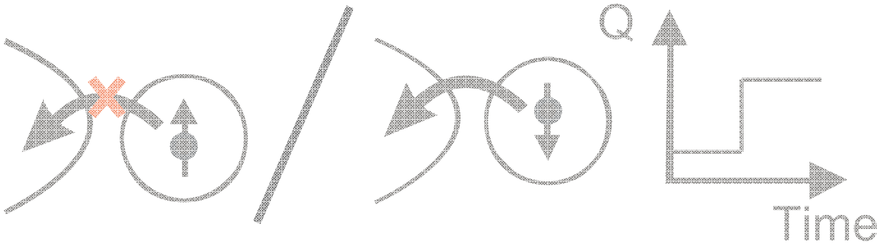
Well-defined qubit states

- *Confine single electrons*



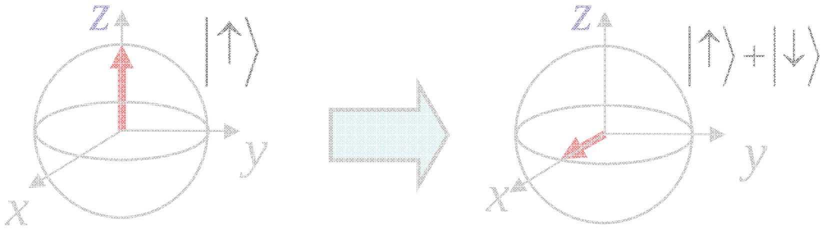
Initialize and read-out

- *Spin to charge conversion*



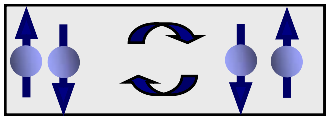
1-qubit gate

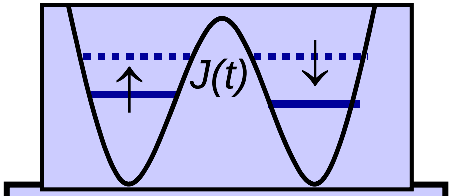
- *Electron spin resonance*



2- qubit gate

- *Exchange interaction*

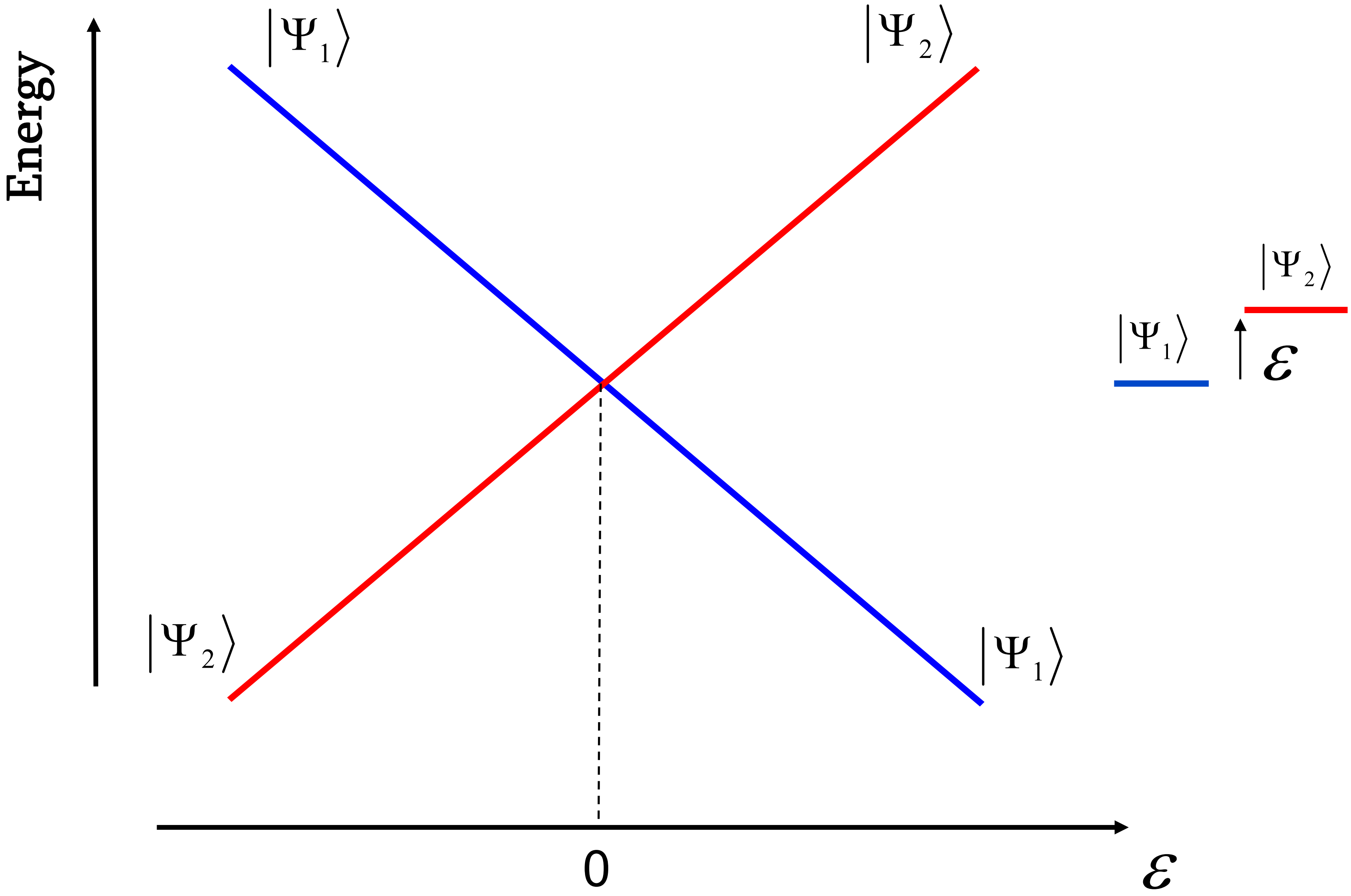




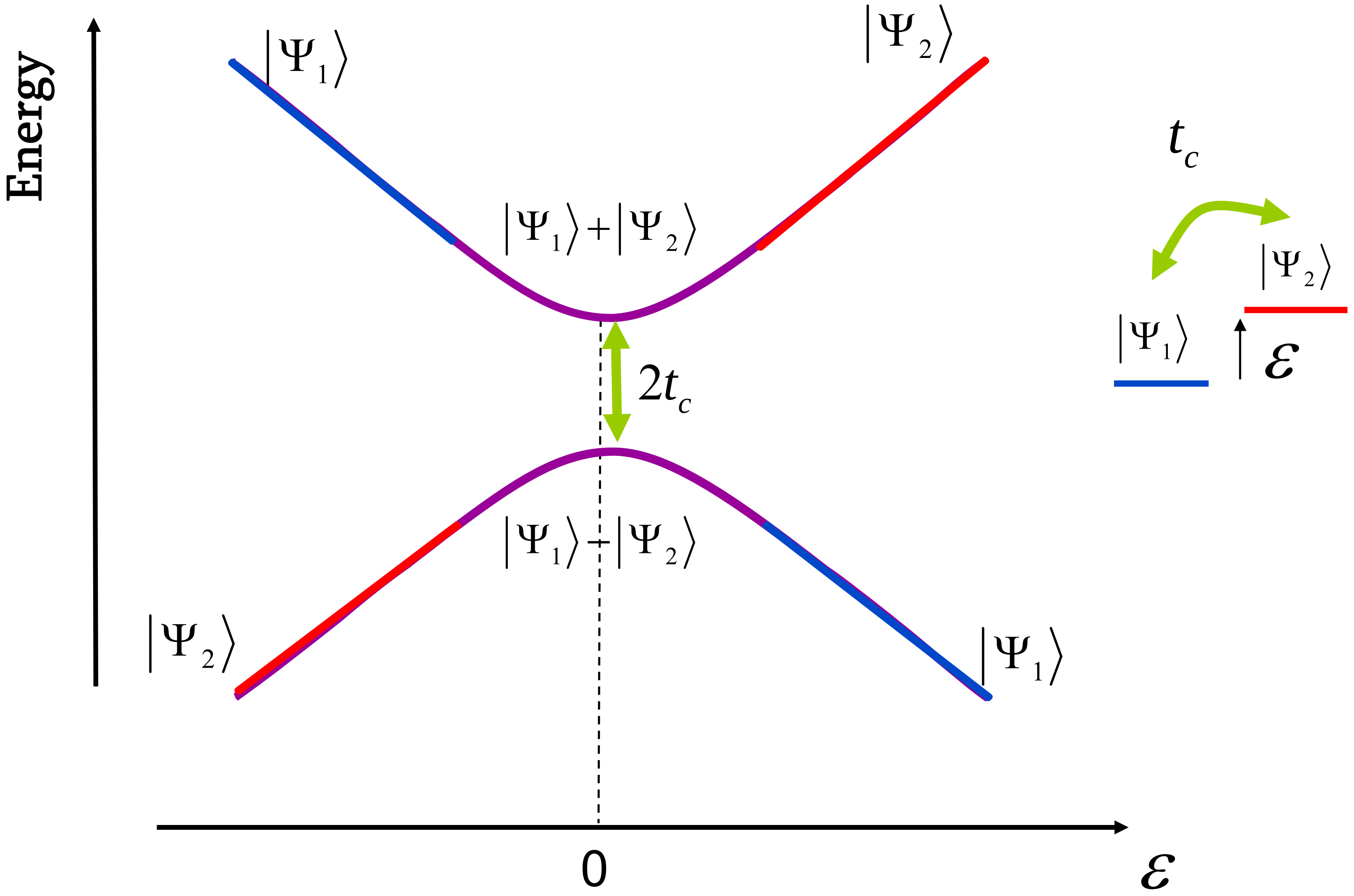
$H_s(t) = J(t)\, \boldsymbol{S_L} \cdot \boldsymbol{S_R}$

Petta *et al.*, Science ('05)

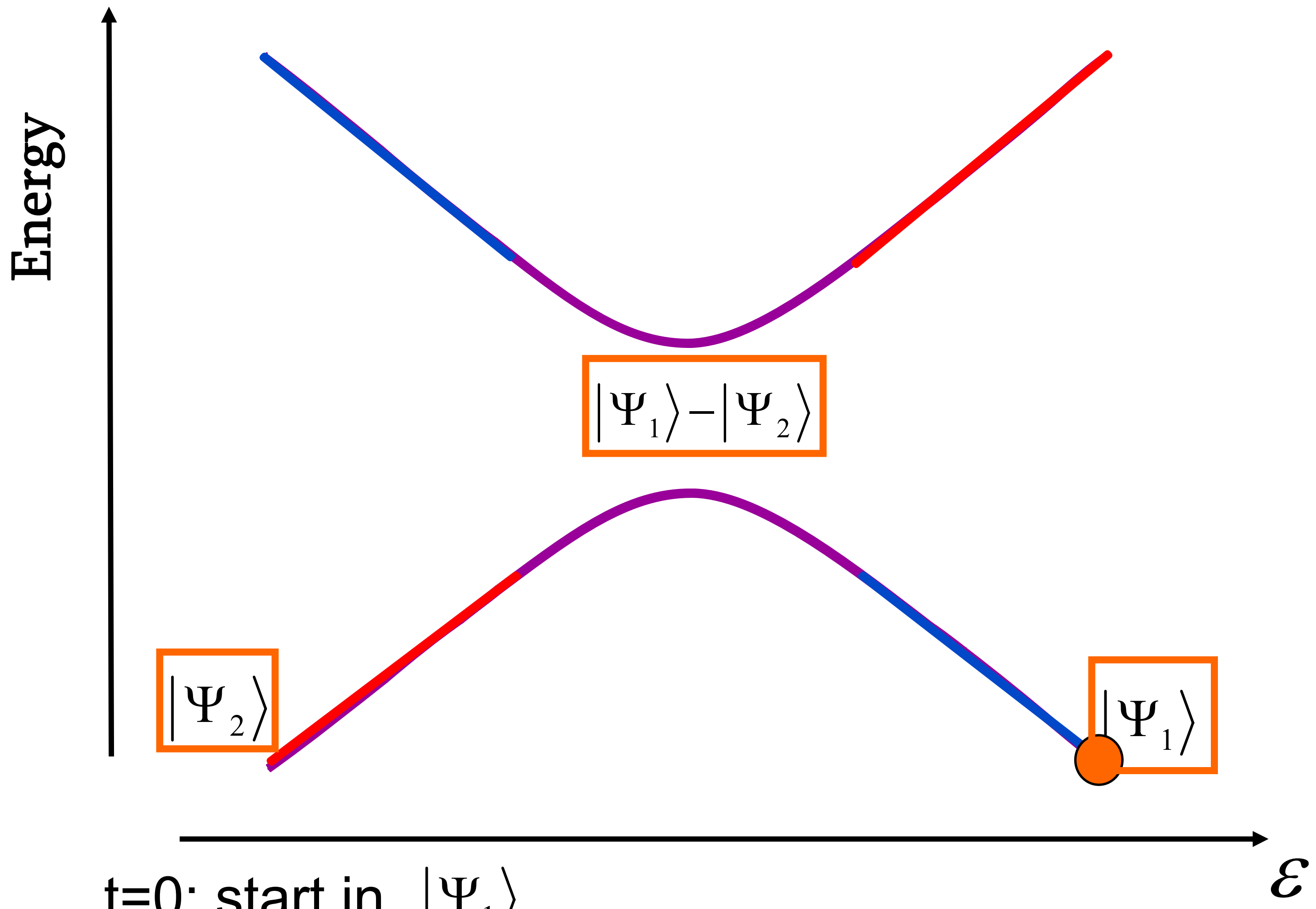
Crossing


Energy
$|\Psi_1\rangle$
$|\Psi_2\rangle$
$|\Psi_2\rangle$
$|\Psi_1\rangle$
0
ε
$|\Psi_2\rangle$
$|\Psi_1\rangle$
ε

