Cavity Quantum Electrodynamics with Superconducting Circuits

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Outline

- Cavity Quantum Electrodynamics
- Quantum Electrical Circuits
 - Harmonic Oscillators
 - Qubits
- Circuit Quantum Electrodynamics
 - The Basics
 - Resonant and Dispersive Circuit QED Experiments
- Quantum Information Processing
 - Single Qubit Control and Read-Out in Circuit QED
 - Quantum Geometric Phases
 - Two-Qubit Gates

Cavity Quantum Electrodynamics



Jaynes-Cummings Hamiltonian

$$H = \hbar\omega_r \left(a^{\dagger}a + \frac{1}{2}\right) + \frac{\hbar\omega_a}{2}\sigma^z + \hbar g(a^{\dagger}\sigma^- + a\sigma^+) + H_{\kappa} + H_{\gamma}$$

strong coupling limit $(g = dE_0/\hbar > \gamma, \kappa, 1/t_{\text{transit}})$

Eidgenössische Technische Hochschule Zürich Swiss Federal Institute of Technology Zurich D. Walls, G. Milburn, Quantum Optics (Spinger-Verlag, Berlin, 1994)

Dressed States Energy Level Diagram

$$H = \hbar \omega_r \left(a^{\dagger} a + \frac{1}{2} \right) + \frac{\hbar \omega_a}{2} \sigma^z + \hbar g (a^{\dagger} \sigma^- + a \sigma^+)$$

i i i
e:

$$- \omega_r = \Delta = 0$$

$$|2\rangle - \frac{2g\sqrt{2}}{2} - |1\rangle$$

$$|2\rangle - \frac{2g}{2g\sqrt{2}} - |1\rangle$$
bling limit:

$$|0\rangle - \frac{dE_0}{2g} - \frac{1}{2g} - \frac{1}{2g} - \frac{1}{2g}$$

Jaynes-Cummings Ladder

Atomic cavity quantum electrodynamics reviews: H. Mabuchi, A. C. Doherty Science 298, 1372 (2002) J. M. Raimond, M. Brune, & S. Haroche *Rev. Mod. Phys.* 73, 565 (2001)

in resonance:

$$\omega_a - \omega_r = \Delta = 0$$

strong coupli

$$g = \frac{dE_0}{\hbar} > \gamma, \ \kappa$$



Vacuum Rabi Oscillations with Rydberg Atoms





20

60

է (μs)

60

100

۵

Review: J. M. Raimond, M. Brune, and S. Haroche *Rev. Mod. Phys.* 73, 565 (2001)
P. Hyafil, ..., J. M. Raimond, and S. Haroche, *Phys. Rev. Lett.* 93, 103001 (2004)

Vacuum Rabi Mode Splitting with Alkali Atoms





R. J. Thompson, G. Rempe, & H. J. Kimble, *Phys. Rev. Lett.* 68 1132 (1992)
A. Boca, ..., J. McKeever, & H. J. Kimble *Phys. Rev. Lett.* 93, 233603 (2004)



Quantum Electronic Circuits: Artificial Atoms and Photons on a Chip





The Quantum Electronic Circuit Toolkit



Electrical Harmonic Oscillators





Typical parameters for microfabricated LC: $L \sim 0.1 \text{ nH}$ $C \sim 1 \text{ pF} \rightarrow \omega_0/2\pi \sim 15 \text{ GHz}$ $1 \text{ GHz} \sim 50 \text{ mK}$

M. Devoret.

Problem #1: Linear

Eidgenössische Technische Hochschule Zürich Swiss Federal Institute of Technology Zurich Quantum fluctuations in electrical circuits, Elsevier Science (1997)

Harmonic Oscillator: A Linear Many-Level System



Electrical Harmonic Oscillators: Dissipation



Problem #2: Avoid internal and external dissipation



Artificial Atom Toolkit



Josephson Junctions ...

