Reconstructing quantum states efficiently

M. Cramer
on work with
T. Baumgratz and M.B. Plenio

Institut für Theoretische Physik, Universität Ulm

- Full quantum state tomography is essential for present and future quantum devices

Quantum simulator: Preparation of elaborate states and operations on them

- Has the intended state indeed been prepared?
- Do the operations do what they are supposed to?

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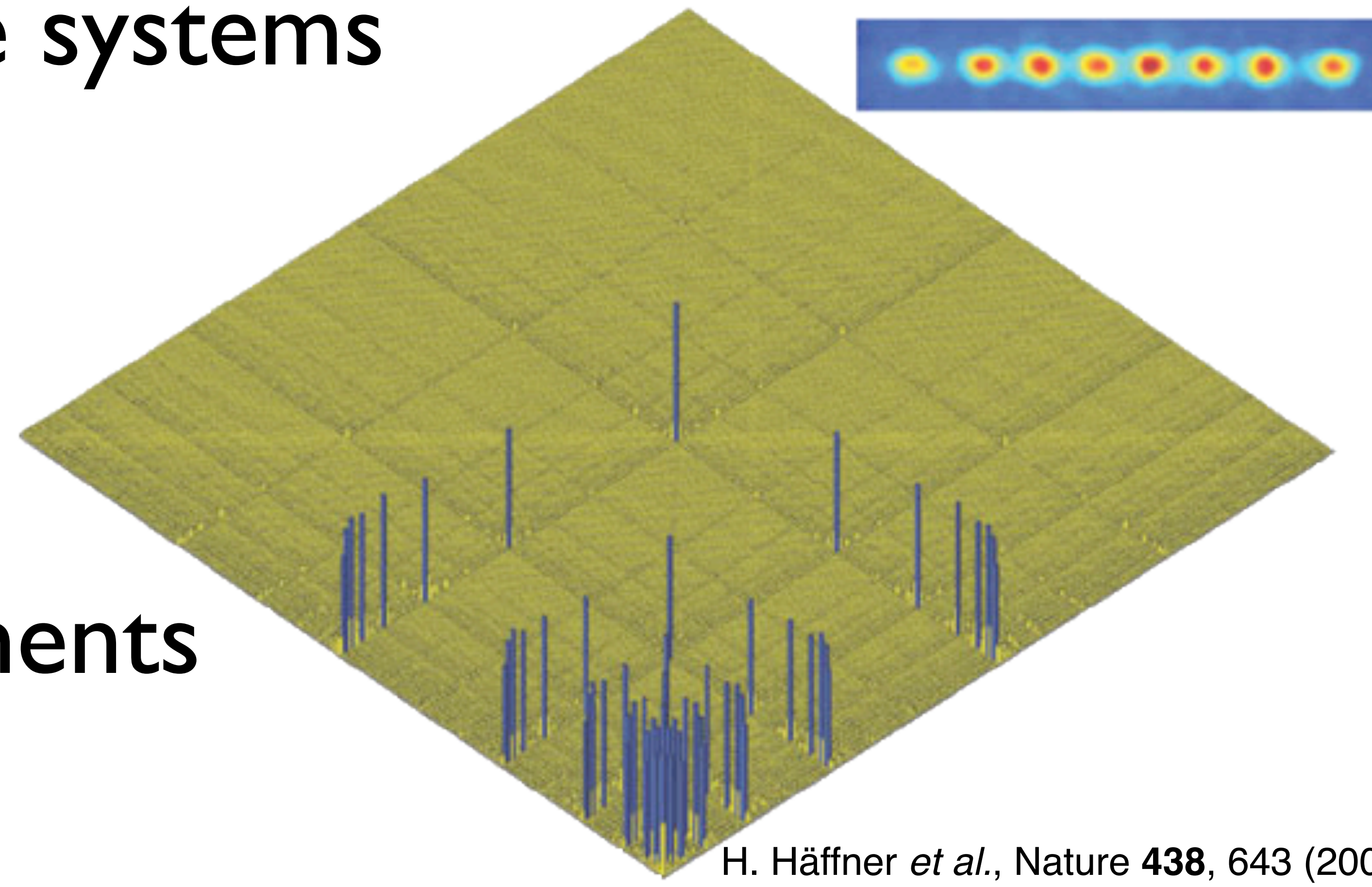

Measure complete basis $\hat{P}_k = \bigotimes_{i=1}^{N} \hat{\sigma}_i^{\alpha_i}$

→ 4^N measurements

H. Häffner *et al.*, Nature **438**, 643 (2005).

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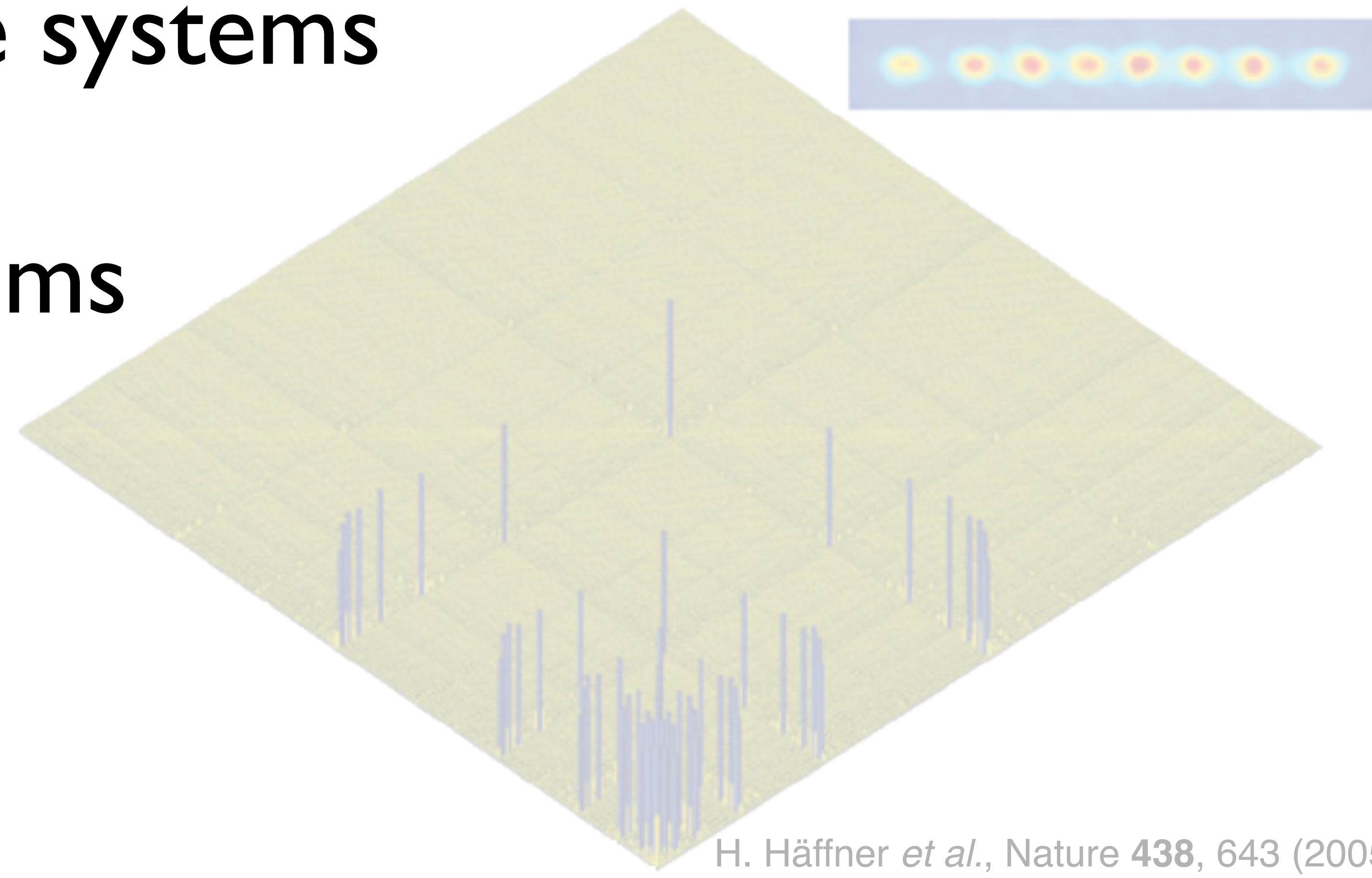

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Quantum state tomography: The problem

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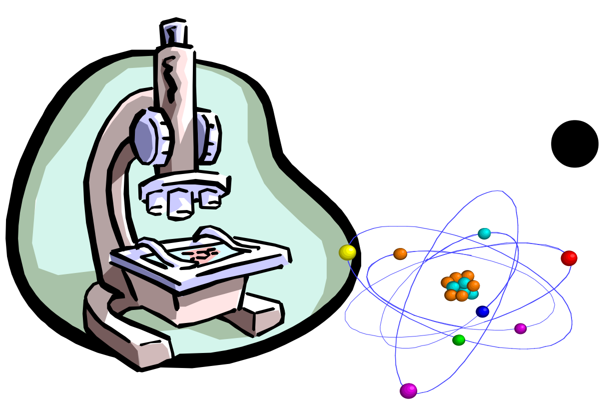
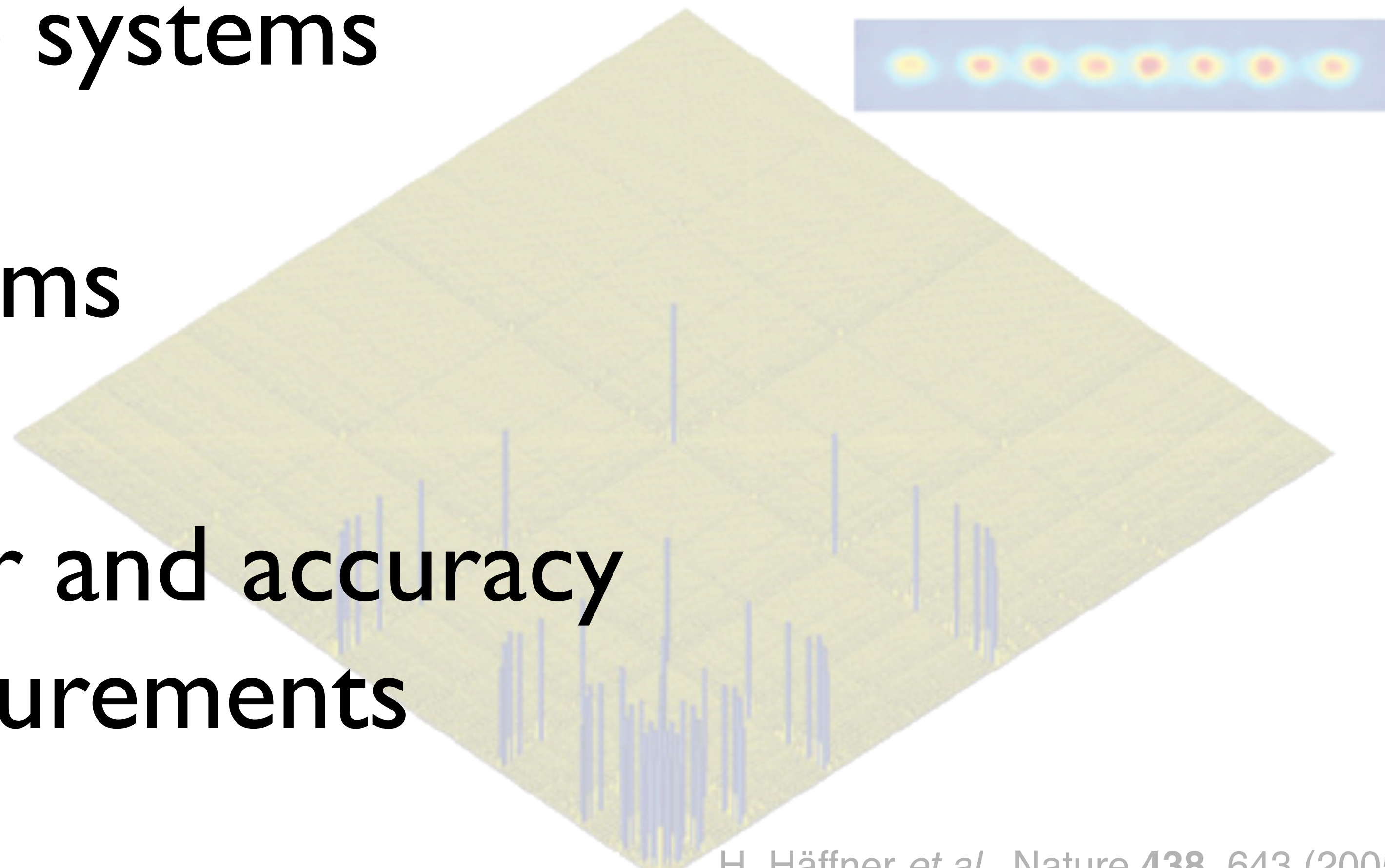