Anticrossing



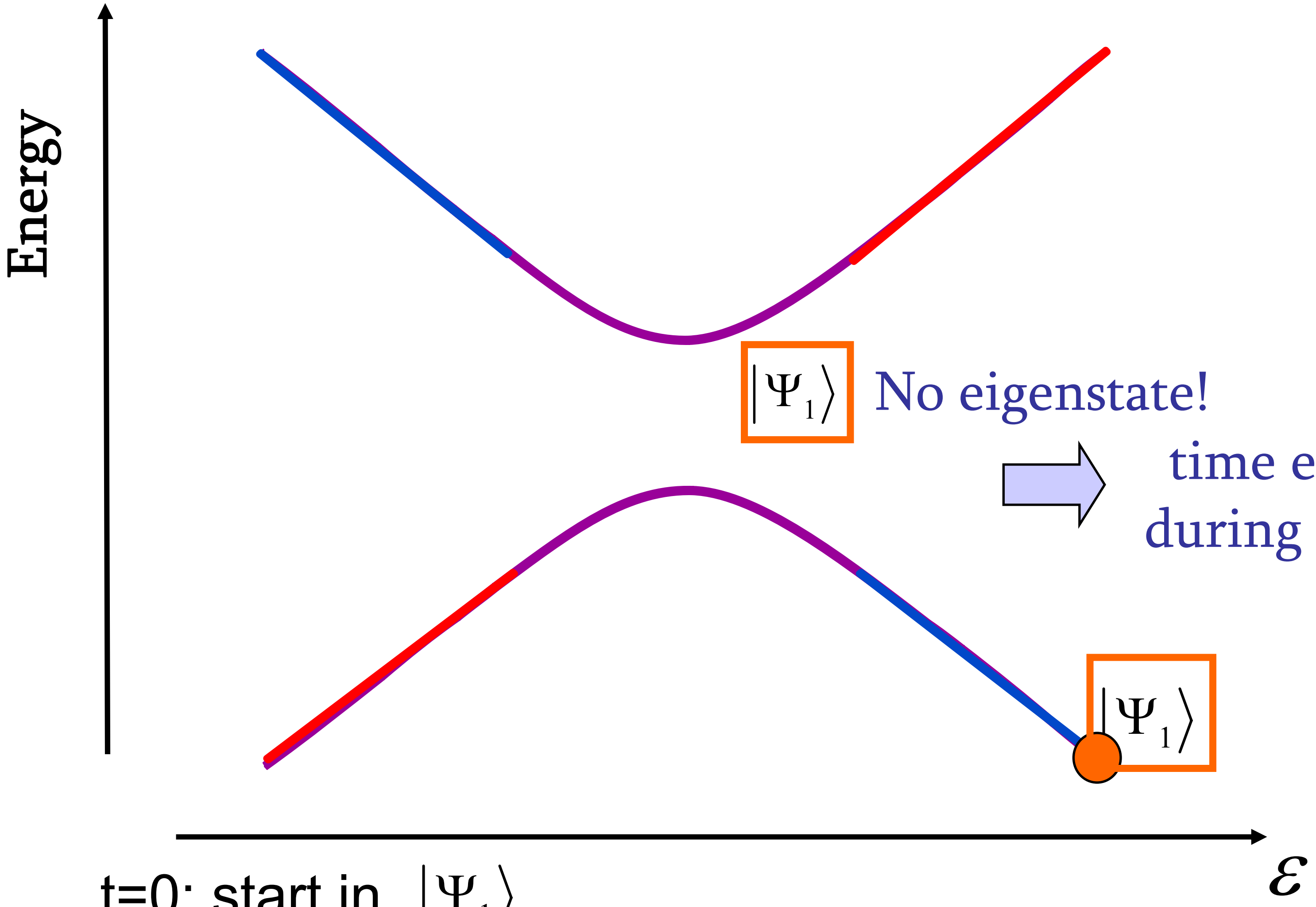
Adiabatic passage



t=0: start in $|\Psi_1\rangle$

vary ε slowly ⇨ stay in eigenstate connecting to initial state

Non-adiabatic passage



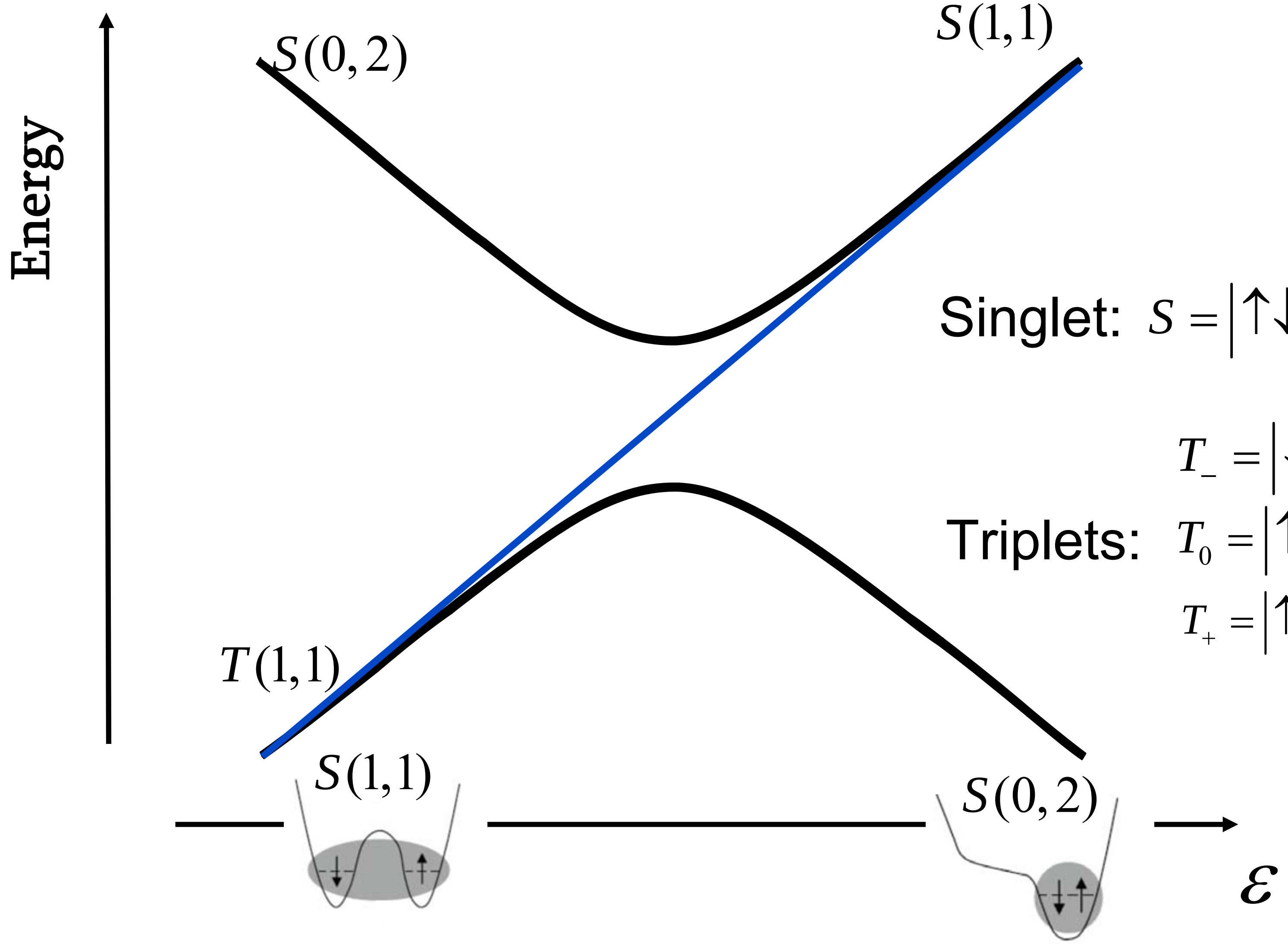
No eigenstate!

⇨ time evolution during wait time

t=0: start in $|\Psi_1\rangle$

vary ε fast ⇨ stay in initial state

Level diagram (0,2)-(1,1) charge transition



Singlet: $S = |\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle$

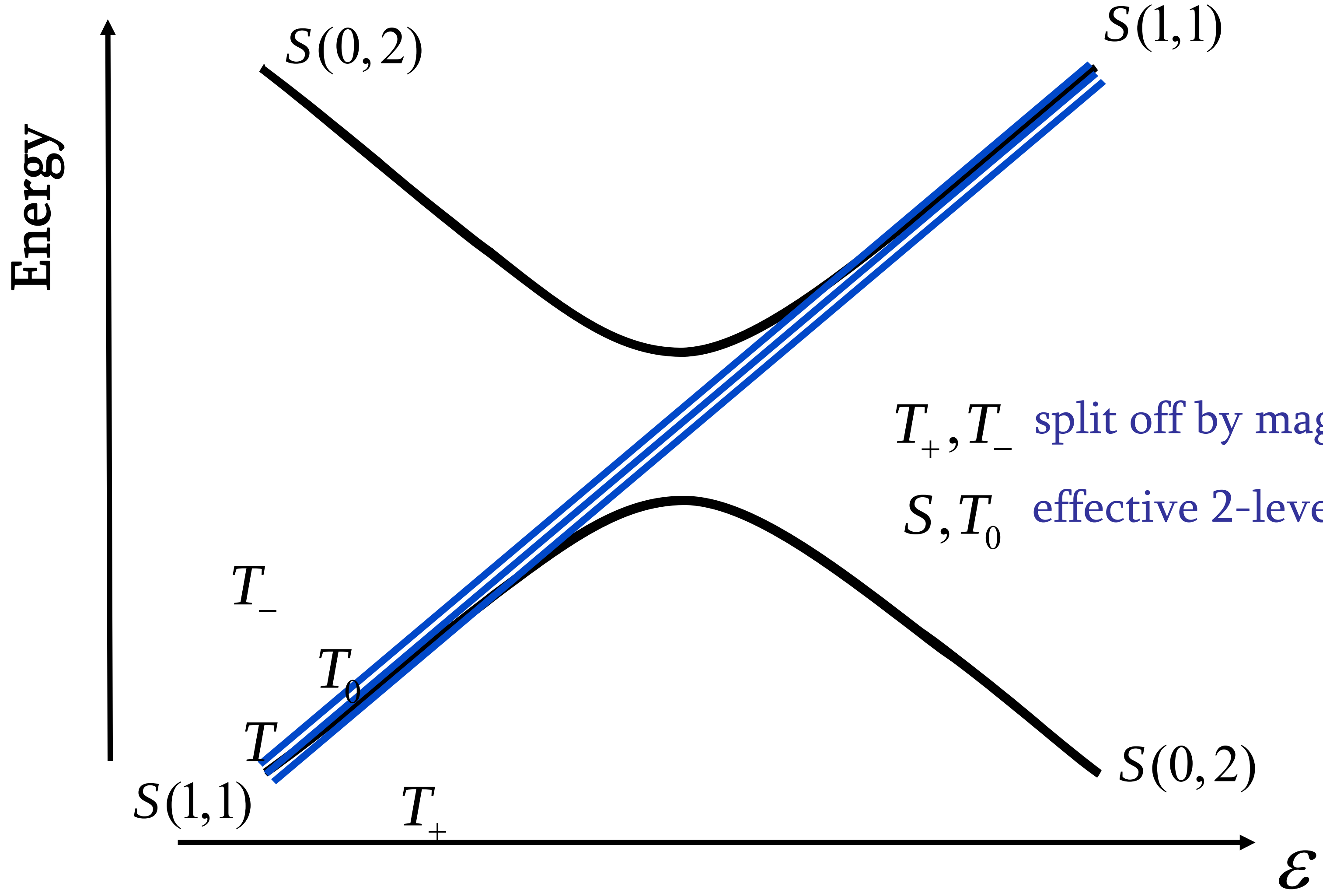
Triplets:

$T_- = |\downarrow\downarrow\rangle$

$T_0 = |\uparrow\downarrow\rangle + |\downarrow\uparrow\rangle$

$T_+ = |\uparrow\uparrow\rangle$

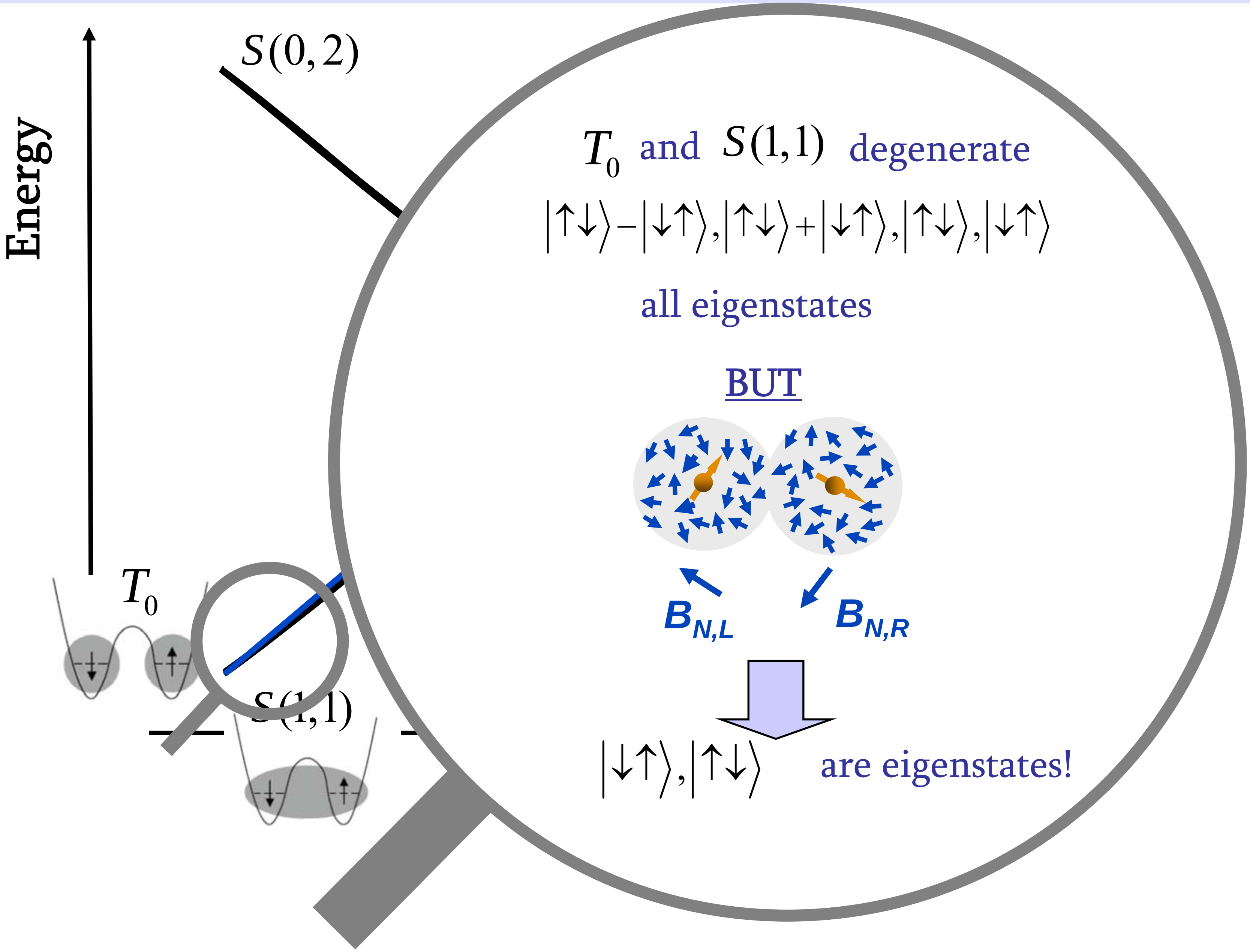
Level diagram (0,2)-(1,1) charge transition



T_+, T_- split off by magnetic field

S, T_0 effective 2-level system

Level diagram (0,2)-(1,1) charge transition



T_0 and $S(1,1)$ degenerate

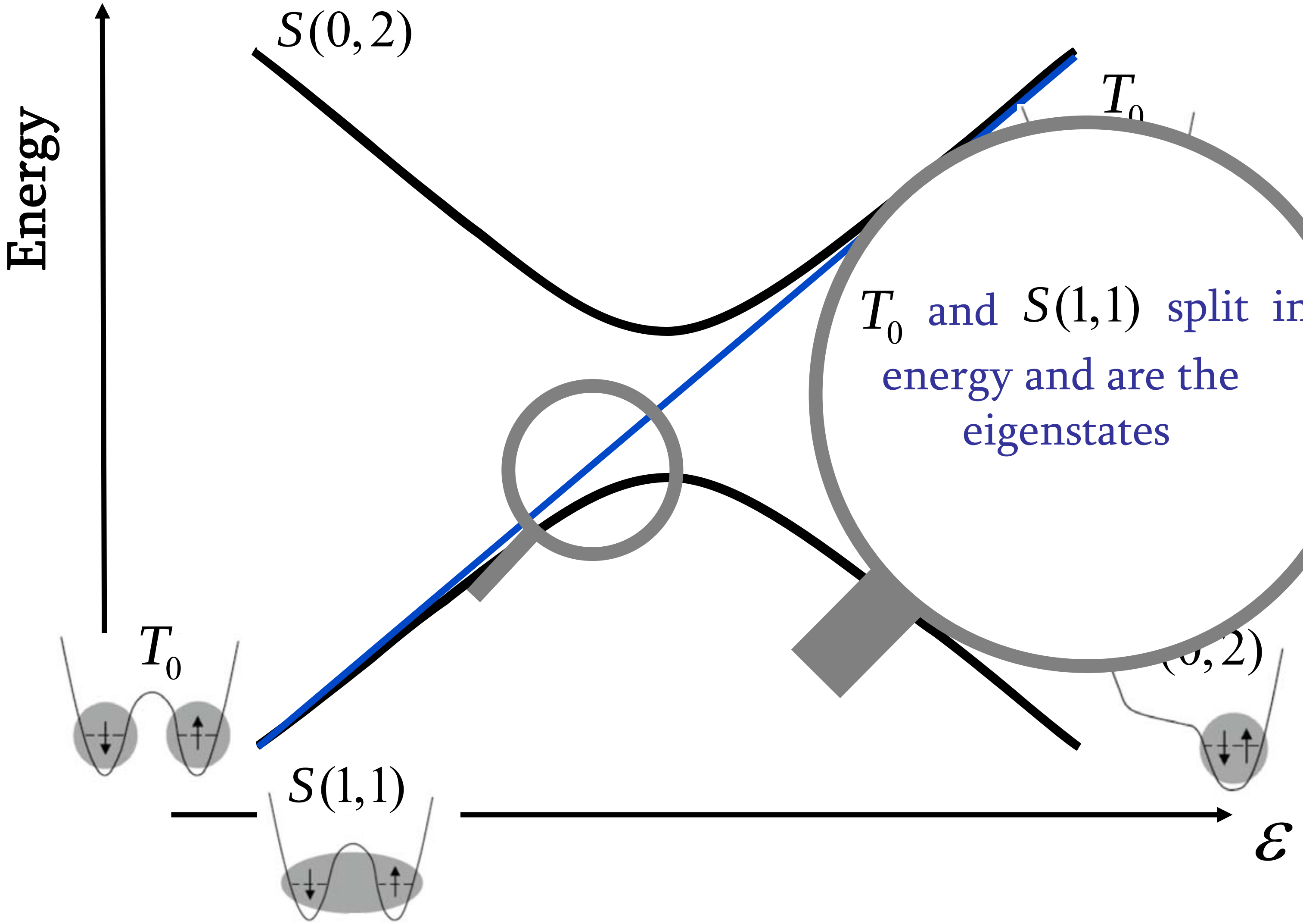
$|\uparrow\downarrow\rangle-|\downarrow\uparrow\rangle, |\uparrow\downarrow\rangle+|\downarrow\uparrow\rangle, |\uparrow\downarrow\rangle, |\downarrow\uparrow\rangle$

all eigenstates

BUT

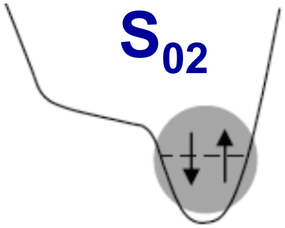
$|\downarrow\uparrow\rangle, |\uparrow\downarrow\rangle$ are eigenstates!