... superconducting non-linear elements:



 $\mathbf{E}_{\mathbf{F}} = \mathbf{E}_{\mathbf{F}} \mathbf{E}_{\mathbf{F}}$

Josephson energy : (tunneling amplitude) E_J junction capacitance: C_J

nonlinear

dissipation-less

A Superconducting Qubit: The Cooper Pair Box



First theoretically suggested:

Shnirman et al. Phys. Rev. Lett. **79**, 2371 (1997) Bouchiat et al. Physica Scripta **176**, 165 (1998)

First experimental realization:

ETH

Y. Nakamura et al. Nature (London) 398, 786 (1999)

The Cooper Pair Box Hamiltonian (for Theorists)

• generic Hamiltonian for an electrical oscillator

$$\mathbf{H} = \mathbf{H}_{\mathsf{el}} + \mathbf{H}_{\mathsf{J}} = \frac{1}{2C}q^2 + \frac{1}{2L}\Phi^2$$

CPB Hamiltonian

$$\mathbf{I}_{CPB} = \mathbf{H}_{el} + \mathbf{H}_{J} = 4E_{C0}(N - n_g)^2 - E_{J0}\cos\Theta$$

$$E_{C0} = (2e^2)/2C_{\Sigma} \qquad E_J = \Phi_0 I_c/2\pi$$

• pick a basis

F

$$[\Theta, N] = i$$
 and $N = i \frac{\partial}{\partial \theta}$, $\Theta = -i \frac{\partial}{\partial n}$

• in the charge basis

$$\mathbf{H}_{\mathsf{CPB}} = \sum_{n} \left[4E_{\mathsf{C0}}(N - n_g)^2 |n\rangle \langle n| - \frac{E_{\mathsf{J}}}{2} (|n\rangle \langle n + 1| + |n + 1\rangle \langle n|) \right]$$

Many Superconducting Qubits



concepts review: M. H. Devoret, A. Wallraff and J. M. Martinis, condmat/0411172 (2004) realizations review: G. Wendin and V.S. Shumeiko, *cond-mat/0508729* (2005)

Thousandfold increase in dephasing times:

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- First coherent oscillations (NEC, 1999) \longrightarrow T₂ ~ 1 ns
- "Sweet spot" (Saclay, 2002) \longrightarrow T₂ ~ 500 ns

 $T_2 \sim 2000 \text{ ns}$

Transmon (Yale, 2007)

How to do Control: Single-Qubit Gates à la NMR



Problem: Charge (and other types of) Noise



Charge fluctuations:

$$n_g \rightarrow n_g + \delta n_g$$

Golden rule:

$$\gamma_1 = T_1^{-1} \propto S_{n_g}(\omega_{01}) + S_{n_g}(-\omega_{01})$$
$$\gamma_\phi = T_\phi^{-1} \propto S_{n_g}(\omega \to 0)$$

Solutions:

- Suppress relaxation by suppressing noise at qubit frequency (circuit QED)
- Suppress phase randomization with flat energy bands (Transmon)

Solution: Reduce Charge Noise Sensitivity



Charge dispersion decreases more rapidly than anharmonicity:

$$\epsilon_m \equiv E_m(n_g = 1/2) - E_m(n_g = 0) \qquad \qquad \alpha_r(n_g = 1/2) \equiv (E_{12} - E_{01}) / E_{01} \\ \simeq (-1)^m E_C \frac{2^{4m+5}}{m!} \sqrt{\frac{2}{\pi}} \left(\frac{E_J}{2E_C}\right)^{\frac{m}{2} + \frac{3}{4}} e^{-\sqrt{8E_J/E_C}} \qquad \qquad \alpha_r(n_g = 1/2) \equiv (E_{12} - E_{01}) / E_{01} \\ \simeq -(8E_J/E_C)^{-1/2}$$

Predicted long dephasing times: J. Koch et al., PRA 76, 042319 (2007)

Measured long dephasing times: J. A. Schreier et al. PRB 77, 180502 (2008)

Two Versions of the Cooper Pair Box



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M. Goppl, P. Leek (Quantum Device Lab, ETHZ, 2007)

Circuit Quantum Electrodynamics





Cavity QED with Superconducting Circuits



coherent quantum mechanics with individual photons and qubits ...