- Number and accuracy of measurements



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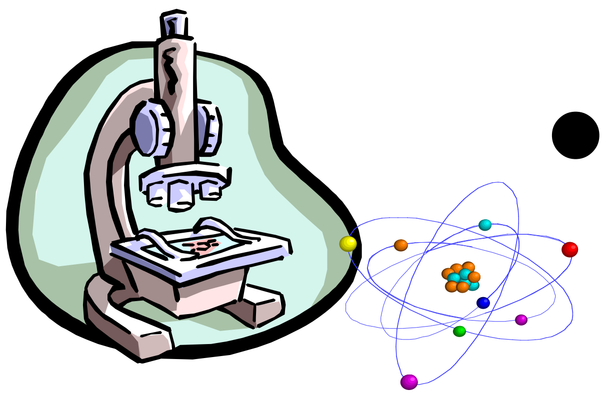
- Find compatible state
- Storage space



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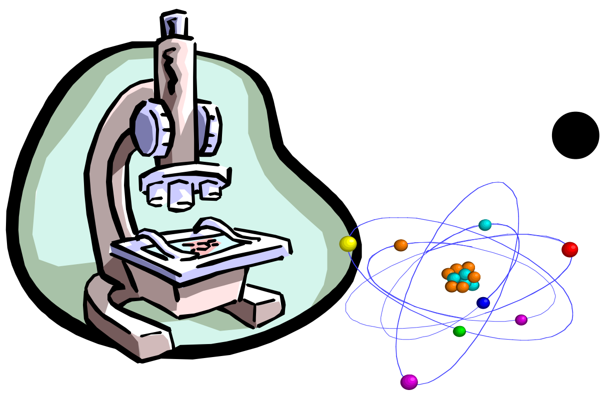
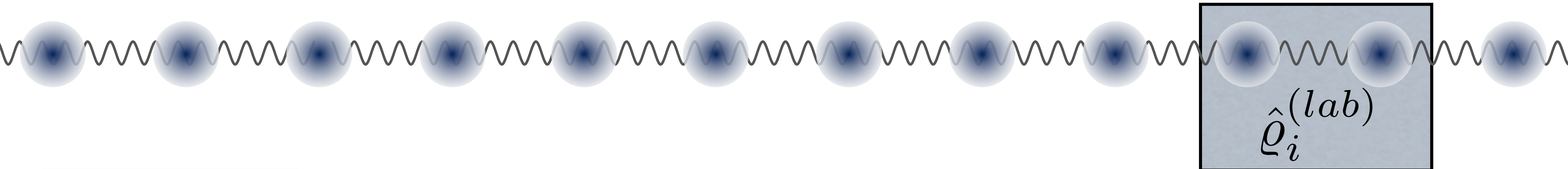


- Number and accuracy of measurements → Take only $\sim N$



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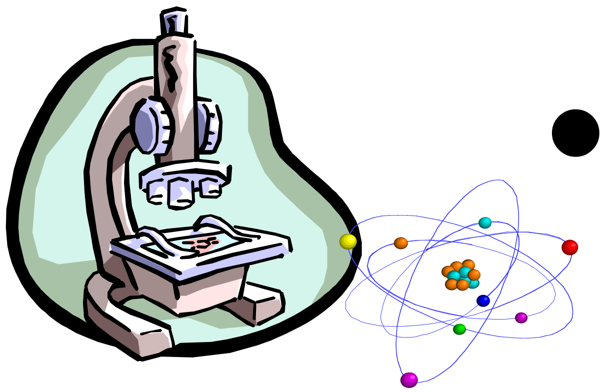
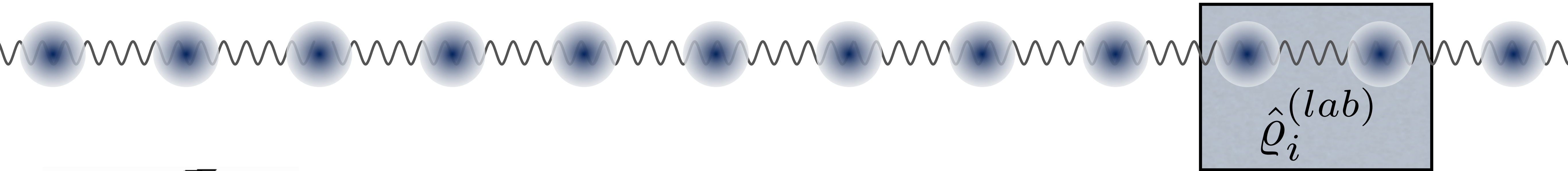


- Number and accuracy of measurements → Take only $4^2 N$



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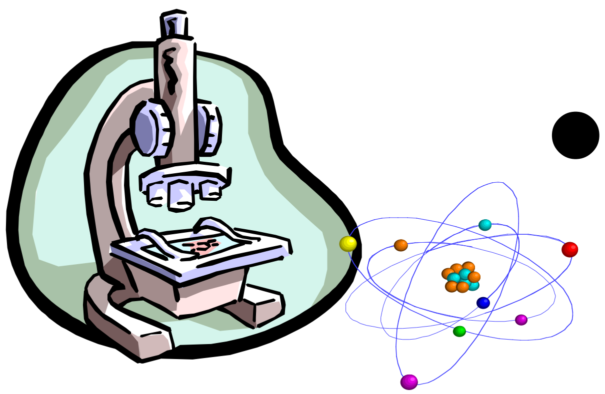
Quantum state tomography: A solution?



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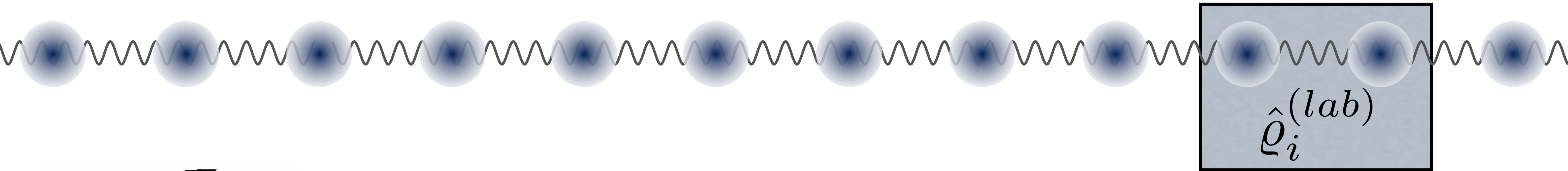

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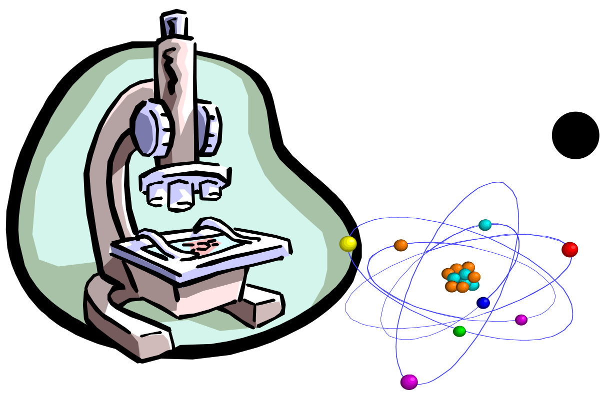

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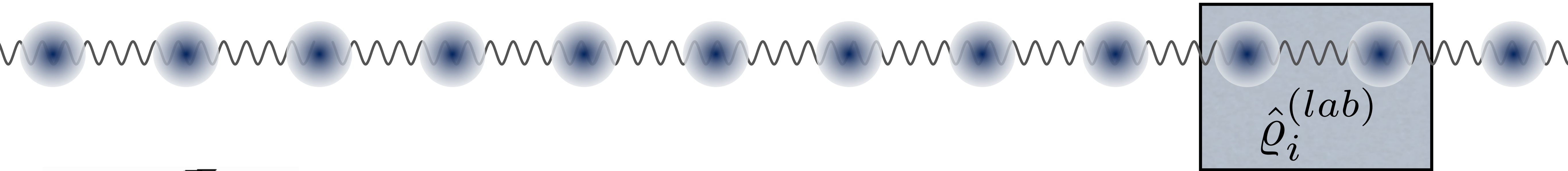

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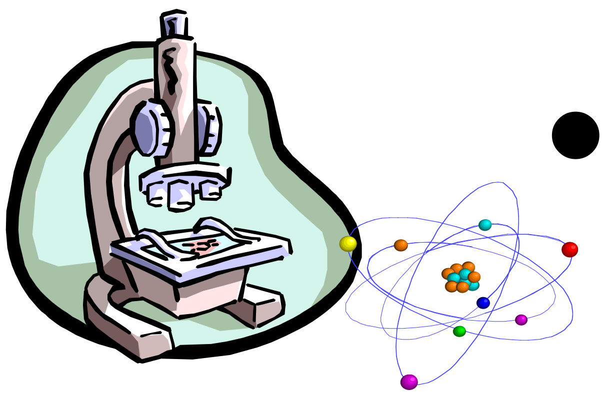

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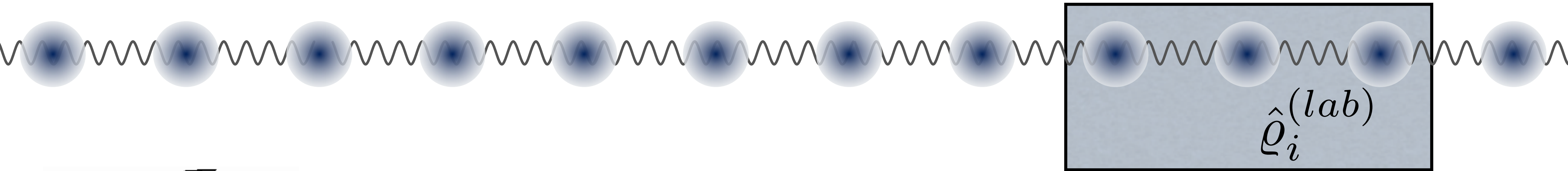

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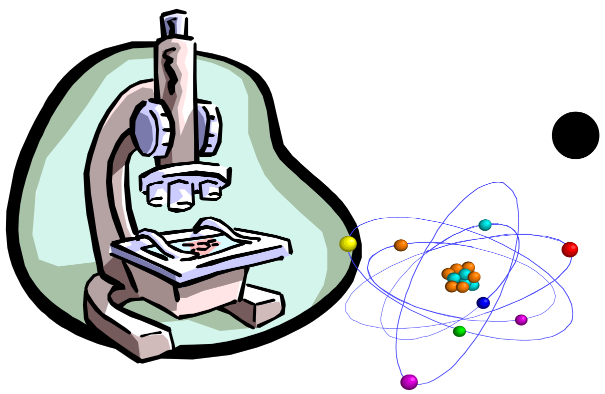

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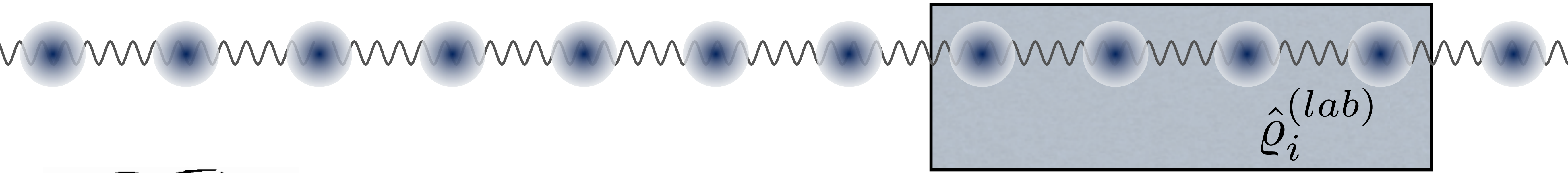

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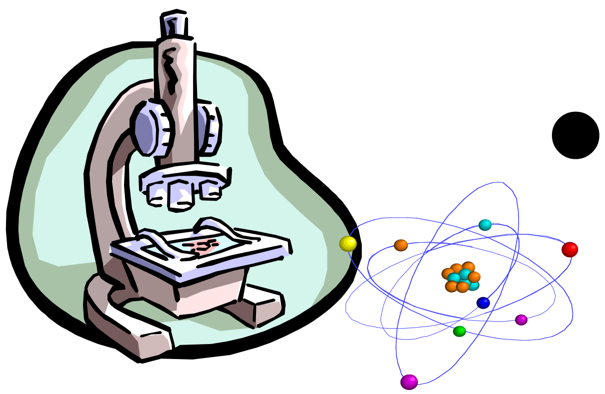

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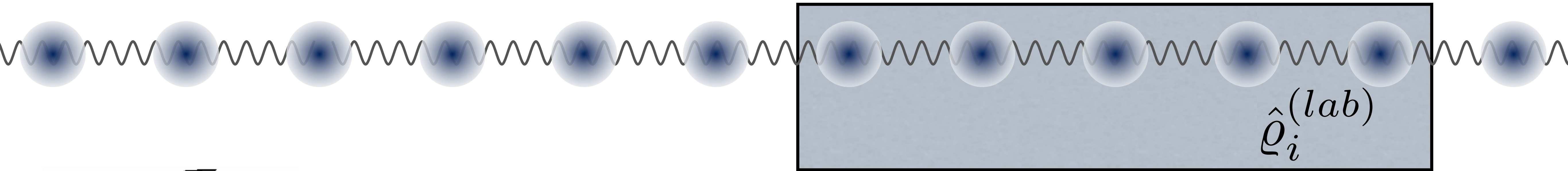