Level diagram (0,2)-(1,1) charge transition

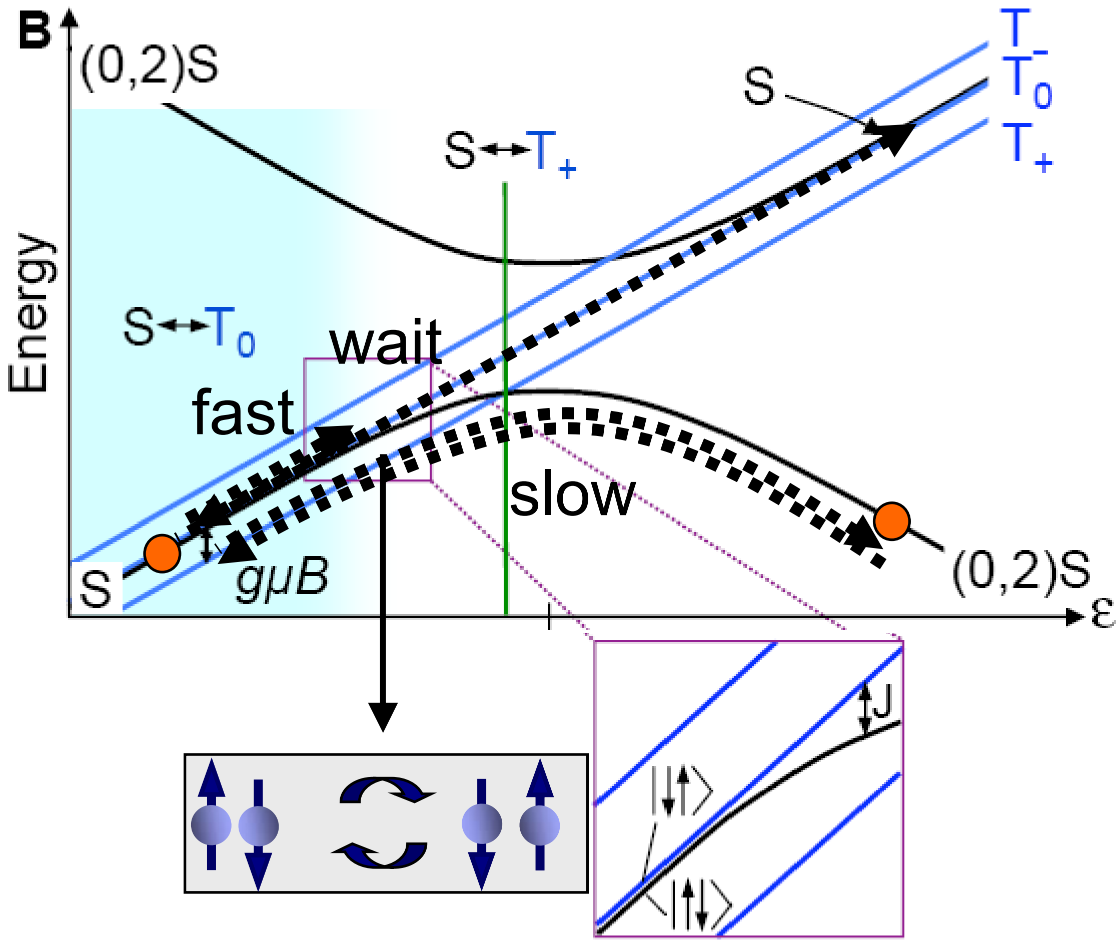


T_0 and $S(1,1)$ split in energy and are the eigenstates

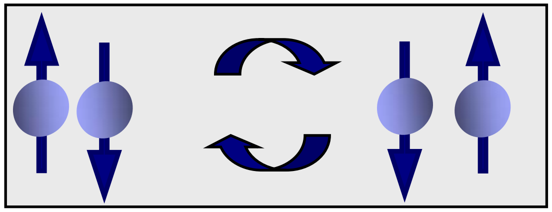
Coherent exchange of two spins - scheme



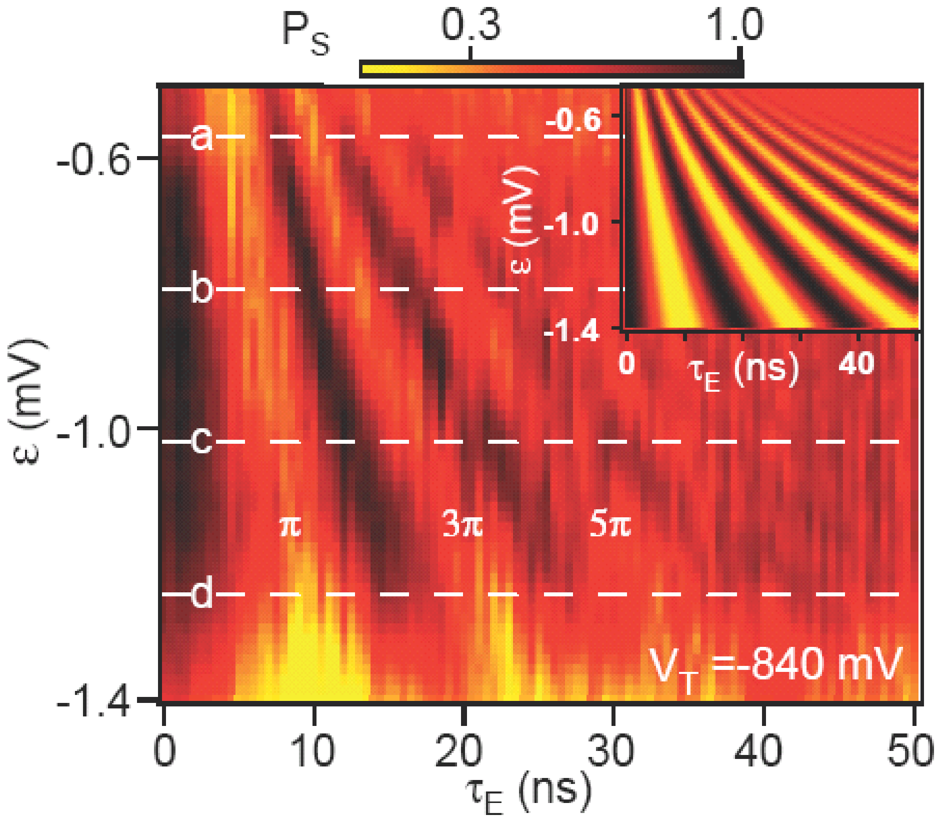
Evolution

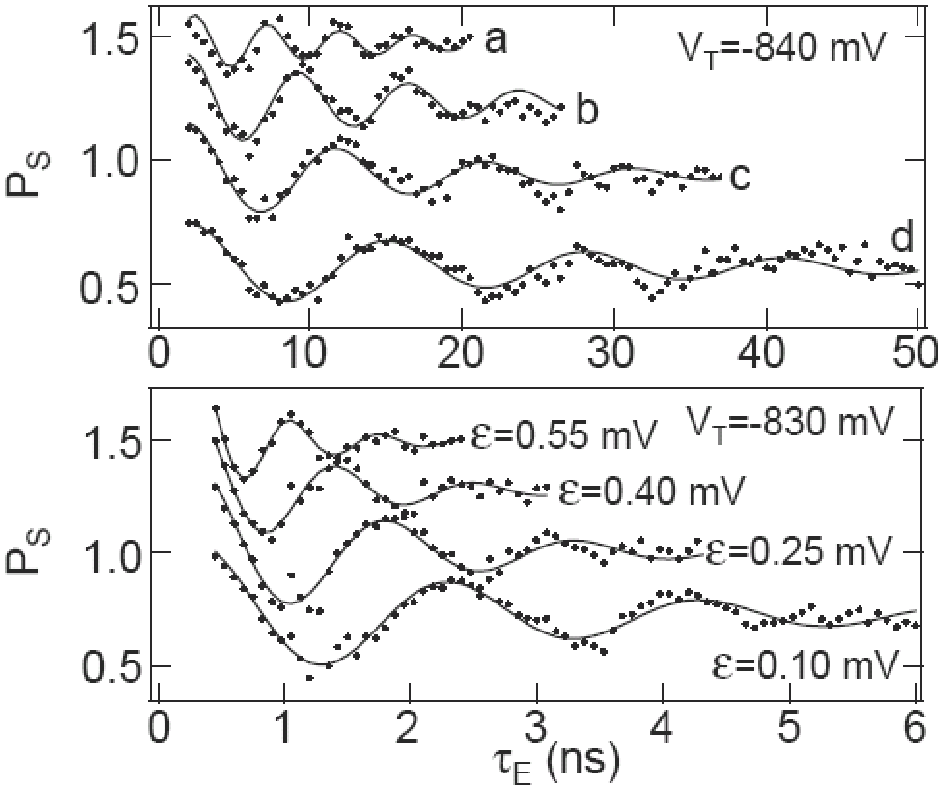


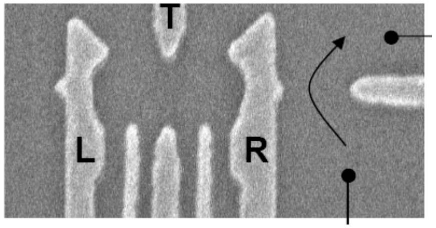
Coherent exchange of two spins



Petta, Johnson, Taylor, Laird, Yacoby, Lukin, Marcus, Hanson, Gossard, *Science* 2005







- $\sqrt{SWAP}$ in as little as 180 ps
- three oscillations visible, independent of period

Spin qubits in quantum dots

Loss & DiVincenzo, PRA 1998

Well-defined qubit states
- *Confine single electrons*

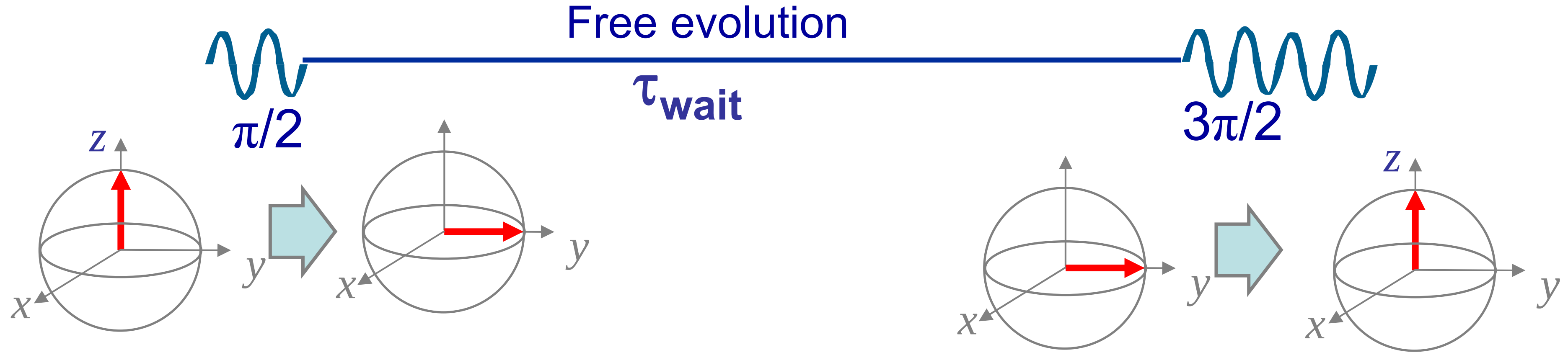
Initialize and read-out
- *Spin to charge conversion*

1-qubit gate
- *Electron spin resonance*

2-qubit gate
- *Exchange interaction*

Study decoherence (T_2)

“Apparent” dephasing

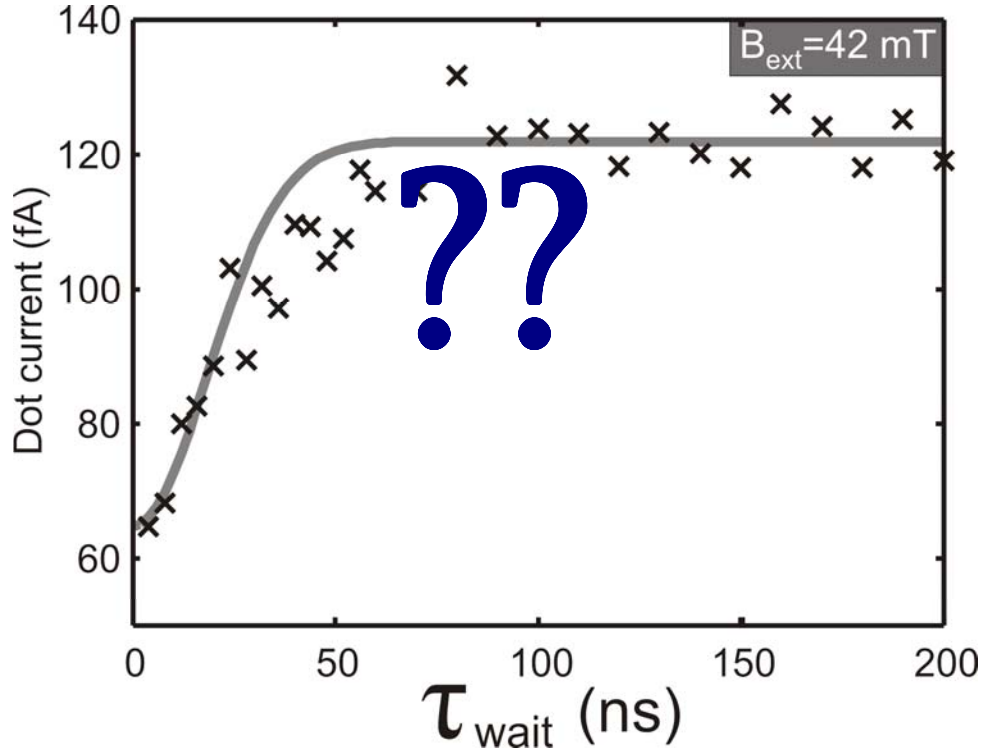


Free evolution

τ_{wait}

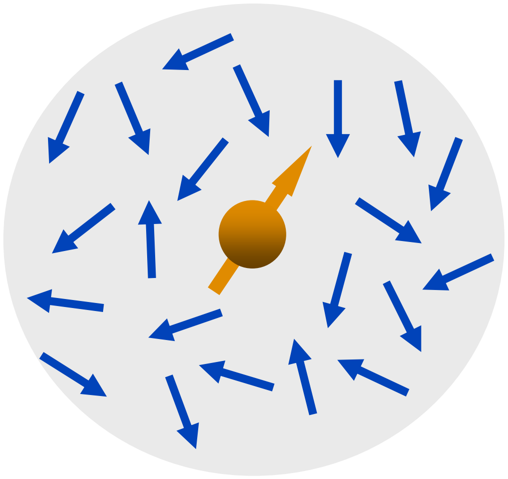
$\pi/2$

$3\pi/2$



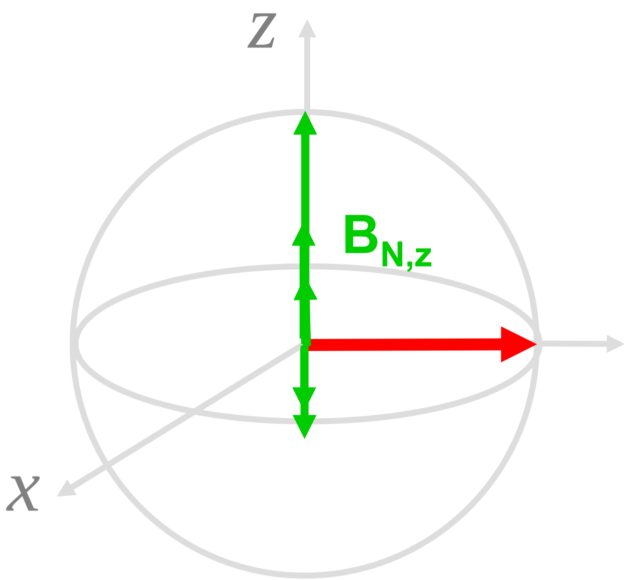
Koppens et al., *PRL* (’08)