... in superconducing circuits:

many proposals:

- discrete LC circuits: Y. Makhlin, G. Schön, and A. Shnirman, Rev. Mod. Phys. 73, 357 (2001).
 - O. Buisson and F. Hekking, in *Macroscopic Quantum Coherence and Quantum Computing*, edited by D. V. Averin, B. Ruggiero, and P. Silvestrini (Kluwer, New York, 2001).
 - large junctions:
 - F. Marquardt and C. Bruder, Phys. Rev. B **63**, 054514 (2001).
 - F. Plastina and G. Falci, Phys. Rev. B 67, 224514 (2003).
 - A. Blais, A. Maassen van den Brink, and A. Zagoskin, Phys. Rev. Lett. 90, 127901 (2003).
 - IES: W. Al-Saidi and D. Stroud, Phys. Rev. B 65, 014512 (2001).
 - C.-P. Yang, S.-I. Chu, and S. Han, Phys. Rev. A 67, 042311 (2003).
 - J. Q. You and F. Nori, Phys. Rev. B 68, 064509 (2003).





Cavity QED with Superconducting Circuits



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Circuit Quantum Electrodynamics



elements

- the cavity: a superconducting 1D transmission line resonator with large vacuum field E_0 and long photon life time $1/\kappa$
- the artificial atom: a Cooper pair box with large dipole moment *d* and long coherence time 1/γ



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Vacuum Field in 1D Cavity



cross-section of transm. line (TEM mode):



 $10\,\mu{
m m}$

voltage across resonator in vacuum state (n = 0)

harmonic oscillator

$$H_r = \hbar \omega_r \left(a^{\dagger} a + \frac{1}{2} \right)$$

 $imes 10^6$ larger than E_0 in 3D microwave cavity

for
$$\omega_r/2\pipprox 6\,{
m GHz}$$
 ($C\sim 1\,{
m pF}$), $bpprox 5\,\mu{
m m}$

 $V_{0,\rm rms} = \sqrt{\frac{\hbar\omega_r}{2C}} \approx 1\,\mu\rm{V}$

 $E_0 = \frac{V_{0,\mathrm{rms}}}{h} \approx 0.2 \,\mathrm{V/m}$

Storing Photons and Controlling their Life Time













photon lifetime (quality factor) controlled by coupling capacitor C_{in/out}

Resonator Quality Factor and Photon Lifetime



resonance frequency:

$$\nu_r = 6.04 \,\mathrm{GHz}$$

quality factor:

$$Q = \frac{\nu_r}{\delta\nu_r} \approx 10^4$$

photon decay rate:

$$\frac{\kappa}{2\pi} = \frac{\nu_r}{Q} \approx 0.8 \,\mathrm{MHz}$$

photon lifetime:

$$T_{\kappa} = 1/\kappa \approx 200 \,\mathrm{ns}$$

Energy Levels of a Superconducting Qubit

transmon:



Cg

 $\bigotimes_{\mathbf{B}}$

F

controllable by:

- electric fields E
- magnetic fields B

circuit diagram:

Transmon energy levels vs. E



M long excited state life time $T_1 = 1/\gamma$

B-Field Dependence of Energy Levels

spectroscopic measurement of transition frequency vs. magnetic field B:





J. Koch *et al.*, Phys. Rev. A **76**, 042319 (2007)

Strong Coupling Cavity QED Circuit





Resonant Vacuum Rabi Mode Splitting ...

... with one photon (n = 1):

very strong coupling:



forming a 'molecule' of a qubit and a photon

first demonstration: A. Wallraff, ... and R. J. Schoelkopf, *Nature (London)* **431**, 162 (2004) Eldgenössische Technische Hochschule Zürich Swiss Federal Institute of Technology Zurich

How to Measure Single Microwave Photons

• average power to be detected

 $\rightarrow \langle n=1 \rangle \hbar \omega_r \kappa / 2 \approx P_{RF} = -140 \,\mathrm{dBm} = 10^{-17} \,\mathrm{W}$