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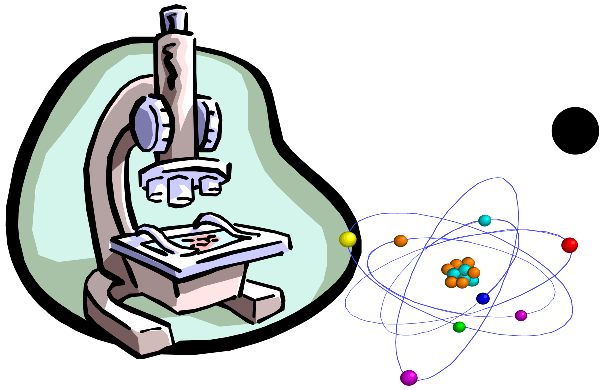

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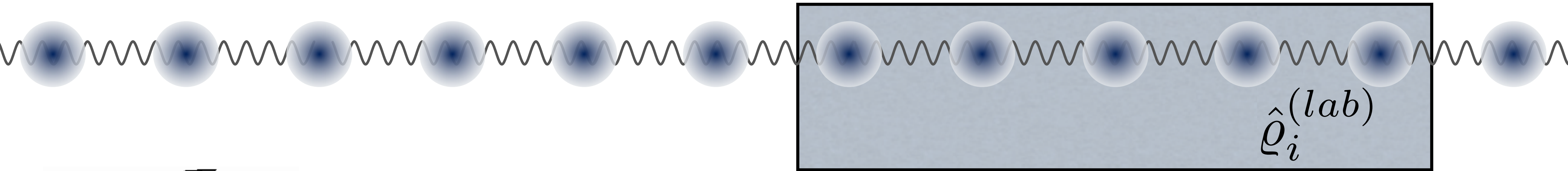
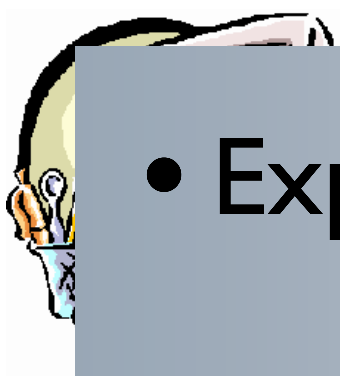
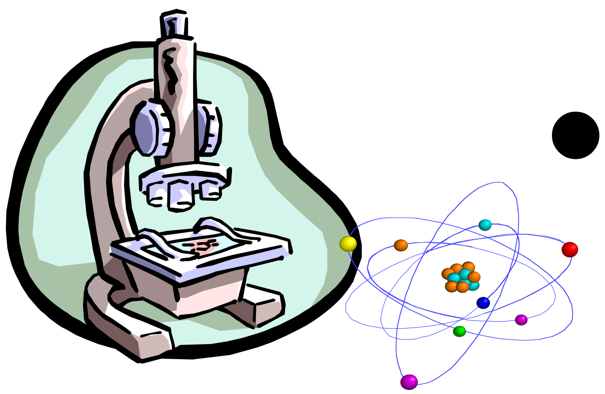




- Experiment: $\|\hat{\varrho}_i^{(lab)} - \hat{\varrho}_i^{(est)}\| \leq \epsilon_i$

S.T. Flammia, D. Gross, S.D. Bartlett, R. Somma, arXiv:1002.3839



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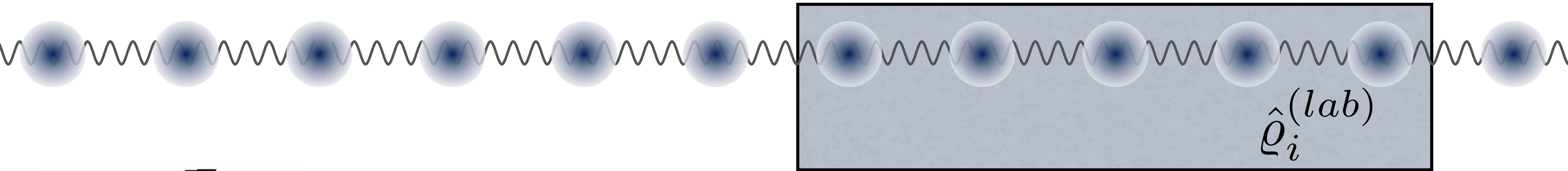

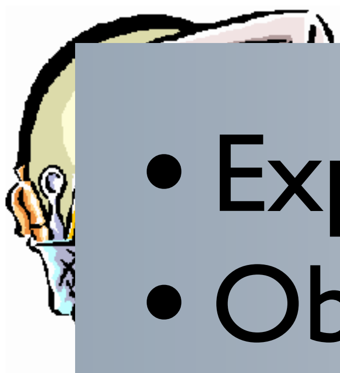

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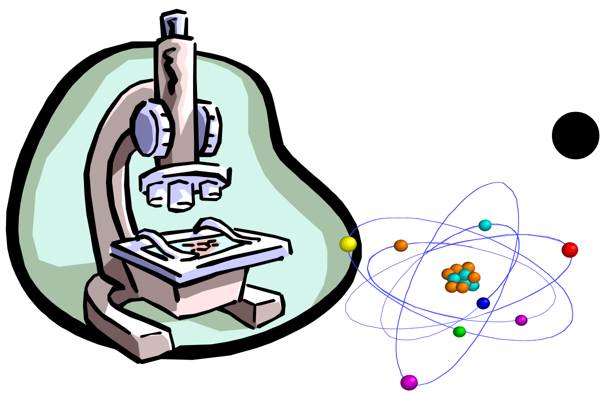

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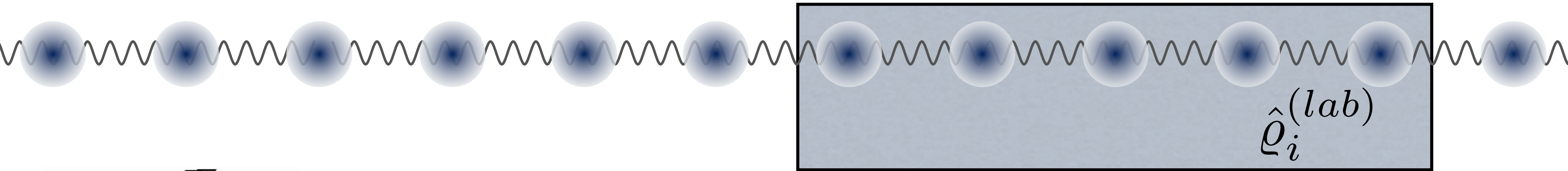

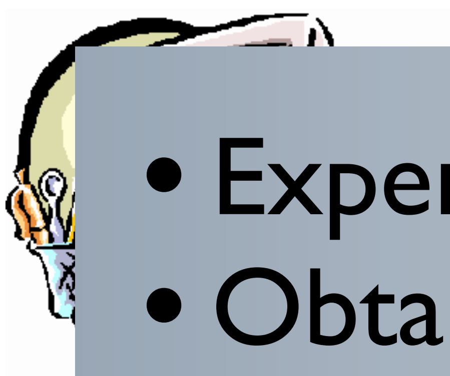

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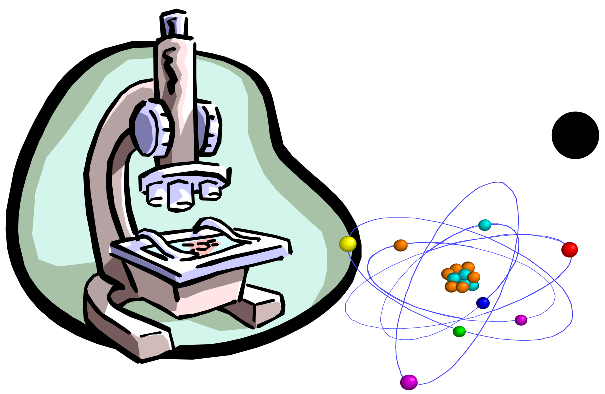

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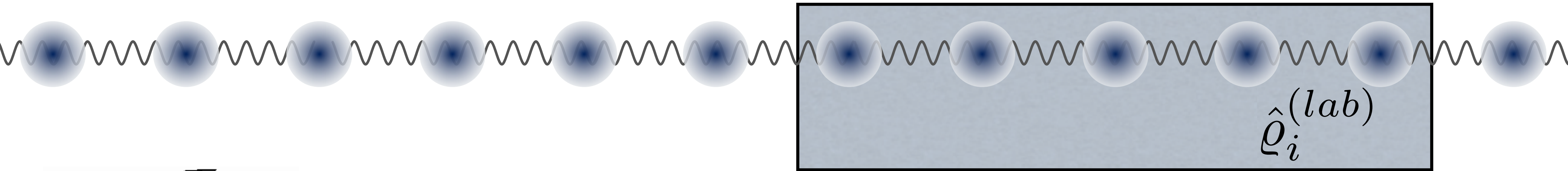

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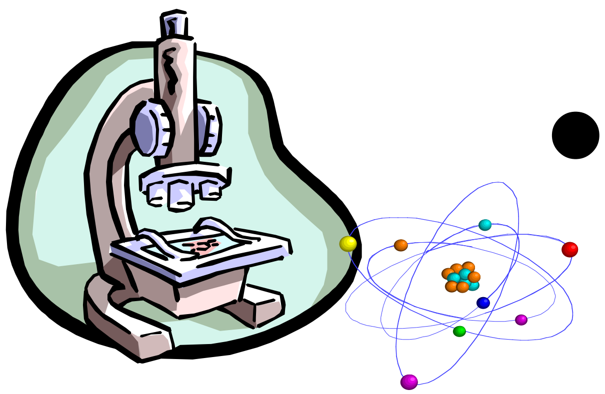

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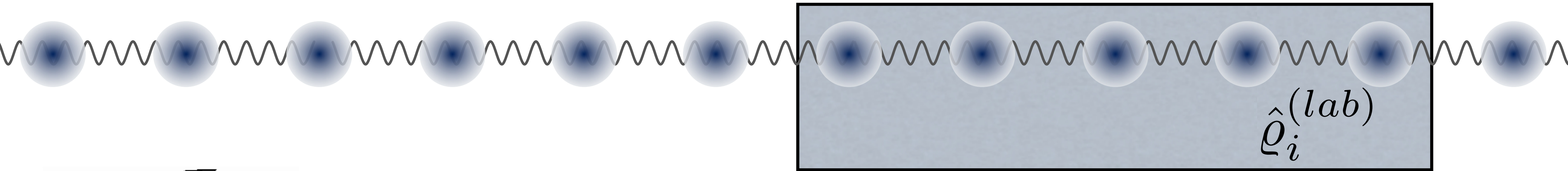

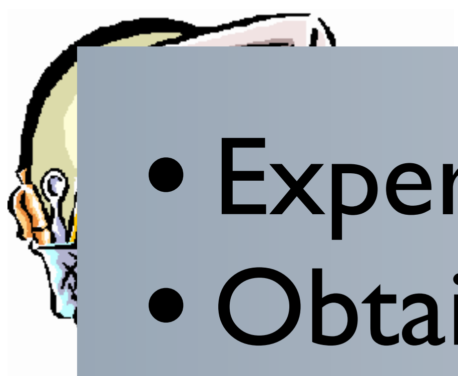

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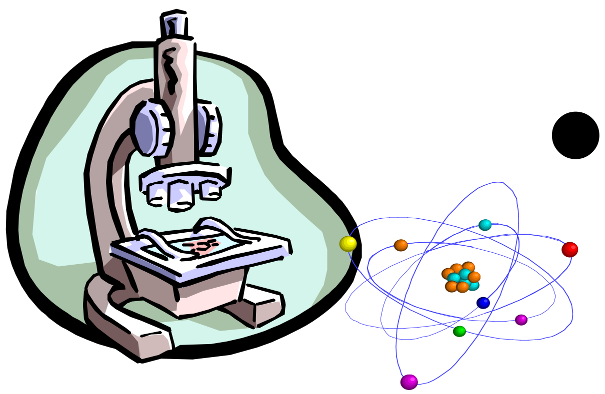

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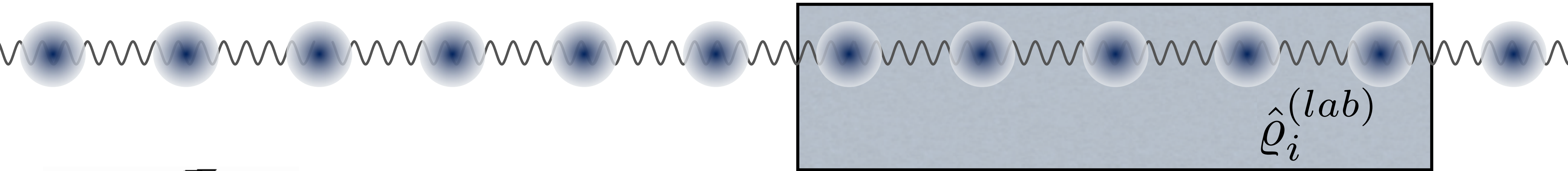