Decoherence: statistical Overhauser field



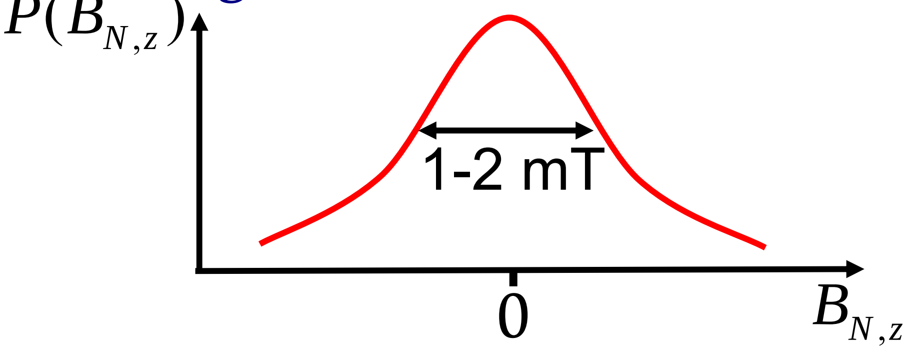
effective magnetic field
due to hyperfine coupling

$$H = g\mu_B S_z (B_{ext} + B_{N,z})$$



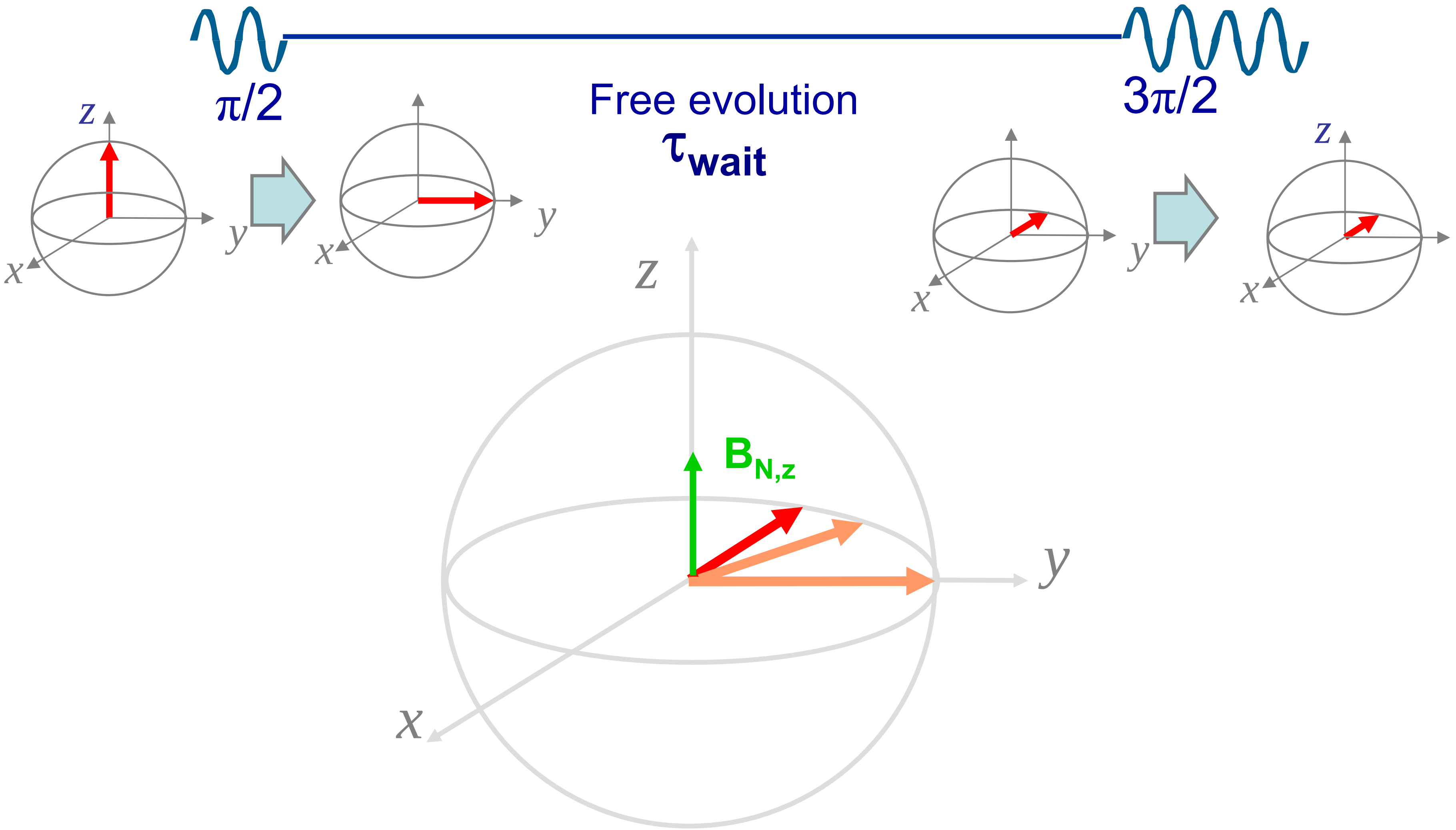
nuclear field gives an offset in
the rotating frame

fluctuating following a
gaussian distribution



e.g. Merkulov *et al.*, PRB ‘02

Effect of the nuclear field



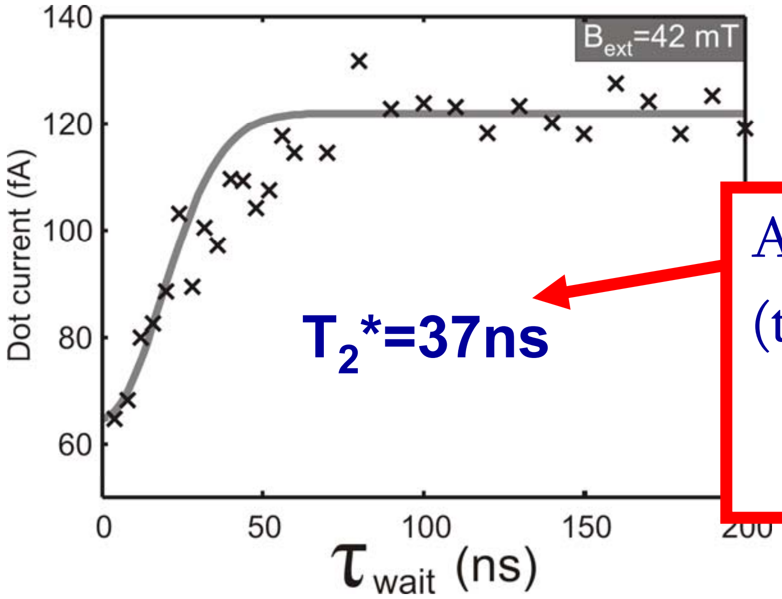
spin precesses during free evolution

“Apparent” dephasing

$\pi/2$

Free evolution
τ_{wait}

$3\pi/2$



$T_2^*=37\text{ns}$

Apparent dephasing
(time-averaged over
experimental runs)

Mechanism: fluctuating nuclear field in z-direction (of 1.5mT)

Koppens et al., *PRL* (’08)

Ramsey fringes

Free evolution

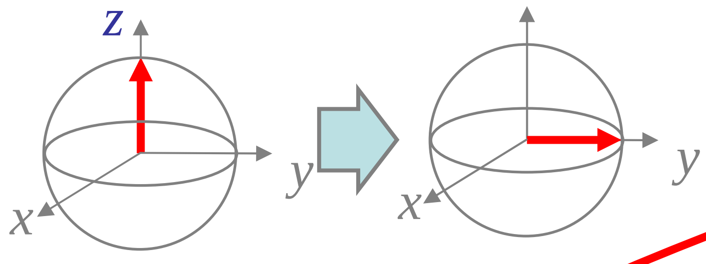
$\pi/2$

τ_{wait}
fixed and short

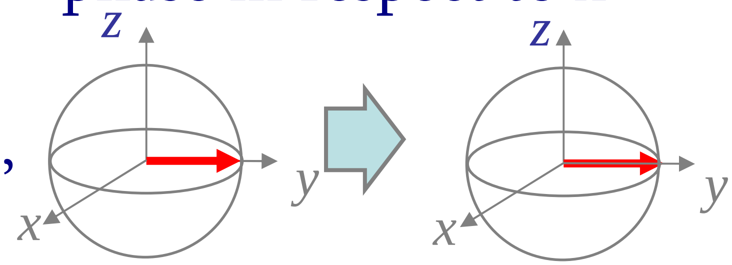
$3\pi/2$

rotate around x

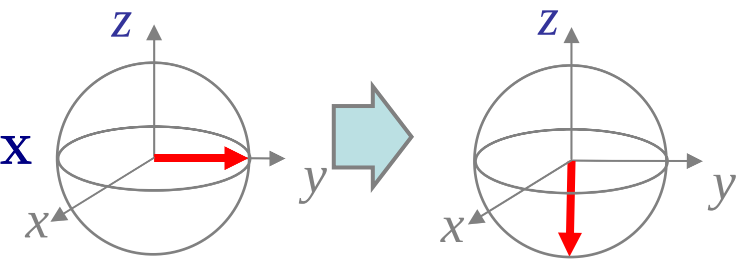
rotate around axis with relative phase in respect to x

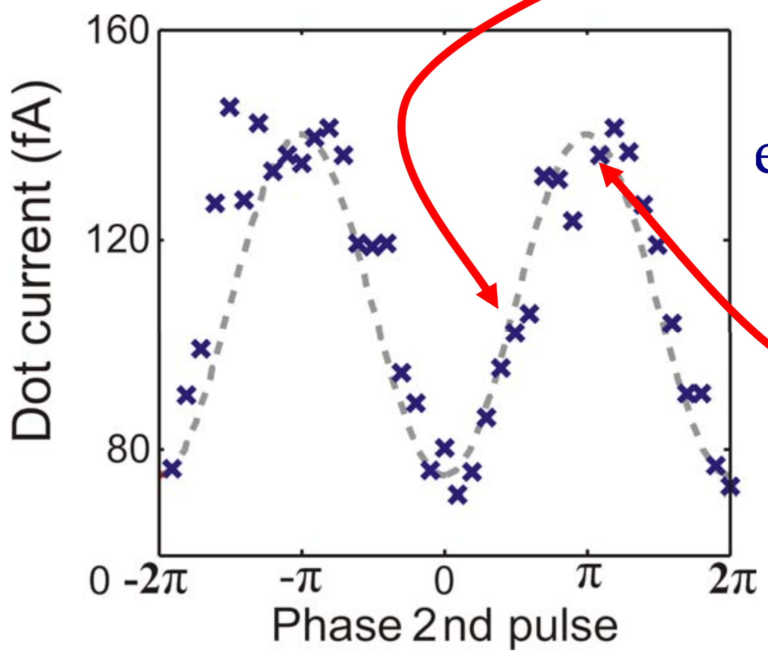


e. g. around y,
phase π/2



e. g. around –x
phase π



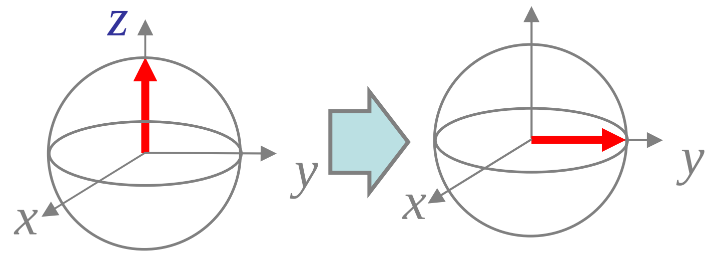


τ_{wait}=10ns

Koppens et al., *PRL* ('08)

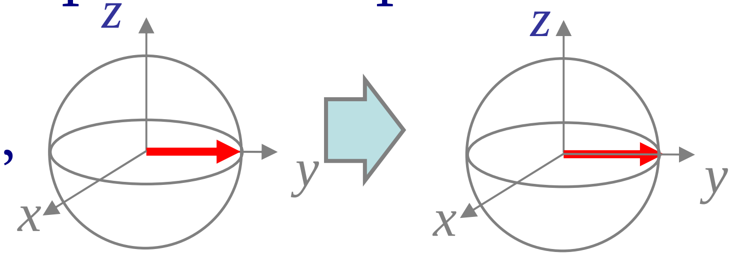
Ramsey fringes

rotate around x

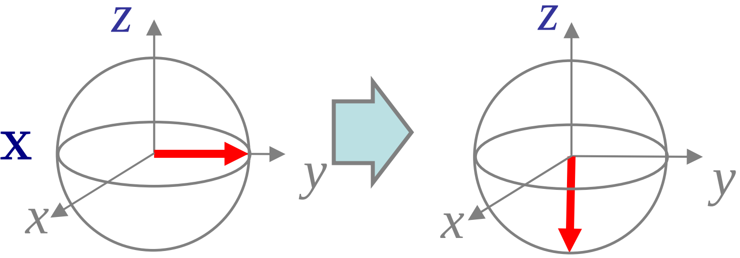


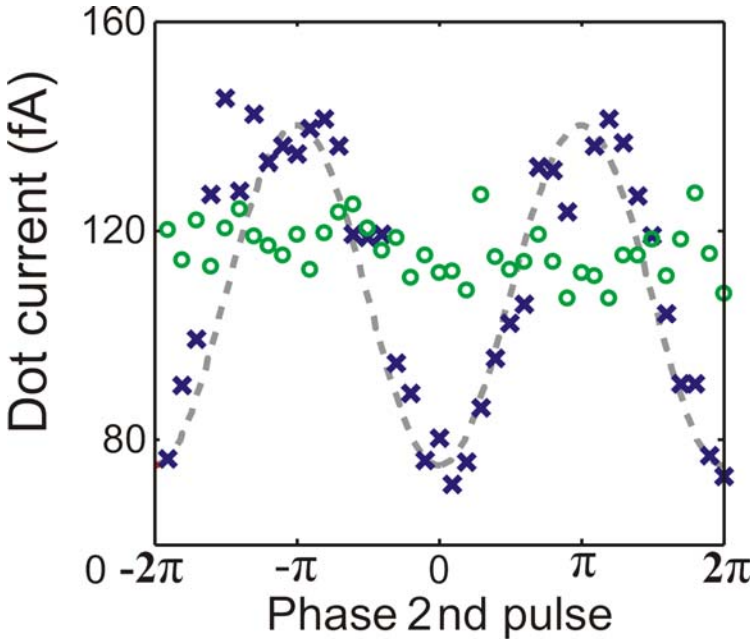
rotate around axis with relative phase in respect to x