- efficient with cryogenic low noise HEMT amplifier ($T_N = 6 \,\mathrm{K}$)
- prevent leakage of thermal photons (cold attenuators and circulators)

Measurement Setup

cold stage

sample mount

20 mK cryostat



Strong Coupling with Superconducting Circuits

circuit quantum electrodynamics (QED) experiments since 2004:



Yale University (now also ETH Zurich) Nature (London) **431**, 162 (2004)

TU Delft. Nature (London) **431**, 159 (2004) *NTT PRL* **96**, 127006 (2006)





NEC Nature (London) **449**, 588 (2007)

Eidgenössische Technische Hochschule Zürich Swiss Federal Institute of Technology Zurich NIST Boulder (now also at UCSB) Nature (London) **449**, 438 (2007)

The Quantum Nonlinearity of the J-C Ladder

Why to probe the nonlinearity?

- classical interpretation of first doublet
- quantum effects
 - scaling of coupling rate $g_{\rm eff}$ with amplitude (square root of photon number n)
- \sqrt{n} -scaling is a pure quantum effect
 - direct evidence for field quantization
 - studied in time-resolved measurements of the atom but until recently not in spectroscopic experiments





 $|n\pm\rangle = (|g,n\rangle \pm |e,n-1\rangle)/\sqrt{2}$

Climbing the Jaynes-Cummings Ladder

How to climb the ladder?

- cool to ground state |g,0
 angle
- Controllably increase # n of excitations
 - thermal population
 - multi photon transitions

pump and probe spectroscopy



 $|n\pm\rangle = (|g,n\rangle \pm |e,n-1\rangle)/\sqrt{2}$

Two-Photon Pump and Probe Spectroscopy



J. Fink, M. Goeppl, M. Baur, R. Bianchetti, P. Leek, A. Blais, A. Wallraff, *Nature (London)* **454**, 315 (2008)

Resonant Vacuum Rabi Mode Splitting ...



• |nangle
ightarrow |n+
angle is weak

J. Fink, M. Goeppl, M. Baur, R. Bianchetti, P. Leek, A. Blais, A. Wallraff, *Nature (London)* **454**, 315 (2008)

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Resonant Vacuum Rabi Mode Splitting ...



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J. Fink, M. Goeppl, M. Baur, R. Bianchetti, P. Leek, A. Blais, A. Wallraff, *Nature (London)* **454**, 315 (2008)

Sqrt(n) Quantum Nonlinearity



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Cavity QED with Multiple Atoms



Two-'Atom' Cavity QED



- two almost identical qubits
- two almost identical coupling constants $g_{\rm A,B}$
- local flux control $\Phi_{\rm A,B}$



Two Qubit Vacuum-Rabi Mode Splitting



- qubit A in resonance with resonantor $(\nu_{ge} = \nu_r)$ $|g_A, 1\rangle \pm |e_A, 0\rangle$
- qubit B tuned into resonance with local flux Φ_B
- two qubit & resonator coupled states

 $|g_A g_B 1\rangle \pm |e_A g_B 0\rangle \pm |g_A e_B 0\rangle$

- multi-qubit splitting scales as $\sqrt{m}\,\bar{g}$, where m is the number of qubits
- and a little bit of a two-photon (n = 2) state

Dispersive Qubit-Photon Interaction

approximate diagonalization in the dispersive limit $|\Delta| = |\omega_a - \omega_r| \gg g$



Eidgenössische Technische Hochschule Zürich Swiss Federal Institute of Technology Zurich A. Blais et al., PRA 69, 062320 (2004)

Qubit Spectroscopy & AC-Stark Effect



D. I. Schuster et al., Phys. Rev. Lett. 94, 123062 (2005)

Photon Number Dependent 'Quantum' Light Shift



weight of spectroscopic peaks relate to probability ${\cal P}(n)$ of photon number states

Measuring Photon Number Statistics



distinguish between coherent and thermal states



Schuster, Houck, Schreier, Wallraff, Gambetta, Blais, Frunzio, Johnson, Devoret, Girvin, Schoelkopf, *Nature* **445**, 515 (2007)

Qubit Control and Time Resolved Measurements

Rabi Oscillations, Ramsey Fringes, Tomography ...