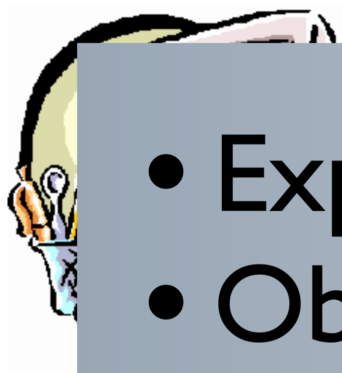

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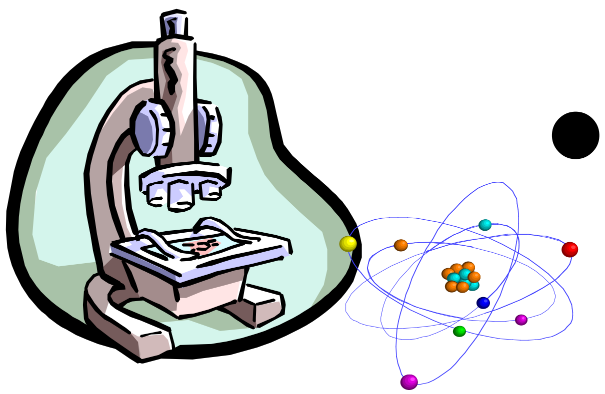

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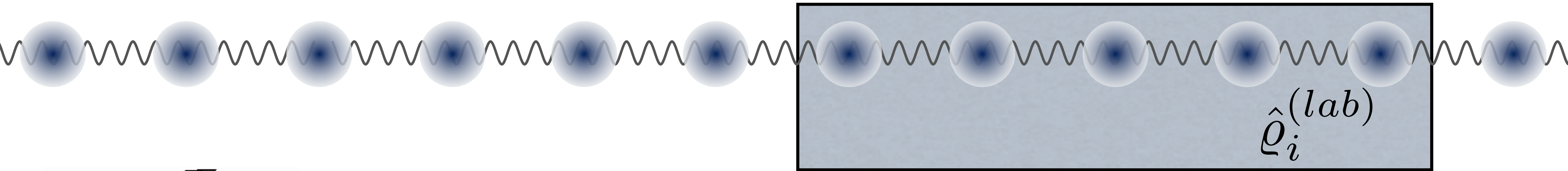

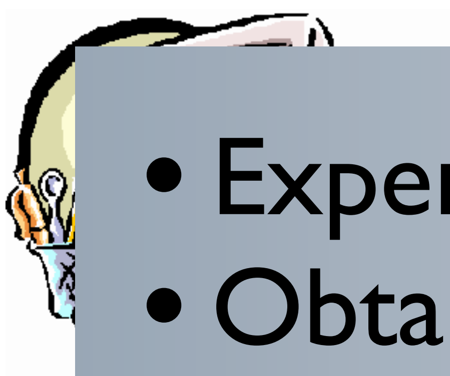

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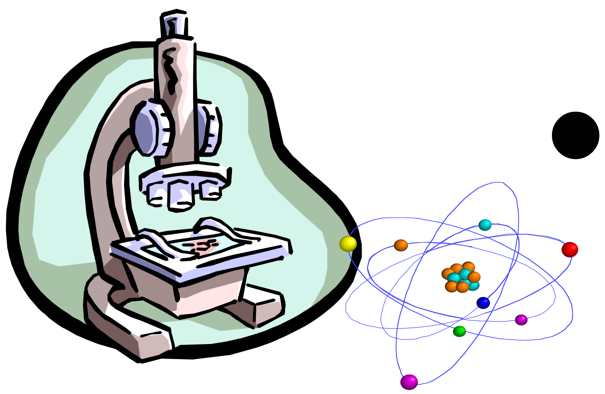

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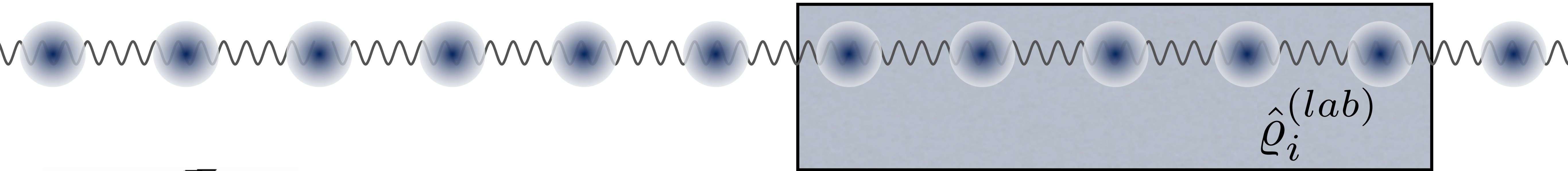

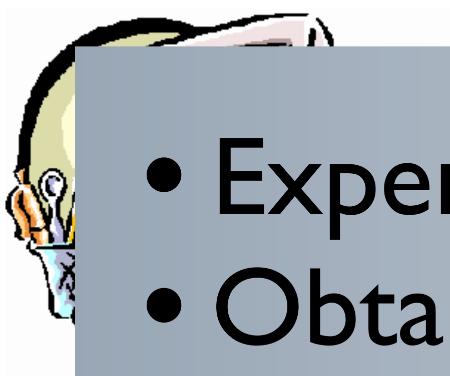

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$$\leq \frac{\sum_i(\epsilon_i + \mathrm{tr}[\hat{h}_i\hat{\varrho}_i^{(est)}])}{\Delta E}$$

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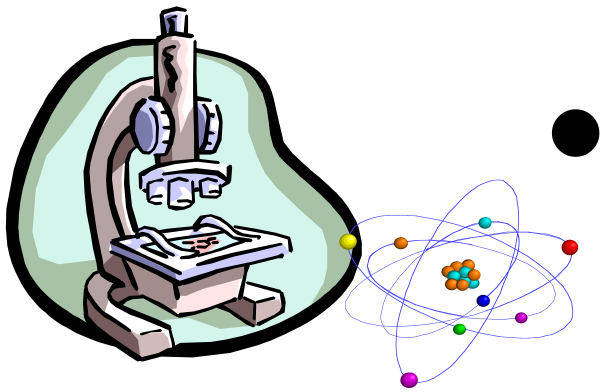

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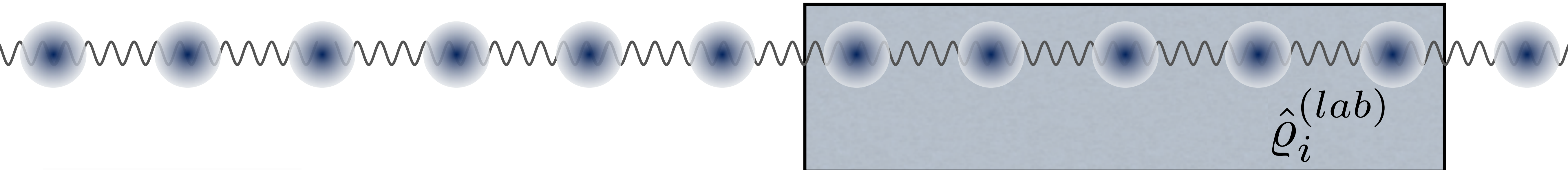

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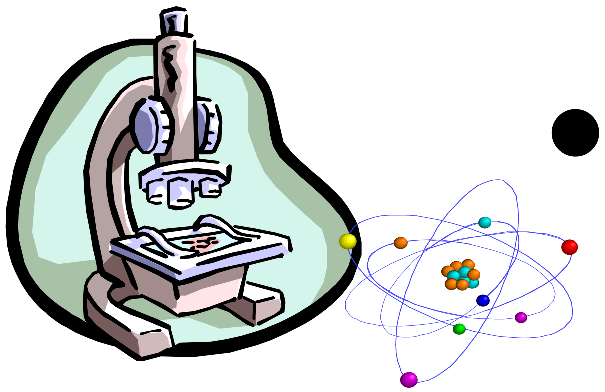

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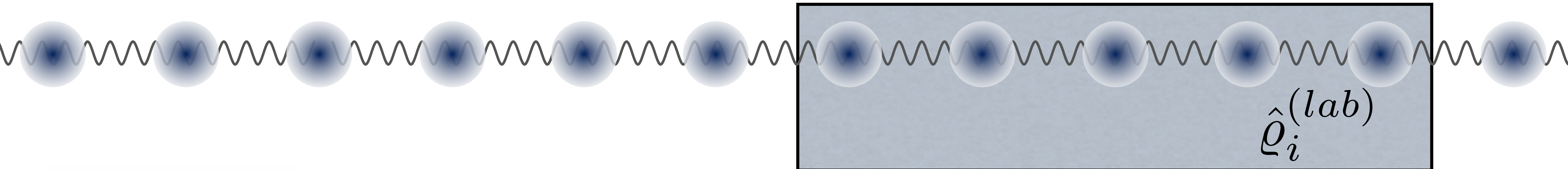
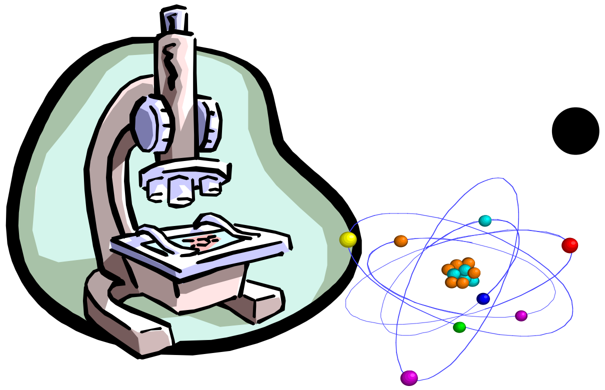


- Find compatible state
- Storage space
- Find parent Hamiltonian, ensure uniqueness of ground state and compute gap efficiently



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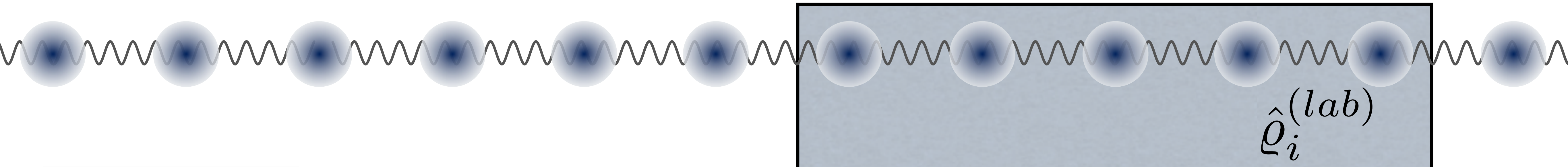


$\hat{\varrho}_i^{(lab)}$

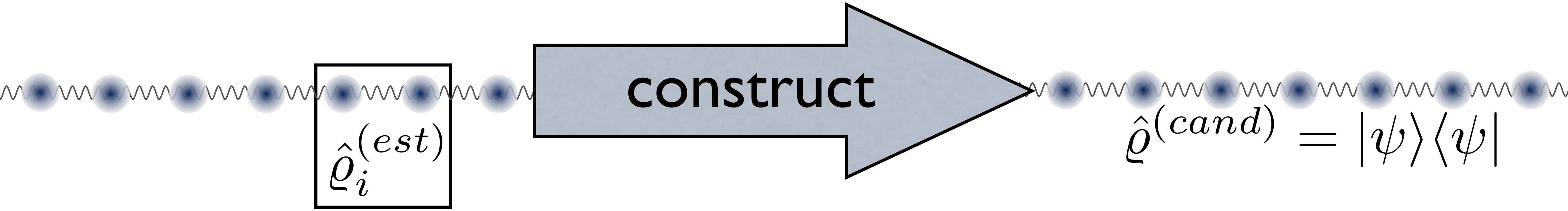
- Find compatible state
- Storage space
- Find parent Hamiltonian, ensure uniqueness of ground state and compute gap efficiently

$$\|\hat{\varrho}_i^{(lab)} - \hat{\varrho}_i^{(est)}\| \leq \epsilon_i$$

$|\psi\rangle$ and $1 - \langle\psi|\hat{\varrho}_{lab}|\psi\rangle \leq \frac{\sum_i(\epsilon_i + \mathrm{tr}[\hat{h}_i \hat{\varrho}_i^{(est)}])}{\Delta E}$

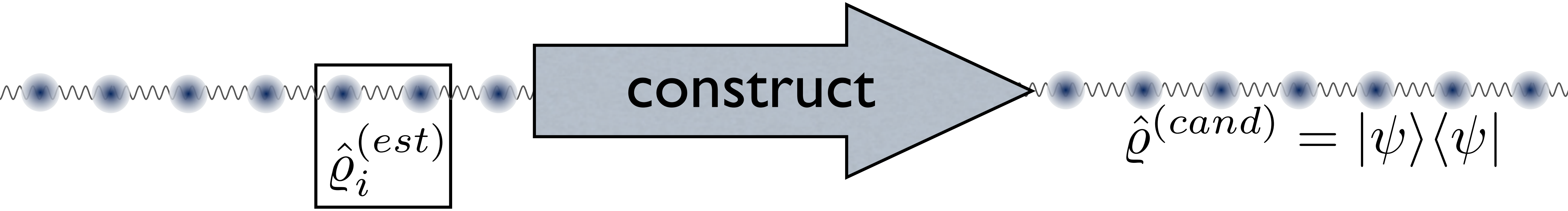
- Find compatible state

- Find compatible state, i.e., $|\psi\rangle$ such that $\hat{\varrho}_i^{(est)} = \hat{\varrho}_i^{(cand)}$



Quantum state tomography: A solution?