e. g. around y, phase $\pi/2$



e. g. around –x phase π



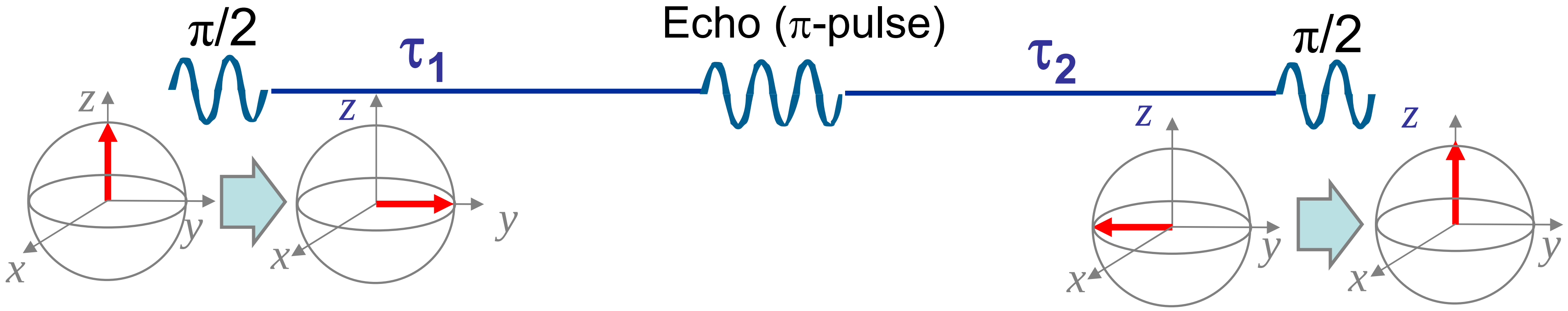


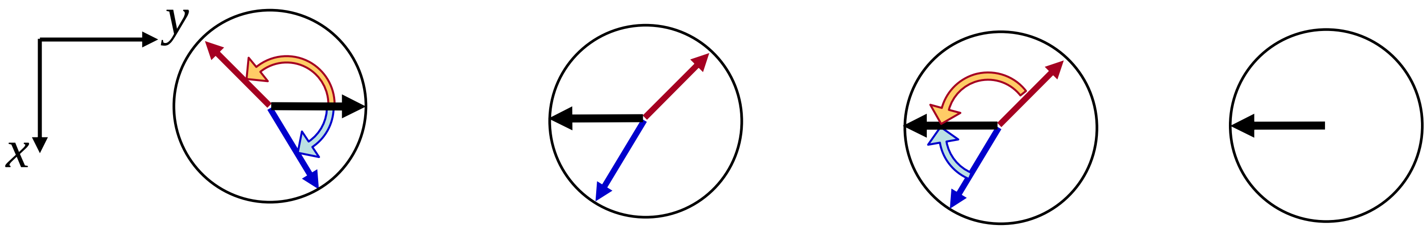
✖ τ_{wait}=10ns

○ τ_{wait}=150ns

Koppens et al., *PRL* ('08)

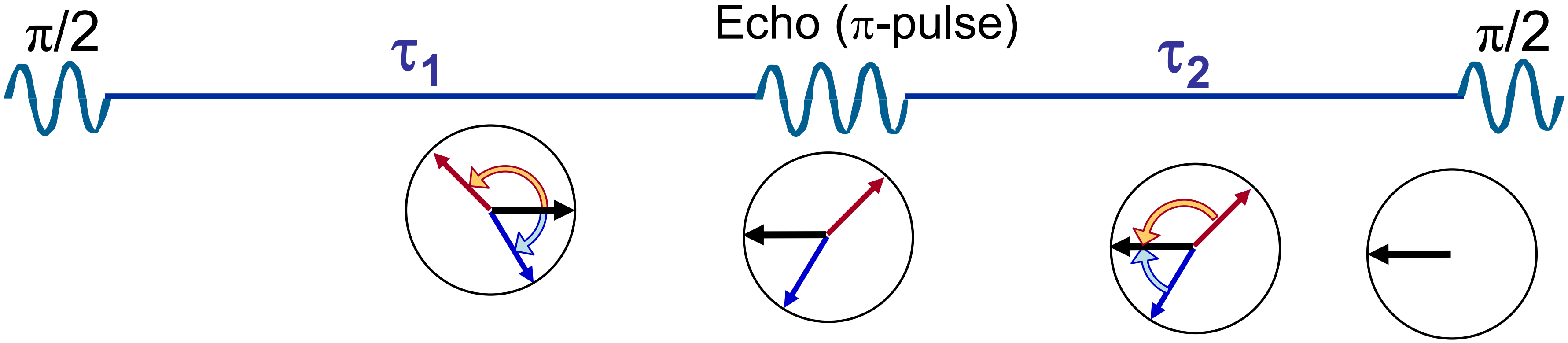
Spin echo – unwind precession around nuclear field

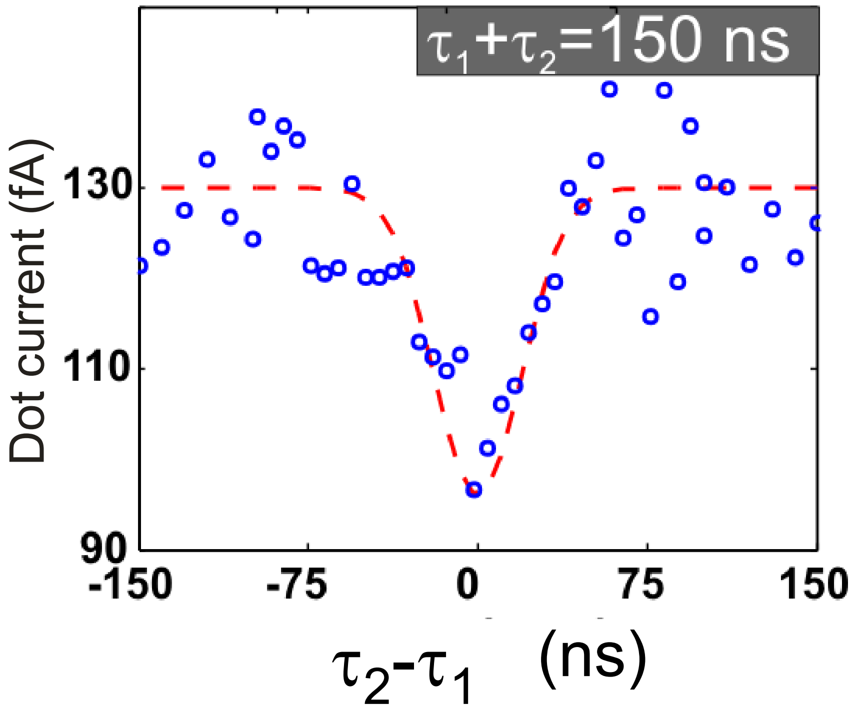




refocusing for $\tau_1 = \tau_2$

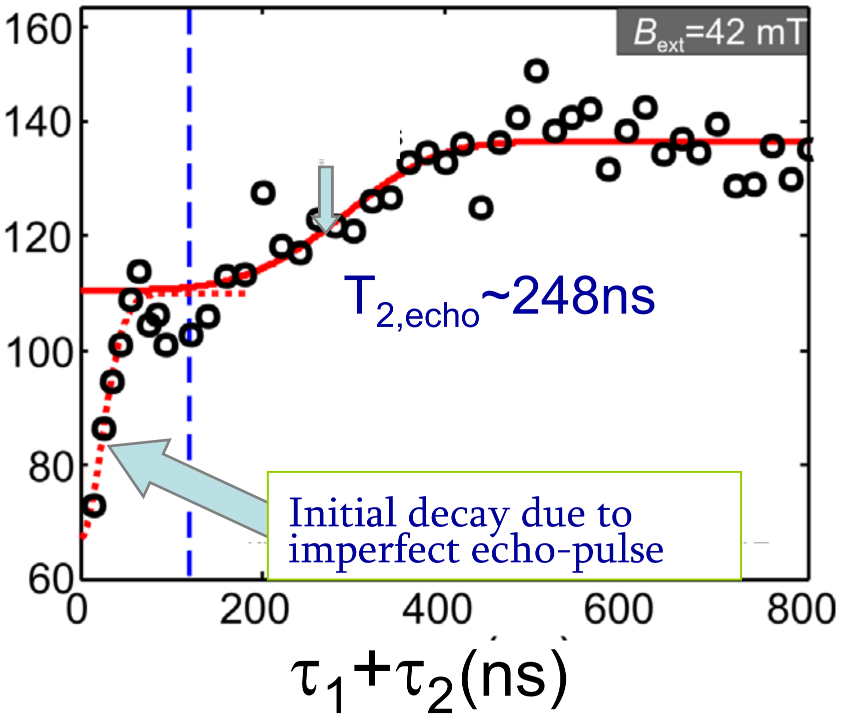
Spin echo – unwind precession around nuclear field





refocusing for $\tau_1 = \tau_2$

Spin echo decay (τ_1=τ_2)

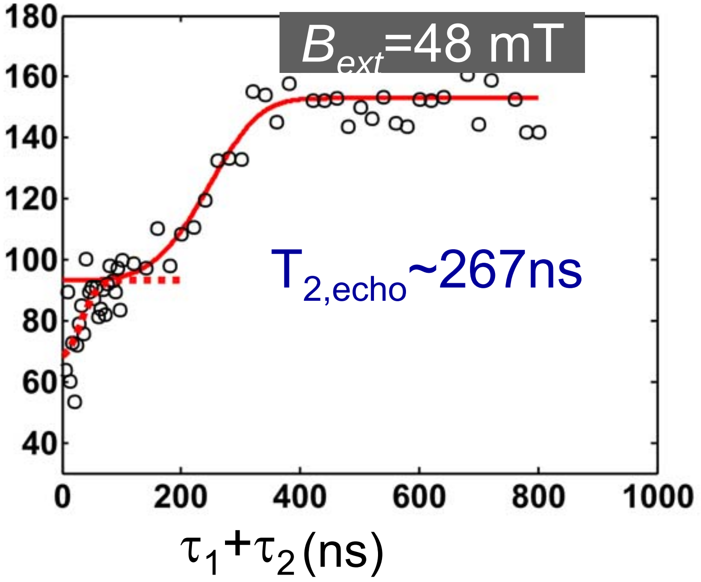


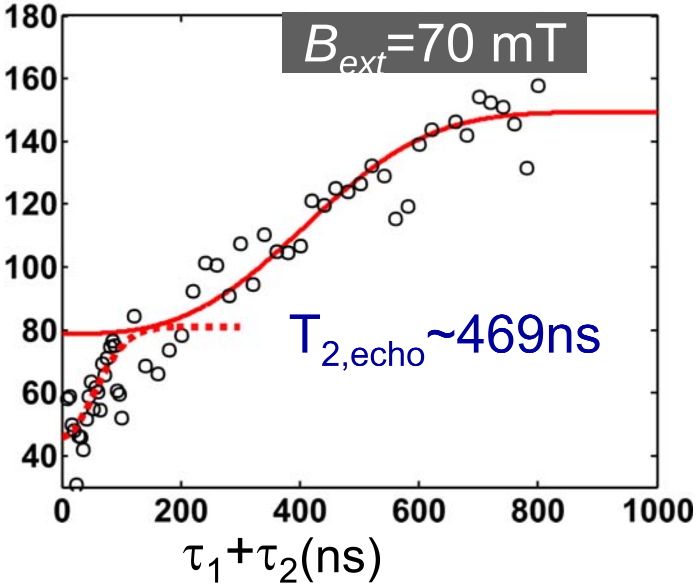
$T_{2,echo}$~248ns

Initial decay due to imperfect echo-pulse

Koppens et al., *PRL* ('08)

$T_{2,echo}$ increases with increasing field

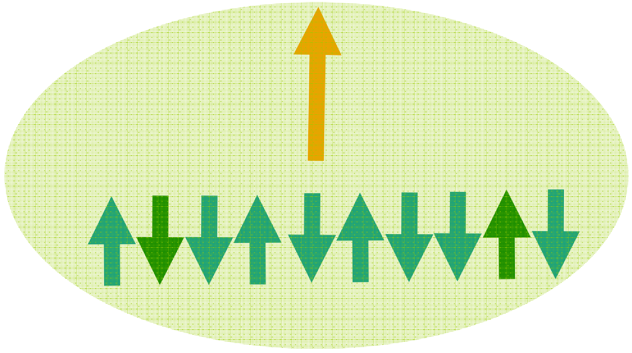




Koppens et al., *PRL* ('08)

Possible mechanism

$$A_i\vec{S}\cdot\vec{I}_i = \overbrace{A_i S_z I_z}^{\text{Overhauser field}} + \overbrace{S_+I_- + S_-I_+}^{\text{Flip-flop}}$$



Virtual state: $g_e\mu_B B » g_n B\mu_N$

Theory:
Coish et al., PRB 70, (2004)
Witzel et al., PRL. 98 (2007)
Yao et al., PRB 74, (2006)

Nuclear spin dynamics

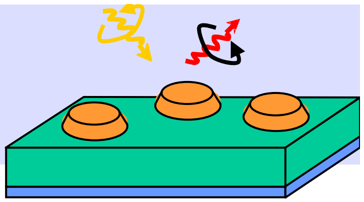
- nuclear-nuclear flip-flops through direct dipole-dipole coupling (> 100 ms)
- electron-nuclear flip-flops (strongly suppressed for B ≠ 0)
- nuclear-nuclear flip-flops through two virtual electron-nuclear flip-flops