Qubit Control and Readout



decay

measurement properties:

- continuous
- dispersive
- quantum non-demolition
- in good agreement with predictions

Wallraff, Schuster, Blais, ... Girvin, and Schoelkopf, *Phys. Rev. Lett.* **95**, 060501 (2005)

Rabi Oscillations (weak cont. measurement)





- high control fidelity
- high read-out fidelity
- good understanding of field-qubit interaction

High Fidelity Control & Read Out



• high visibility $95 \pm 5\%$

detailed understanding of qubit/read-out interaction

Quantum State Tomography



Single Qubit Coherence: Ramsey Fringes



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A. Wallraff et al., Phys. Rev. Lett. 95, 060501 (2005)

Spin Echo Experiment

pulse scheme:



result:





• refocusing

- elimination of low frequency fluctuations
- increased effective coherence time

Lars Steffen et al. (2007)

Circuit QED and Quantum Optics



with large vacuum fields long photon lifetimes artificial atom with large dipole moment long coherence time

Quantum Computation with Circuit QED



The ETH Zurich Quantum Device Lab

with funding from:

Eldgenössische Technische Hochschule Zürich Swiss Federal Institute of Technology Zurich

and the second

Schweizerischer Nationalfonds zur Förderung der wissenschaftlichen Forschung Fonds national suisse de la recherche scientifique Swiss National Science Foundation Fondo nazionale svizzero per la ricerca scientifica



lab started in April 2006

The Yale Circuit QED Team



David Schuster Alexandre Blais Rob Schoelkopf Steve Girvin Andreas Wallraff and also: Jay Gambetta Andrew Houck Joseph Schreier Blake Johnson Jerry Chow Hannes Majer Luigi Frunzio Michel Devoret

with funding from:











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Circuit QED Publications

circuit QED proposal:

- Blais, Huang, Wallraff, Girvin, Schoelkopf, PRA 69, 062320 (2004)
- strong coupling & vacuum Rabi mode splitting:
- Wallraff, Schuster, Blais, Frunzio, Huang, Majer, Kumar, Girvin, Schoelkopf, *Nature* **431**, 162 (2004)
- Fink, Goeppl, Baur, Bianchetti, Leek, Blais, Wallraff, *Nature* **454**, 315 (2008)
- high visibility Rabi oscillations & coherence time measurements:
- Wallraff, Schuster, Blais, Frunzio, Majer, Girvin, and Schoelkopf, *PRL* **95**, 060501 (2005)
- ac Stark shift, number splitting & measurement induced dephasing:
- Schuster, Wallraff, Blais, Frunzio, Huang, Majer, Girvin, Schoelkopf, PRL 94, 123062 (2005)
- Gambetta, Blais, Schuster, Wallraff, Frunzio, Majer, Devoret, Girvin, Schoelkopf, PRA 74, 042318 (2006)
- Schuster, Houck, Schreier, Wallraff, Gambetta, Blais, Frunzio, Johnson, Devoret, Girvin, Schoelkopf, *Nature* **445**, 515 (2007)

circuit QED gates, side band transitions:

- Blais, Gambetta, Wallraff, Schuster, Devoret, Girvin, Schoelkopf, PRA 75, 032329 (2007)
- Wallraff, Schuster, Blais, Gambetta, ..., Frunzio, Devoret, Girvin, Schoelkopf, PRL 99, 050501 (2007)
- Majer, Chow, Gambetta, Koch, Johnson, Schreier, Frunzio, Schuster, Houck, Wallraff, Blais, Devoret, Girvin, Schoelkopf, *Nature* **449**, 443 (2007)
- Leek, Fink, Blais, Bianchetti, Goeppl, Gambetta, Schuster, Frunzio, Schoelkopf, Wallraff, Science 318, 1889 (2007)

circuit QED device fabrication:

• Frunzio, Wallraff, Schuster, Majer, Schoelkopf, IEEE Trans. Appl. Supercond. 15, 860 (2005)

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