- Find compatible state, i.e., $|\psi\rangle$ such that $\hat{\varrho}_i^{(est)} = \hat{\varrho}_i^{(cand)}$



measured Entries:

$$\Omega = \{k : \ \hat{P}_k = \mathbb{1} \otimes \hat{\sigma}_i^{\alpha_i} \otimes \hat{\sigma}_{i+1}^{\alpha_{i+1}} \otimes \mathbb{1}\}$$

Entries:

$$p_k = \mathrm{tr}[\hat{P}_k |\psi\rangle\langle\psi|]$$

$$\hat{P}_k = \bigotimes_{i=1}^{N} \hat{\sigma}_i^{\alpha_i}$$

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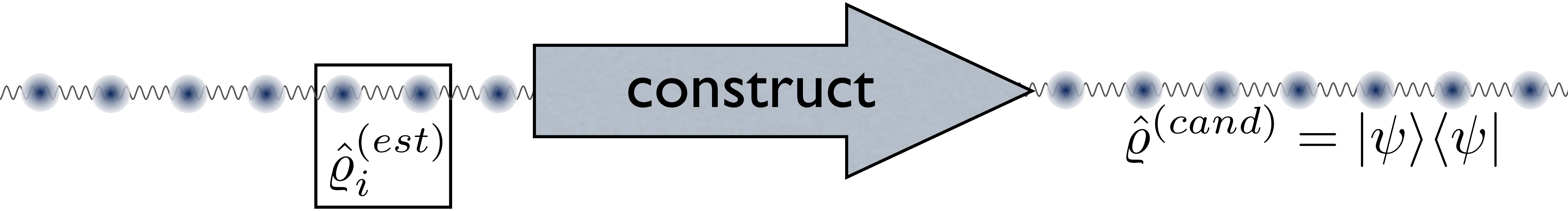

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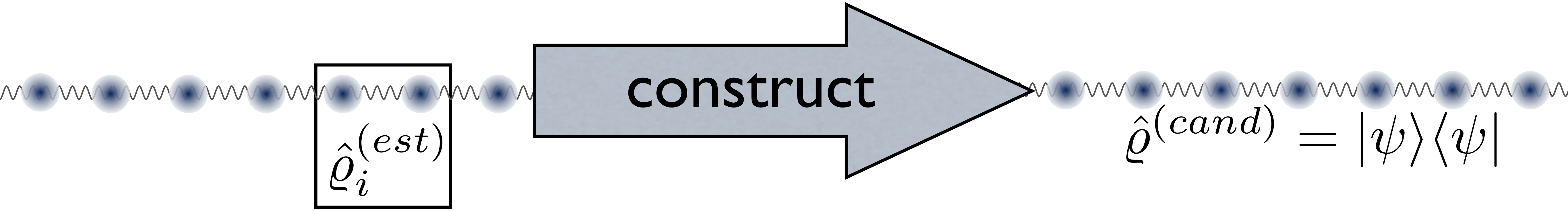
→ Construct matrix from few known entries

- Find compatible state, i.e., $|\psi\rangle$ such that $\hat{\varrho}_i^{(est)} = \hat{\varrho}_i^{(cand)}$



$\hat{\varrho}_i^{(est)}$

$\hat{\varrho}^{(cand)} = |\psi\rangle\langle\psi|$

measured Entries:

$$\Omega = \{k : \; \hat{P}_k = \mathbb{1} \otimes \hat{\sigma}_i^{\alpha_i} \otimes \hat{\sigma}_{i+1}^{\alpha_{i+1}} \otimes \mathbb{1}\}$$

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→ Construct matrix from few known entries

→ *Compressed sensing*

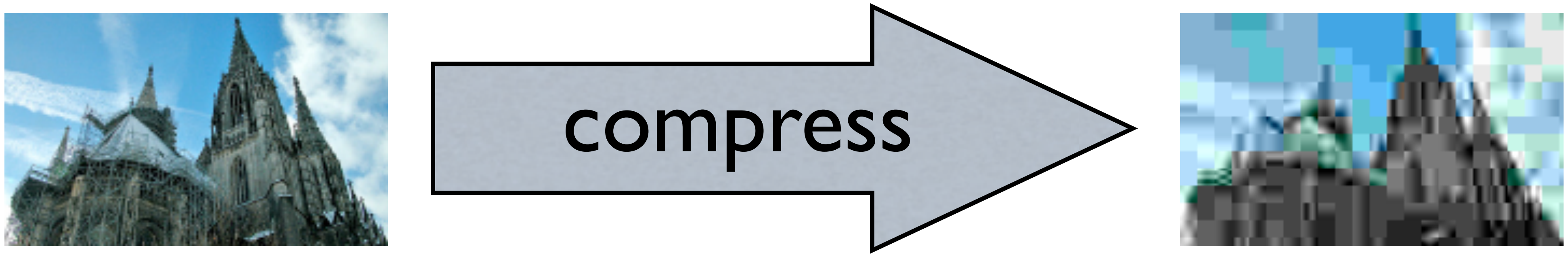
“a technique that may be the hottest topic in applied math today”

“paradigm-busting field in mathematics that’s reshaping the way people work with large data sets”

“Only six years old, compressed sensing has already inspired more than a thousand papers and pulled in millions of dollars in federal grants”

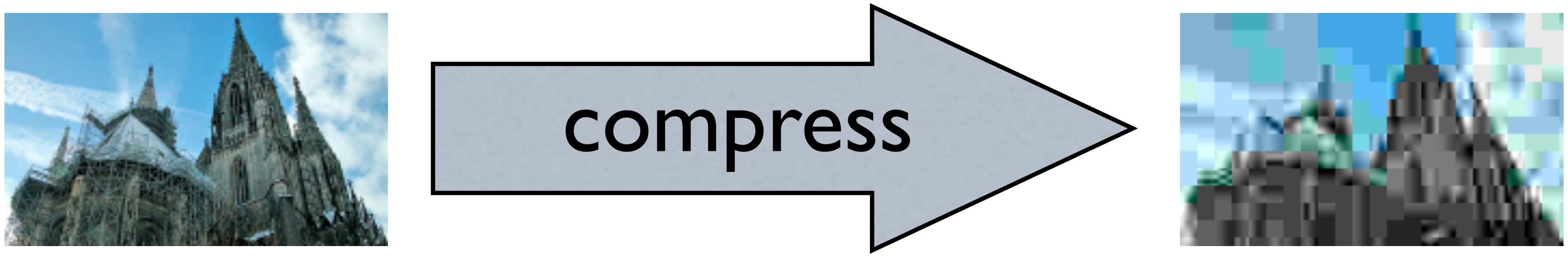
WIRED February 2010

Compressed sensing



Pioneered by Donoho, Candès, Tao; an introduction: E.J. Candes and M.B. Wakin, IEEE Sig. Proc. Mag. **25**, 21 (2008); a quantum version: D. Gross, Y.-K. Liu, S.T. Flammia, S. Becker, J. Eisert, arXiv:0909.3304

Compressed sensing



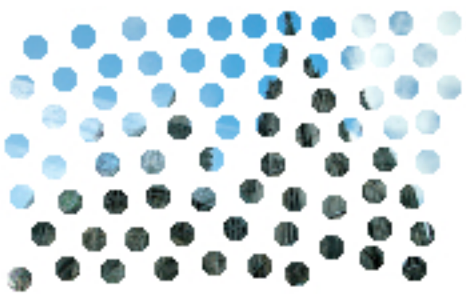
instead:

Pioneered by Donoho, Candès, Tao; an introduction: E.J. Candes and M.B. Wakin, IEEE Sig. Proc. Mag. **25**, 21 (2008); a quantum version: D. Gross, Y.-K. Liu, S.T. Flammia, S. Becker, J. Eisert, arXiv:0909.3304

Compressed sensing

compress

instead:



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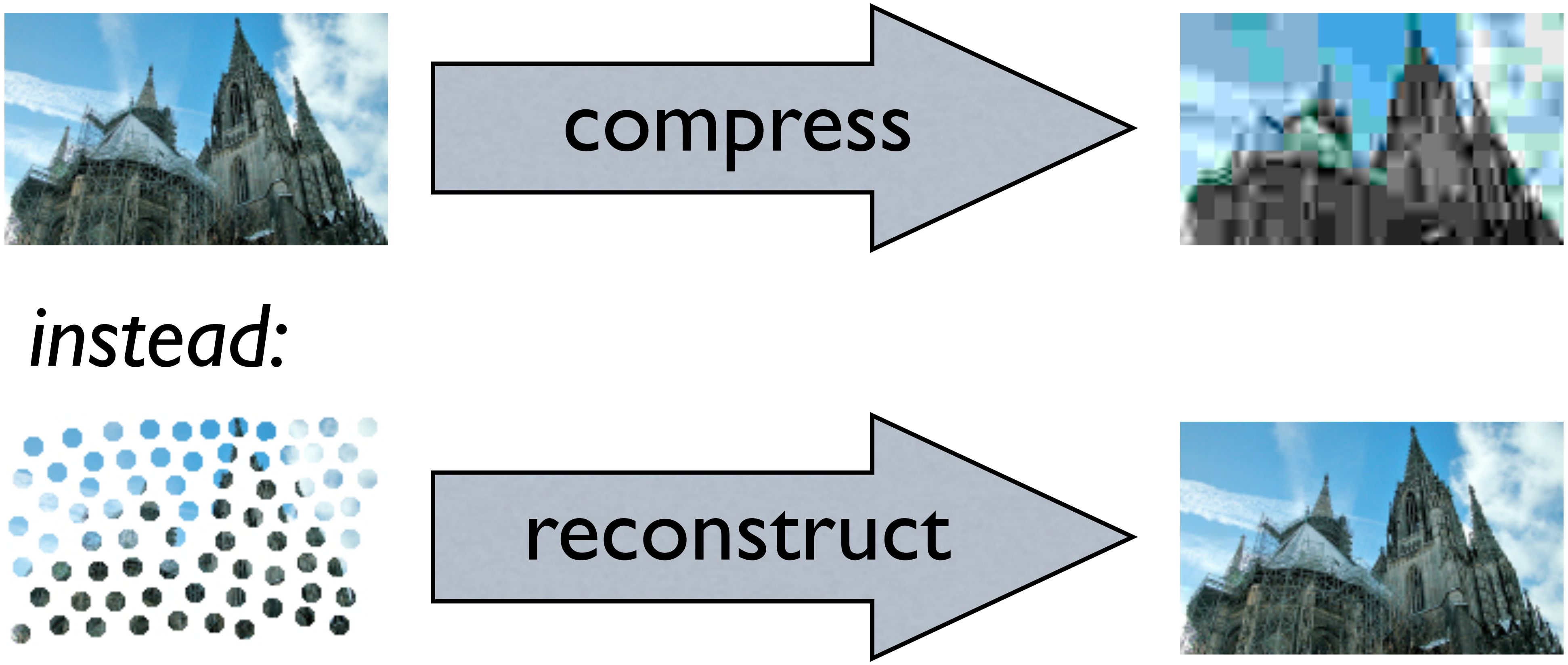

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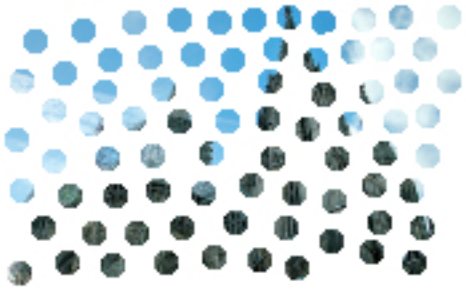


reconstruct

$m \times m$

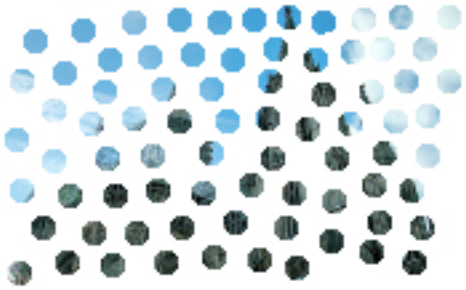
sample size

$\geq Crm^{1.2} \log(m)$

small rank

E.J. Candes and B. Recht, arXiv:0805.4471

Compressed sensing



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perfect reconstruction with very high probability