Theory:
de Sousa, das Sarma, *PRB* 2003
Coish, Loss *PRB* 2004
Witzel, de Sousa, das Sarma, *PRB* 2005

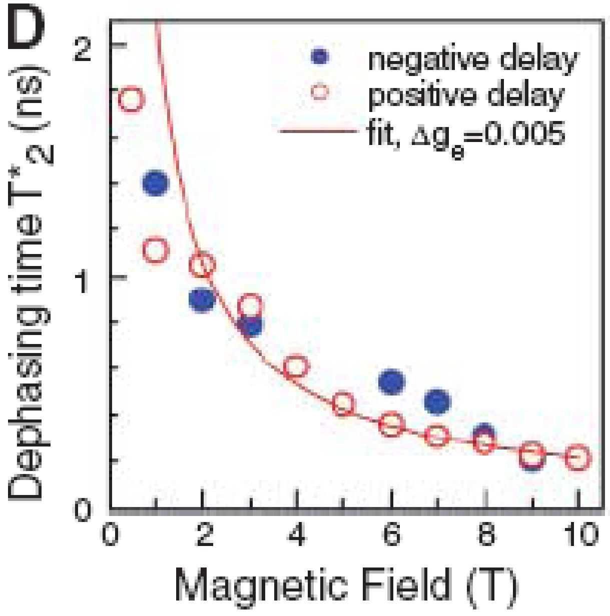
Yao, Liu, Sham, *PRB* 2006
Deng & Hu, *PRB* 2006
…

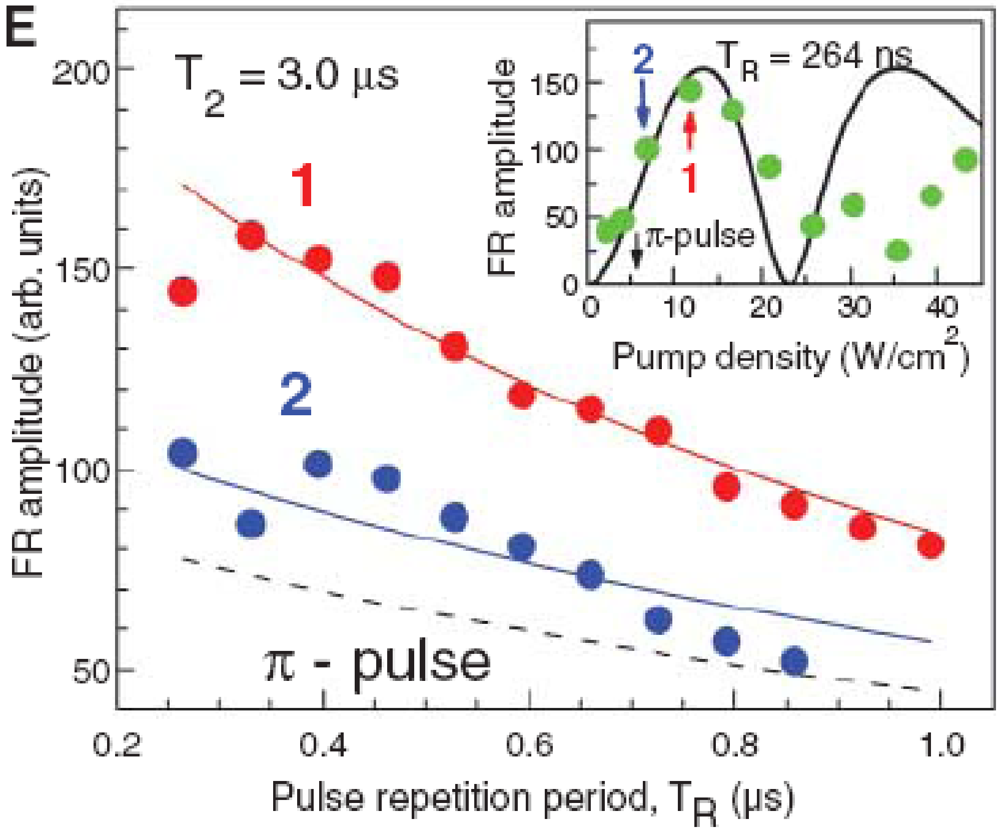
$T_{2,\mathrm{echo}}$ depends on ***timescale*** (t_{nuc}) ***and magnitude*** ($1/T_2^*$) of nuclear field fluctuations

Optical measurement of T_2^* and $T_{2,\mathrm{echo}}$



Ensemble of self-assembled dots, measured via “mode-locking” technique

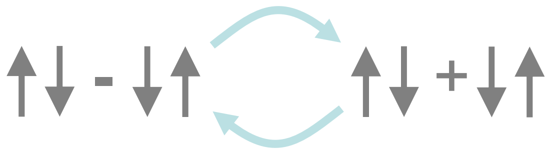


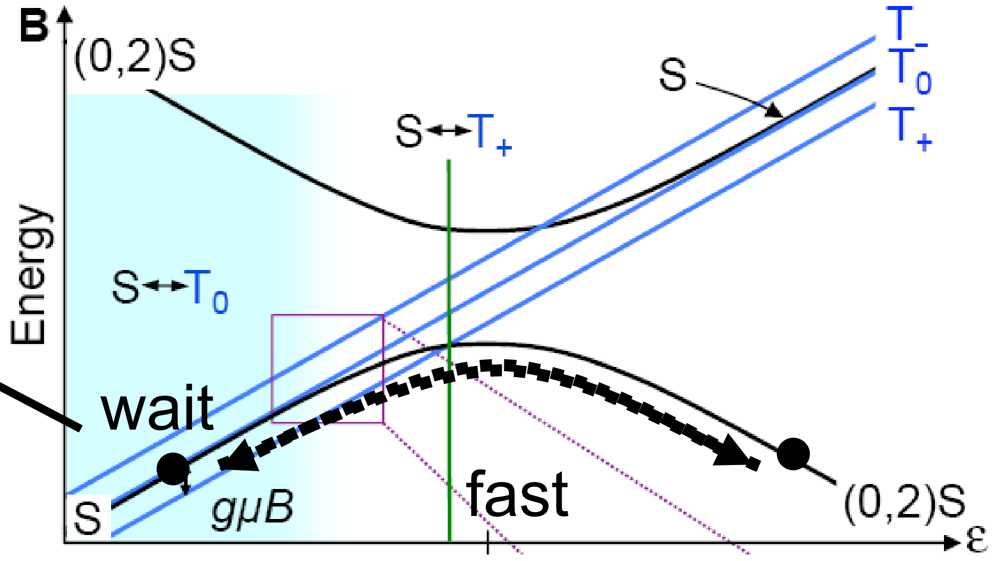


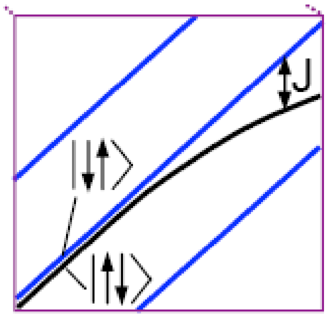
Greilich et al., *Science* (’06)

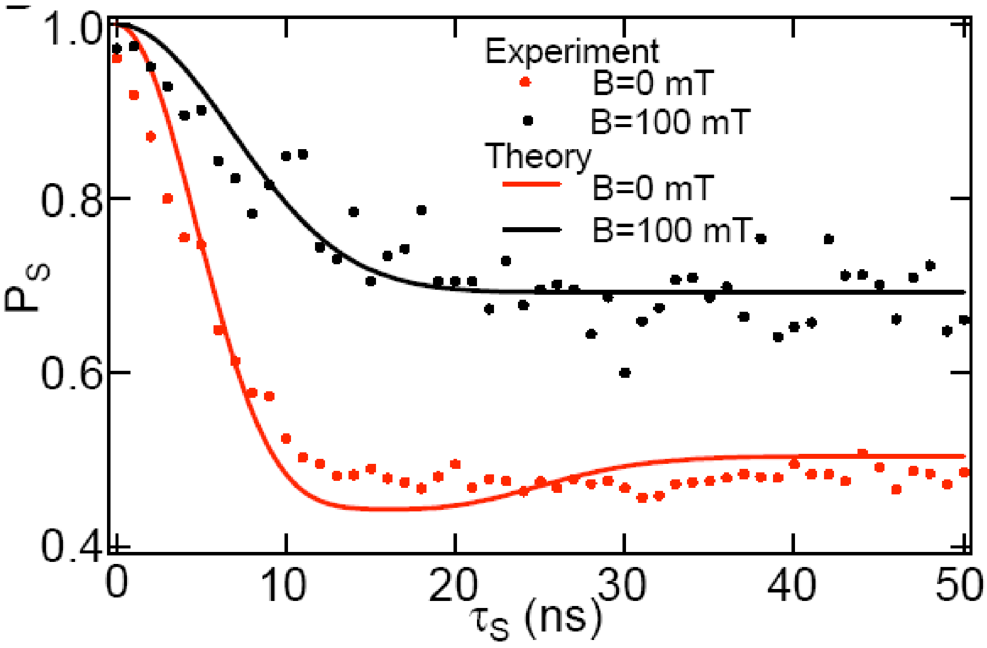
Singlet-triplet T_2^* measurement

singlet dephases to m_z=0 triplet









Petta et al., *Science* ('05)

Spin-echo-type measurement in S-T_0 subspace

↑↓ - ↓↑

dephasing

↑↓ - $e^{i\phi}$ ↓↑

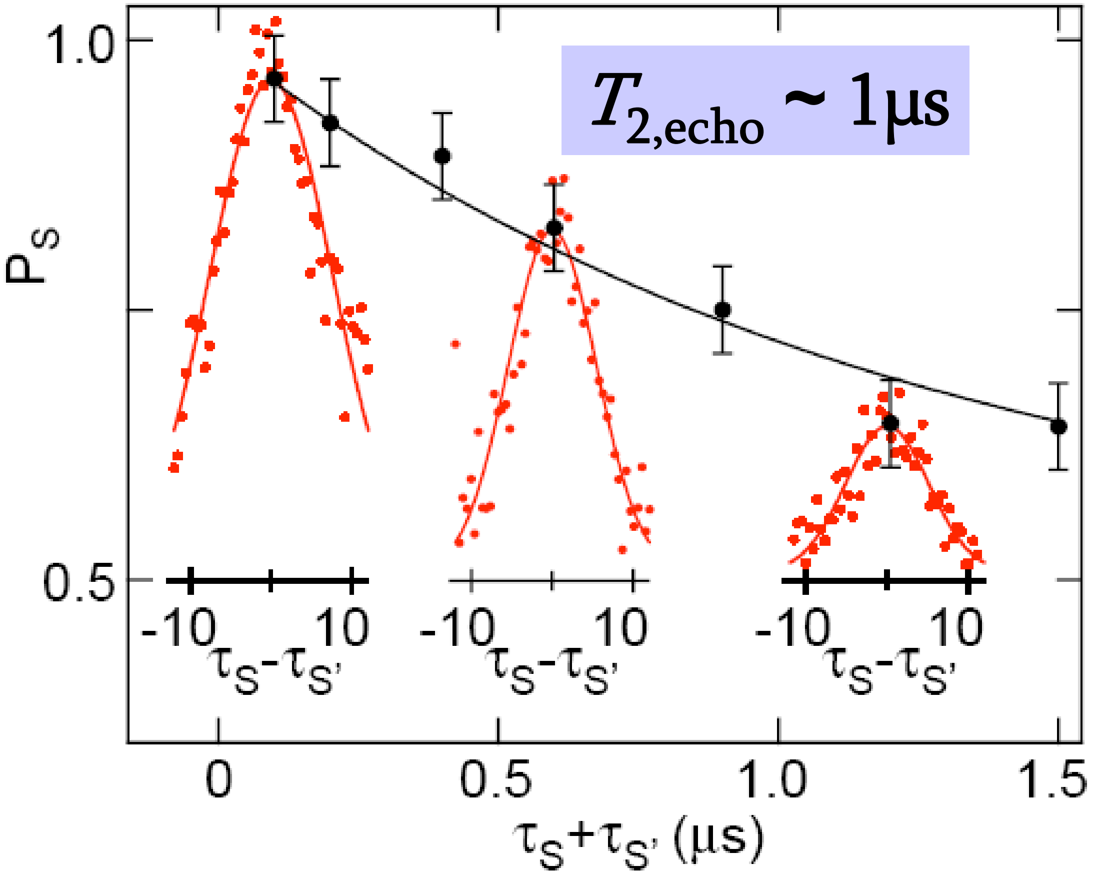
SWAP
("echo"-pulse)

↓↑ - $e^{i\phi}$ ↑↓

same
dephasing

$e^{i\phi}$ ↓↑ - $e^{i\phi}$ ↑↓

~ ↑↓ - ↓↑



Petta et al., *Science* ('05)

Spin qubits in quantum dots – present status

Well-defined qubit states
- *Confine single electrons*

Initialize and read-out
- *Spin to charge conversion*

Initialize in ~ 5 T_1 with 99% fidelity ?