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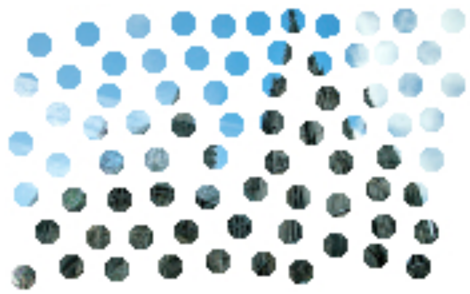
E.J. Candes and B. Recht, arXiv:0805.4471



reconstruct

$= M$

$m \times m$

sample size

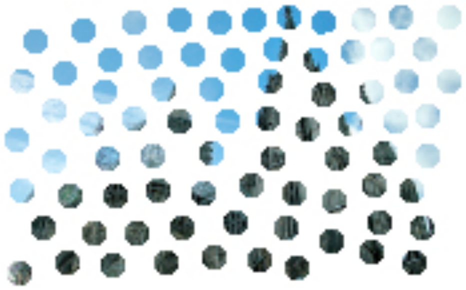
$\geq Crm^{1.2}\log(m)$

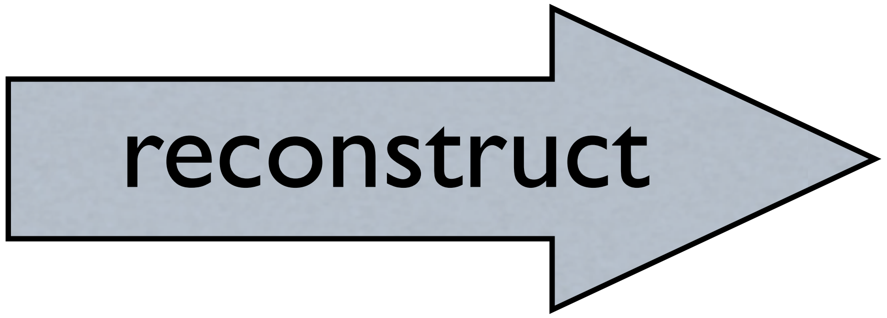
perfect reconstruction with very high probability

small rank

$$\text{minimize } \mathrm{tr}[|A|]$$
$$\text{such that } A_{i,j} = M_{i,j} \text{ for all } (i,j) \in \Omega$$

E.J. Candes and B. Recht, arXiv:0805.4471





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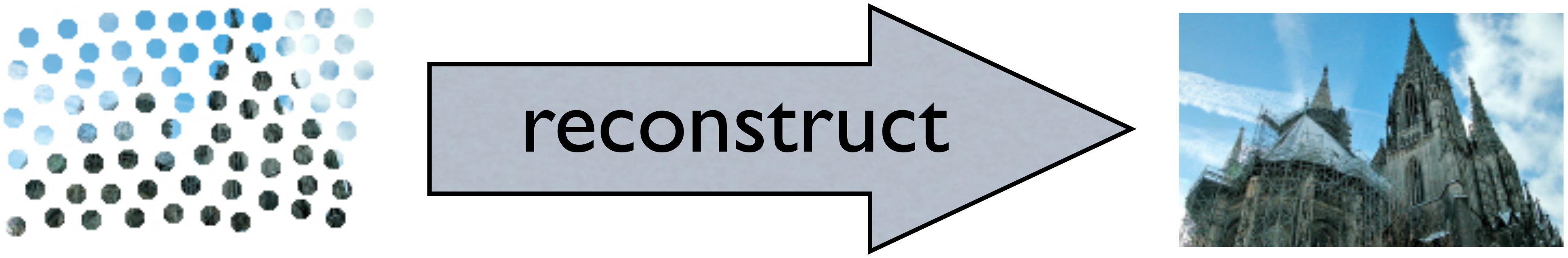

$= M$

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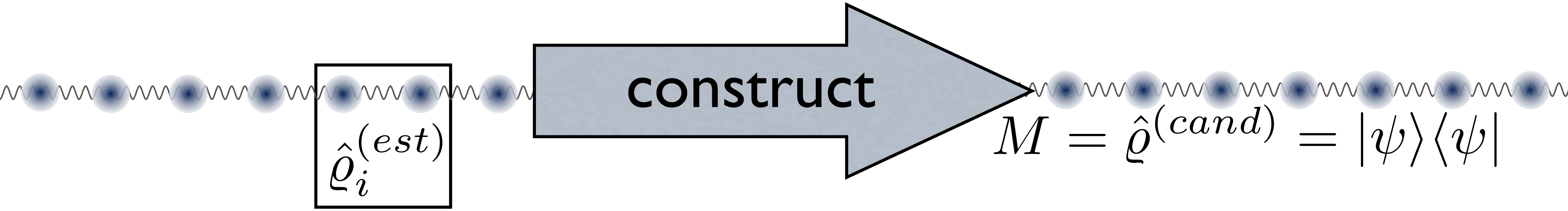
Initialize Y_0 (e.g., by zero matrix), proceed inductively

- Singular value decomposition $Y_{n-1} = U \Sigma V^\dagger$
- Thresholding $X_n = U \max\{0, \Sigma - \mathbb{1}\tau\} V^\dagger$
- $Y_n = Y_{n-1} + \delta_n P_\Omega (M - X_n)$

Converges provably to solution for sufficiently small δ_n and $\tau \gg 1$

J-F. Cai, E.J. Candes, and Z. Shen, arXiv:0810.3286



$\hat{\varrho}_i^{(est)}$

$M = \hat{\varrho}^{(cand)} = |\psi\rangle\langle\psi|$

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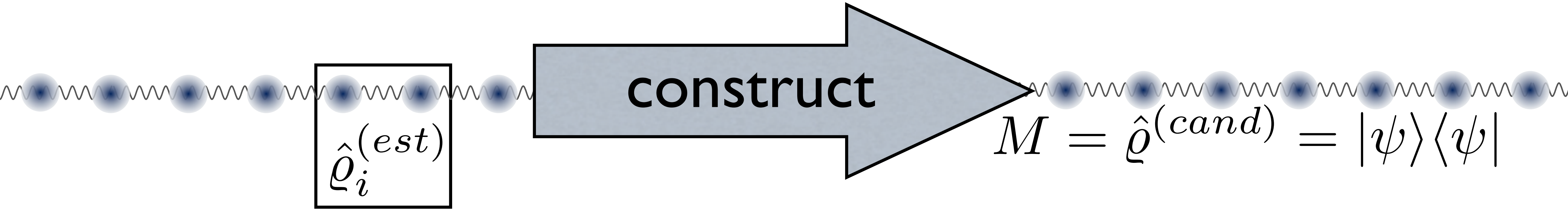

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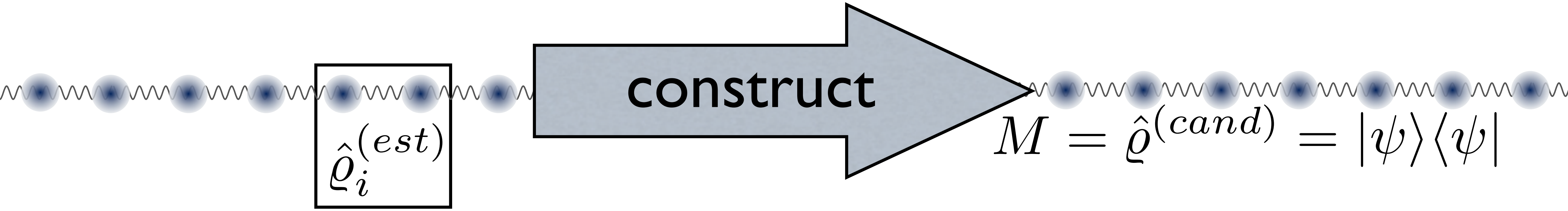
Quantum state tomography: A solution?



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Initialize Y_0 (e.g., by zero matrix), proceed inductively

- Singular value decomposition of $2^N \times 2^N$ matrix
- Thresholding $X_n = U \max\{0, \Sigma - \mathbb{1}\tau\} V^\dagger$
- $Y_n = Y_{n-1} + \delta_n \sum_{k \in \Omega} \frac{p_k - \mathrm{tr}[X_n \hat{P}_k]}{2^N} \hat{P}_k$

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M. Cramer and M.B. Plenio, arXiv:1002.3780

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Instead of threshold, keep largest singular value

M. Cramer and M.B. Plenio, arXiv:1002.3780

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$$= \sum_{k\in\Omega} a_k \hat{P}_k, \quad a_k \in \mathbb{R}$$

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local "Hamiltonian"

M. Cramer and M.B. Plenio, arXiv:1002.3780

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local "Hamiltonian", find ground state,

M. Cramer and M.B. Plenio, arXiv:1002.3780

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local "Hamiltonian", find ground state, compute expectation values efficiently

DMRG, MPS methods

M. Fannes, B. Nachtergaele, and R.F. Werner, Comm. Math. Phys. **144**, 443 (1992),
U. Schollwöck, Rev. Mod. Phys. **77**, 259 (2005),
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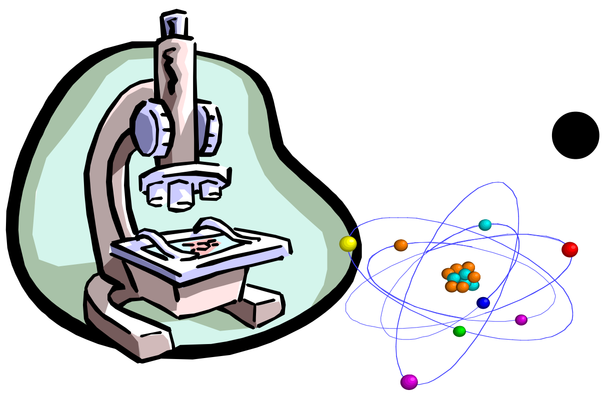
$D \times D$

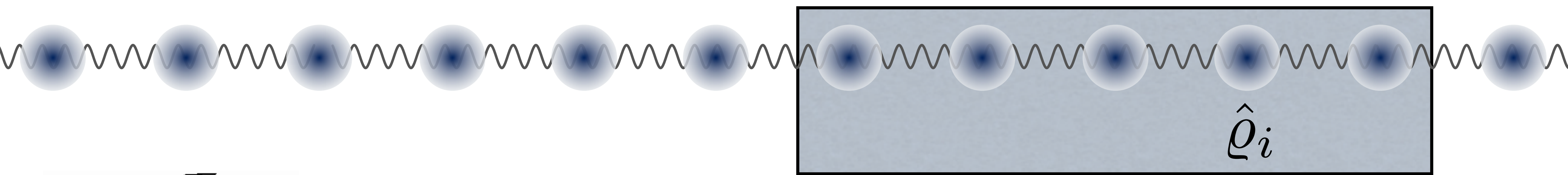
Matrix Product State $|MPS\rangle = \sum_{s_1,\ldots,s_N} \text{tr}\big[A_1[s_1]\cdots A_N[s_N]\big]|s_1\cdots s_N\rangle$

M. Cramer and M.B. Plenio, arXiv:1002.3780



- Number and accuracy of measurements

Take only $\sim N$



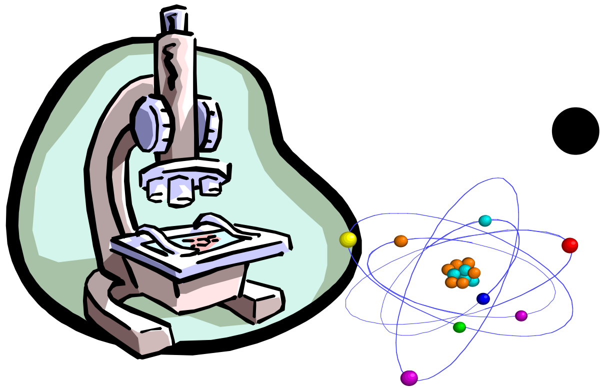
- Find compatible state
- Storage space

MPS-SVT

$2D^2N$

- Find *parent Hamiltonian*, ensure uniqueness of ground state and compute gap efficiently

Quantum state tomography: A solution!



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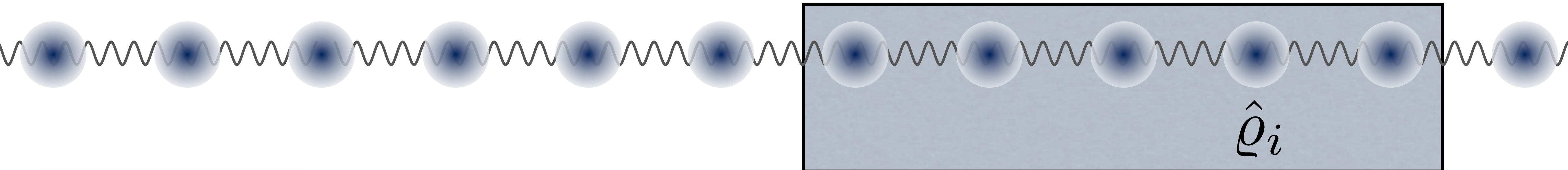

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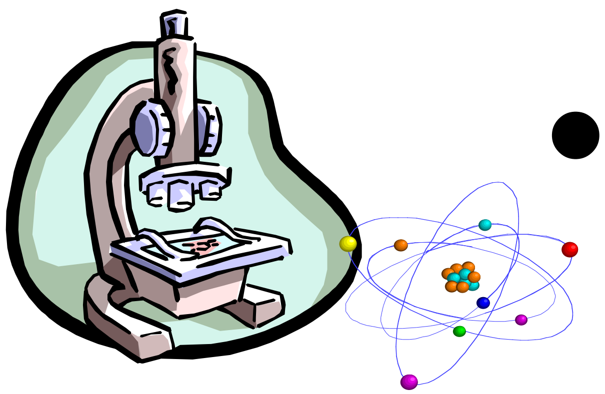

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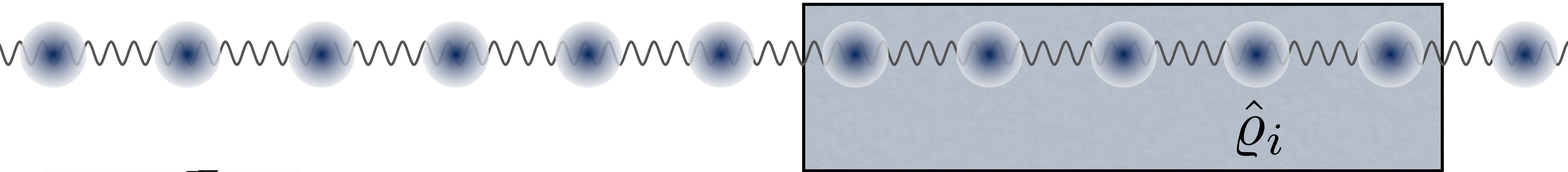

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MPS-SVT

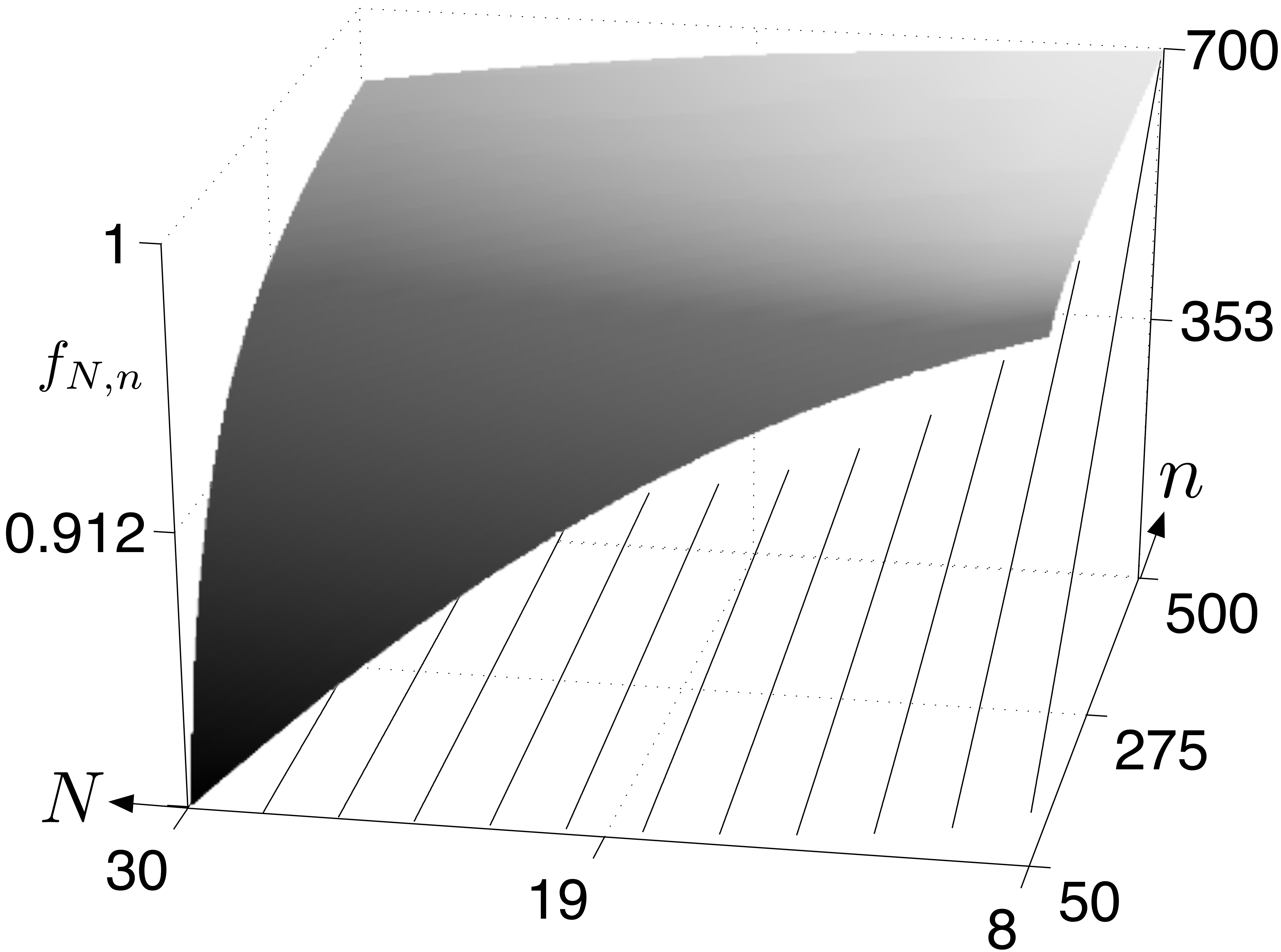
$2D^2N$

How does the algorithm perform?

Ground state critical Ising model $\hat{H} = -\sum_{i=1}^{N-1} \hat{\sigma}_i^x \hat{\sigma}_{i+1}^x - \sum_{i=1}^{N} \hat{\sigma}_i^z$

- "Measure" all $\hat{\varrho}_{i,i+1}$
- Completely determines ground state
- Compute fidelity $f_{N,n} = |\langle gs|X_n\rangle|^2$

M. Cramer and M.B. Plenio, arXiv:1002.3780



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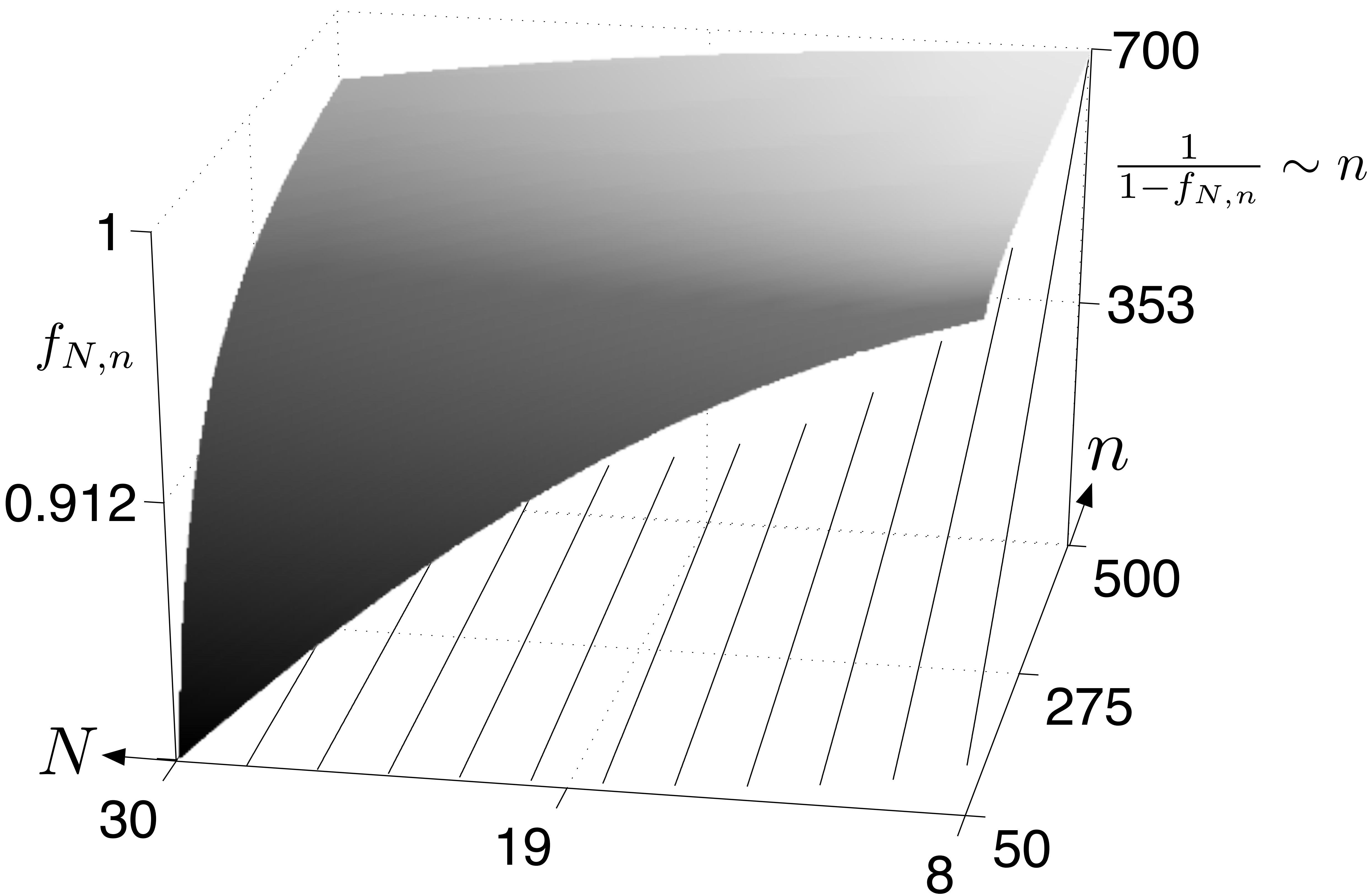