Read-out duration ~ 100 μs; 82-97% fidelity

Energy relaxation
$T_1 \sim 1$ sec

1-qubit gate
- *Electron spin resonance*

gate duration ~ 25 ns; observed 8 periods

2- qubit gate
- *Exchange interaction*

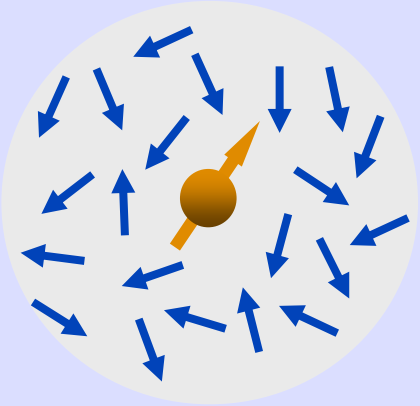
gate duration ~ 0.2 ns; observed 3 periods

Phase coherence
$T_2^* \sim 20$ ns
$T_2 > 1$ ms

See also Hanson et al., *RMP* (‘07)

Main themes in the coming years

Controlling the nuclear spin bath



Nuclear spin bath dynamics

Electron spin acts back on nuclei (flip-flop)
Nuclear polarization can build up

Observations:

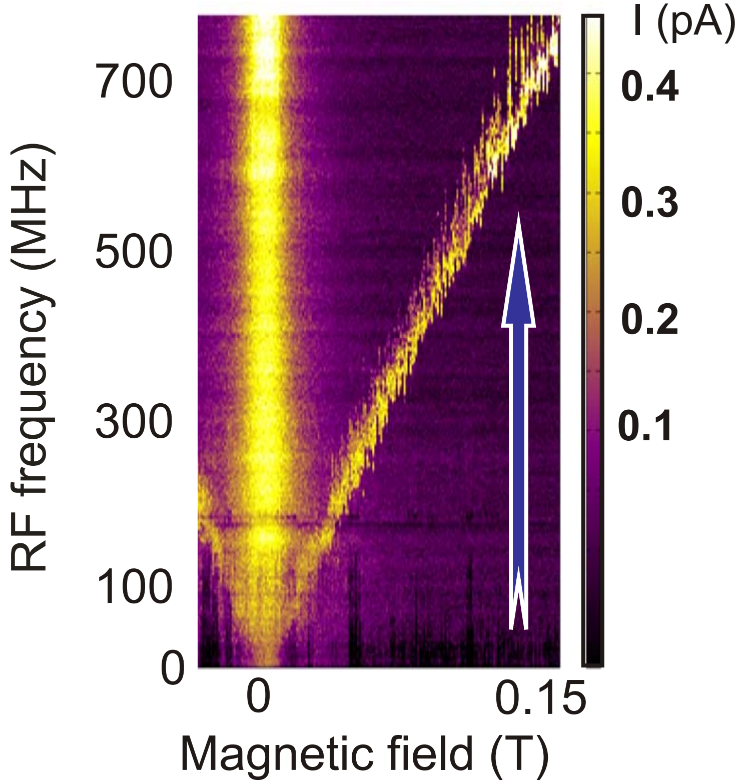
- current hysteretic in magnetic field
- current oscillations

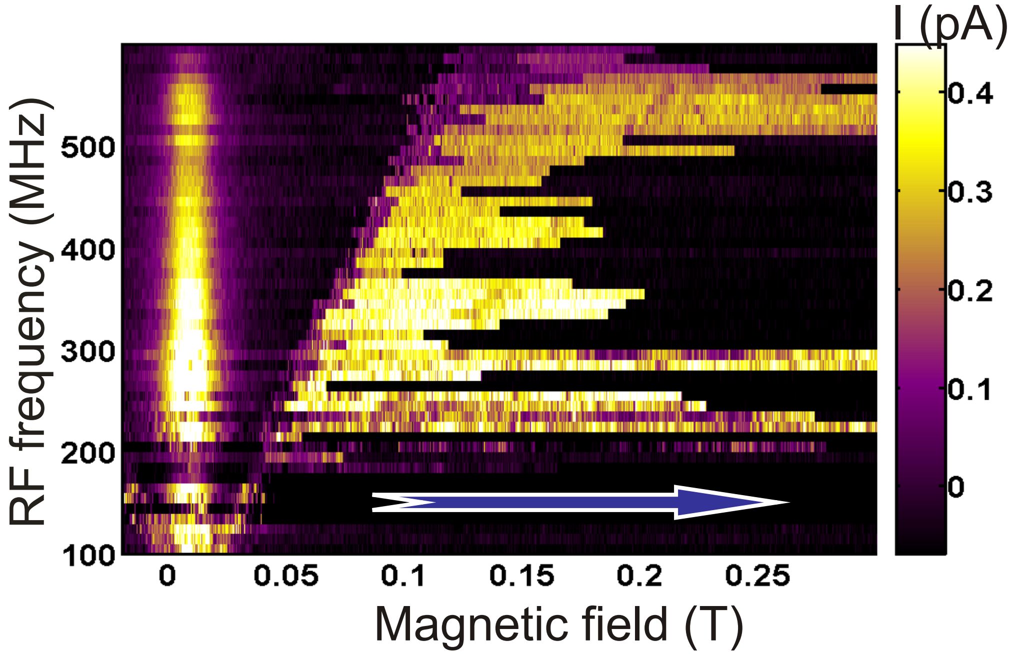
  Ono & Tarucha, PRL 2004
- current fluctuations

  Koppens, Folk, *et al*, Science 2005

Dynamic nuclear spin polarization

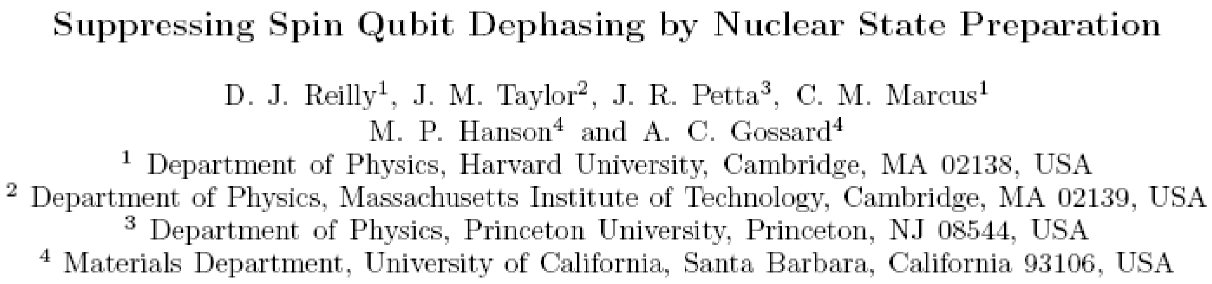
Electron spins act back on nuclear spins





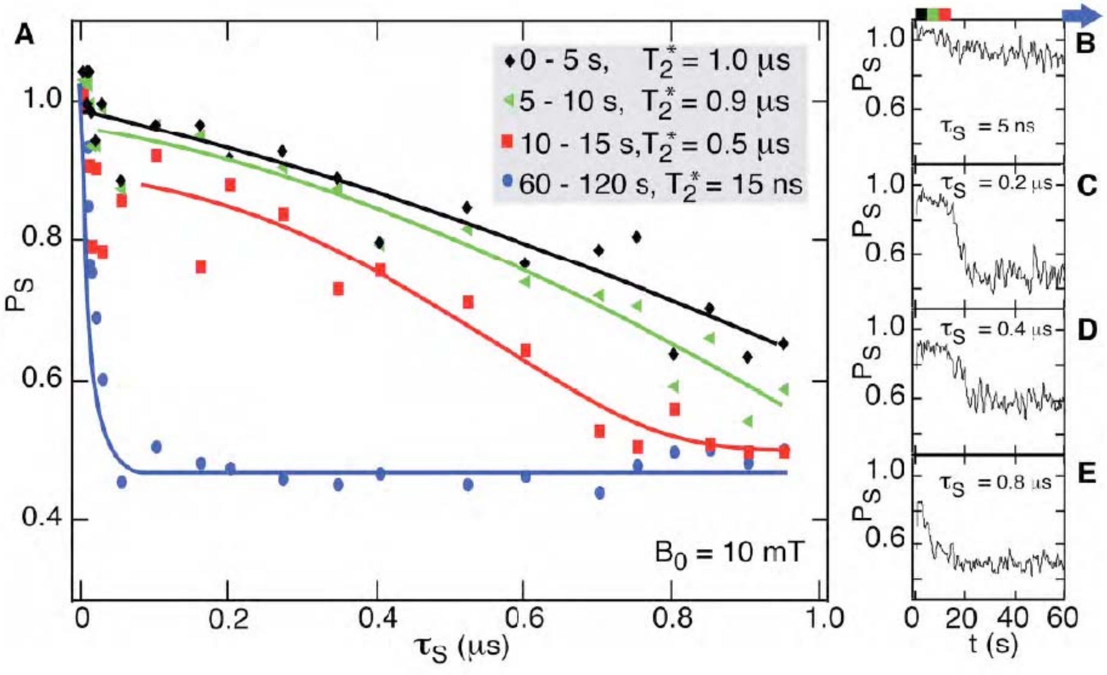
ESR resonance condition: $hf = g\mu_B(B_N + B_{ext})$

Getting grip on the nuclear spin bath



Suppressing Spin Qubit Dephasing by Nuclear State Preparation

D. J. Reilly[1], J. M. Taylor[2], J. R. Petta[3], C. M. Marcus[1]
M. P. Hanson[4] and A. C. Gossard[4]
[1] Department of Physics, Harvard University, Cambridge, MA 02138, USA
[2] Department of Physics, Massachusetts Institute of Technology, Cambridge, MA 02139, USA
[3] Department of Physics, Princeton University, Princeton, NJ 08544, USA
[4] Materials Department, University of California, Santa Barbara, California 93106, USA

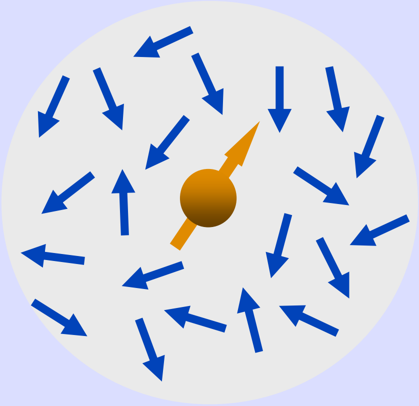


Singlet-triplet T_2^*
enhanced by factor
of ~ 70!

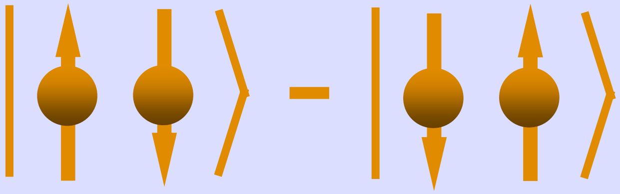
Reilly et al., *Science* ('08)

Main themes in the coming years

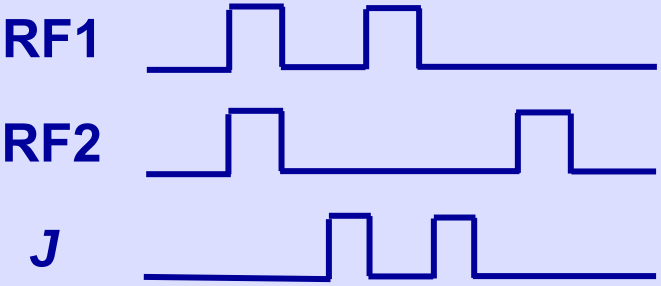
Controlling the nuclear spin bath



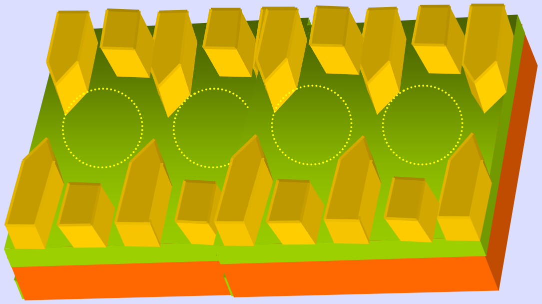
Bell's inequalities



Quantum control of electron spins



Integration and scaling



People and collaborations

"Delft Spin Qubit Team"

Jeroen Elzerman (ETH)
Ronald Hanson (Delft)
Laurens Willems v Beveren (UNSW)
Josh Folk (UBC)
Frank Koppens (Harvard)
Ivo Vink
Christo Buizert
Klaas-Jan Tielrooij (AMOLF)
Tristan Meunier(Grenoble)
Katja Nowack
Lars Schreiber
Floris Braakman
Victor Calado
Tjitte Nooitgedacht
Han Keijzers
Leo Kouwenhoven
Lieven Vandersypen

External collaborations

Loss group (Basel)
Nazarov (Delft)
Rudner & Levitov (MIT)
Wegscheider (Regensburg)
Tarucha group (Tokyo)