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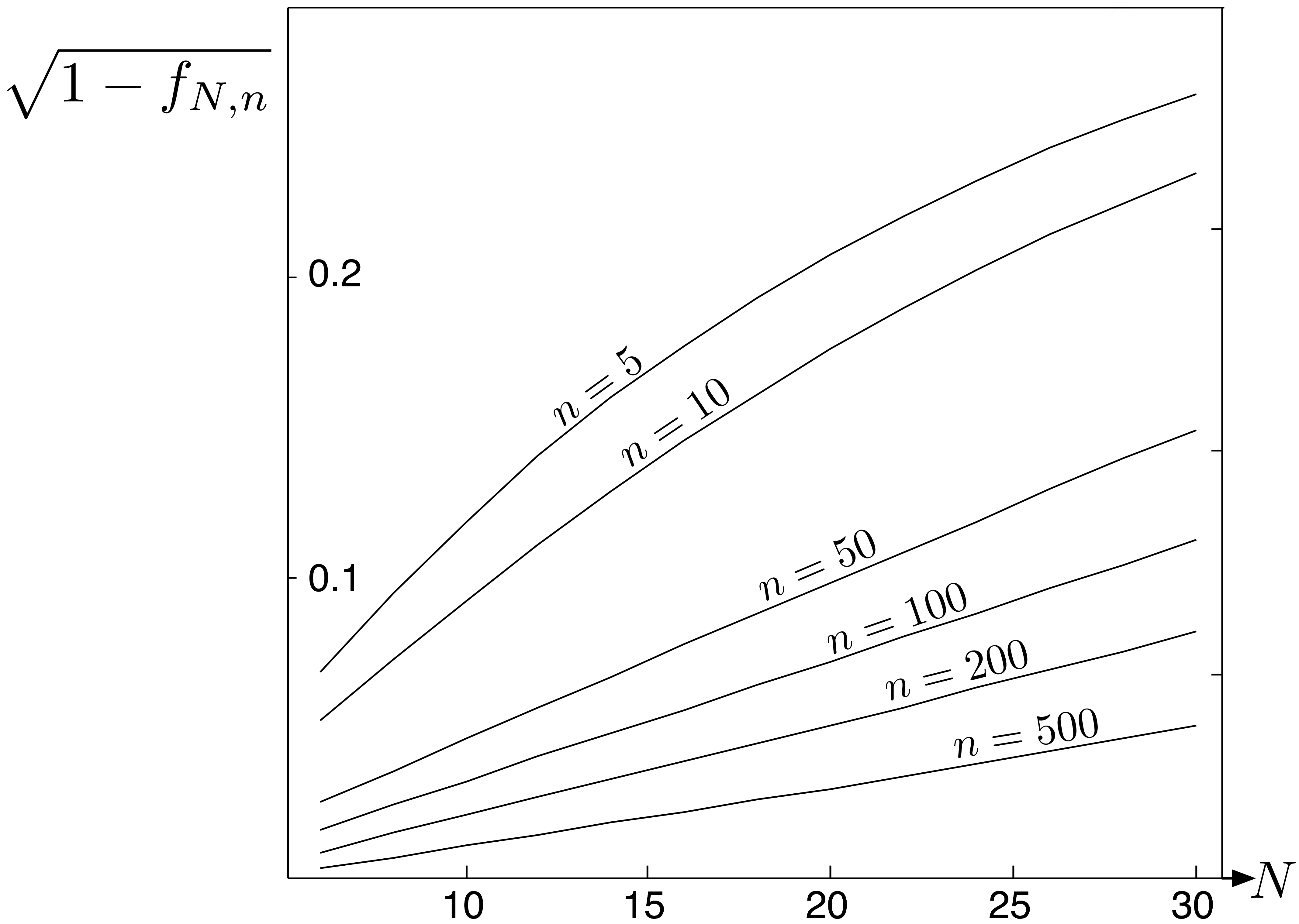
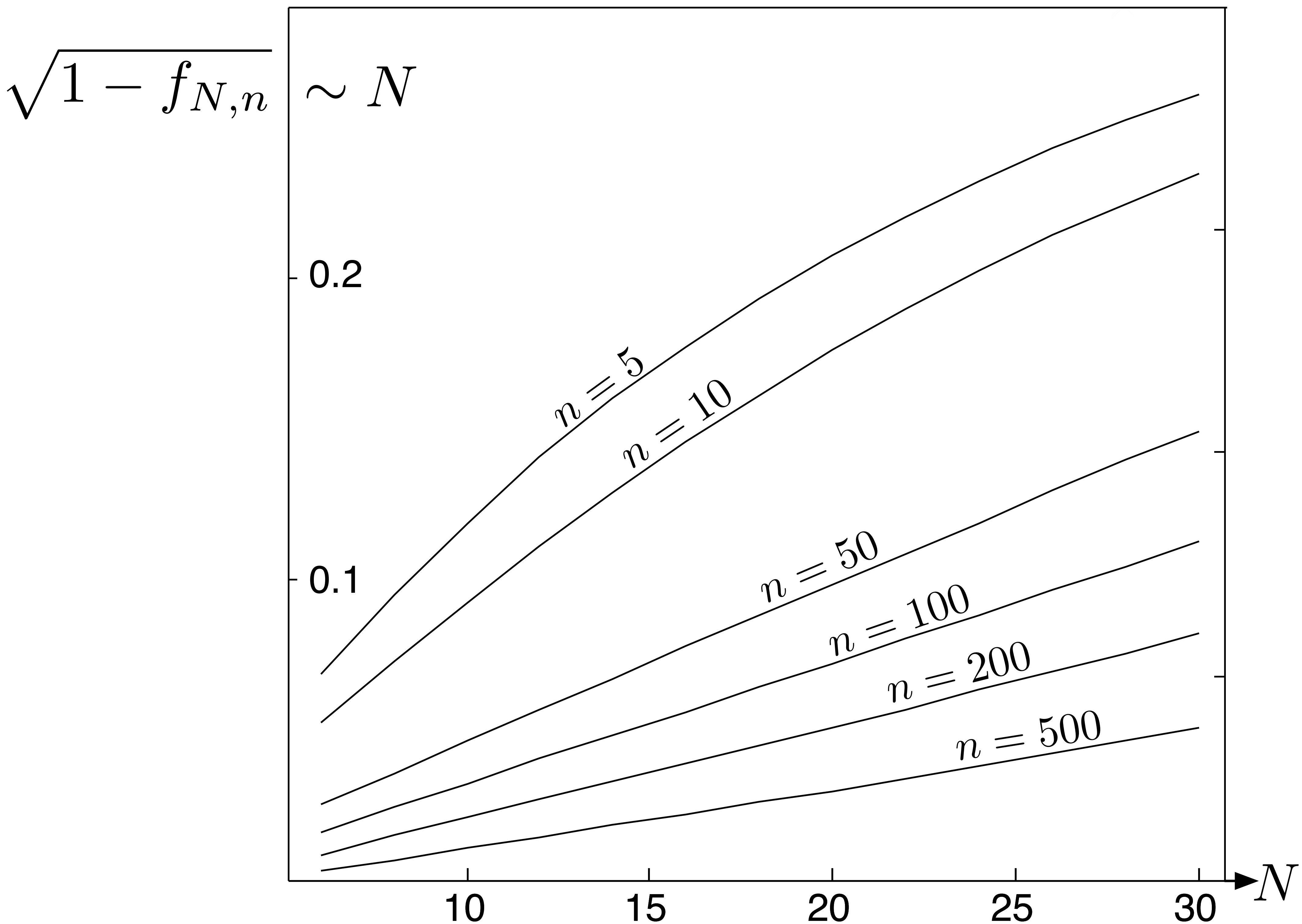


M. Cramer and M.B. Plenio, arXiv:1002.3780



M. Cramer and M.B. Plenio, arXiv:1002.3780

$$\hat{H} = \sum_{i=1}^{N-1} \hat{r}_i^{(i)} \hat{r}_{i+1}^{(i)}$$

hermitian, real and imaginary part of entries uniformly from $[-1,1]$, 1000 realizations for each N, $n=5$

M. Cramer and M.B. Plenio, arXiv:1002.3780

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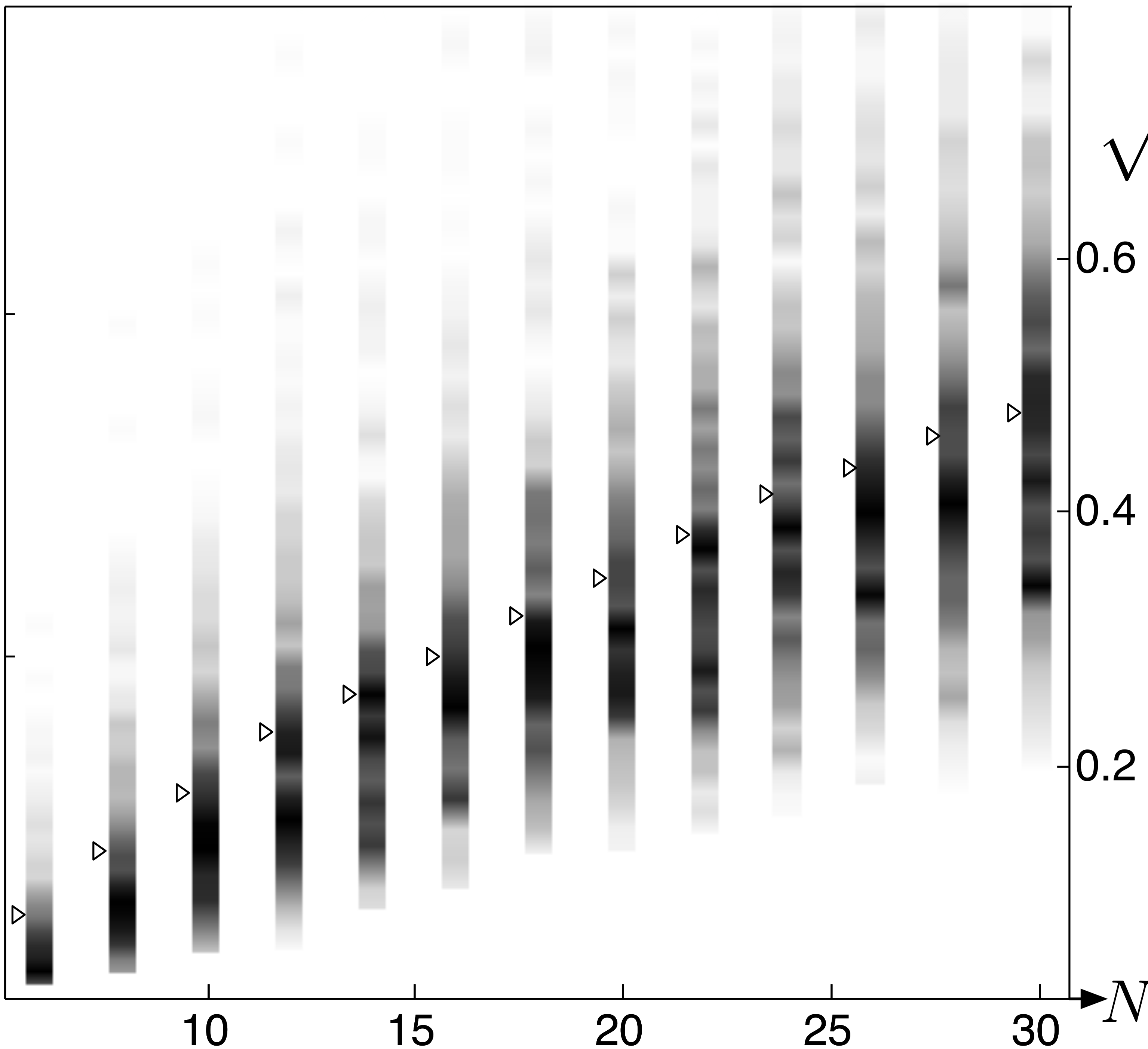
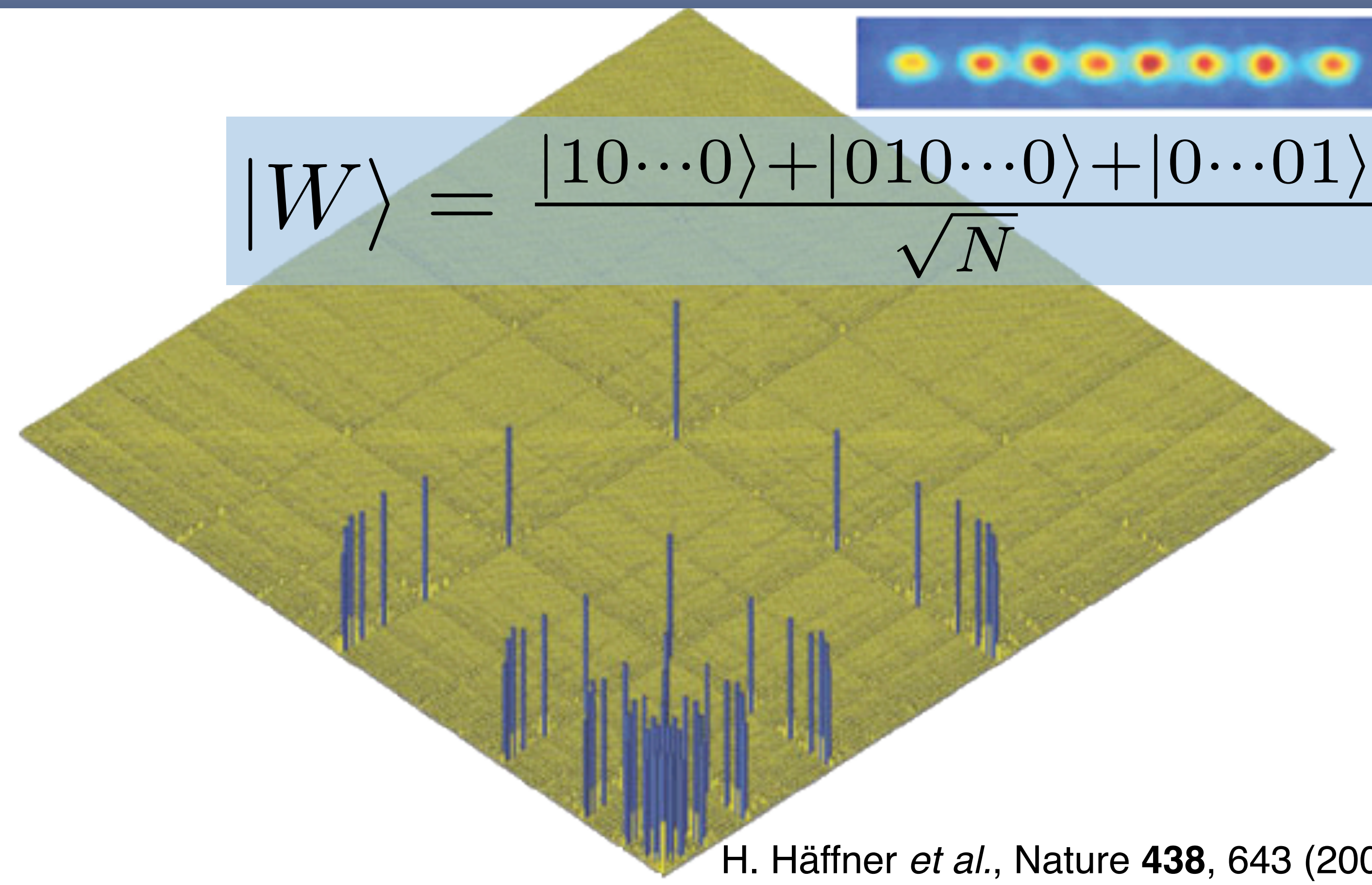


M. Cramer and M.B. Plenio, arXiv:1002.3780

MPS-SVT: Numerical experiments



$$|W\rangle = \frac{|10\cdots0\rangle + |010\cdots0\rangle + |0\cdots01\rangle}{\sqrt{N}}$$

H. Häffner *et al.*, Nature **438**, 643 (2005).

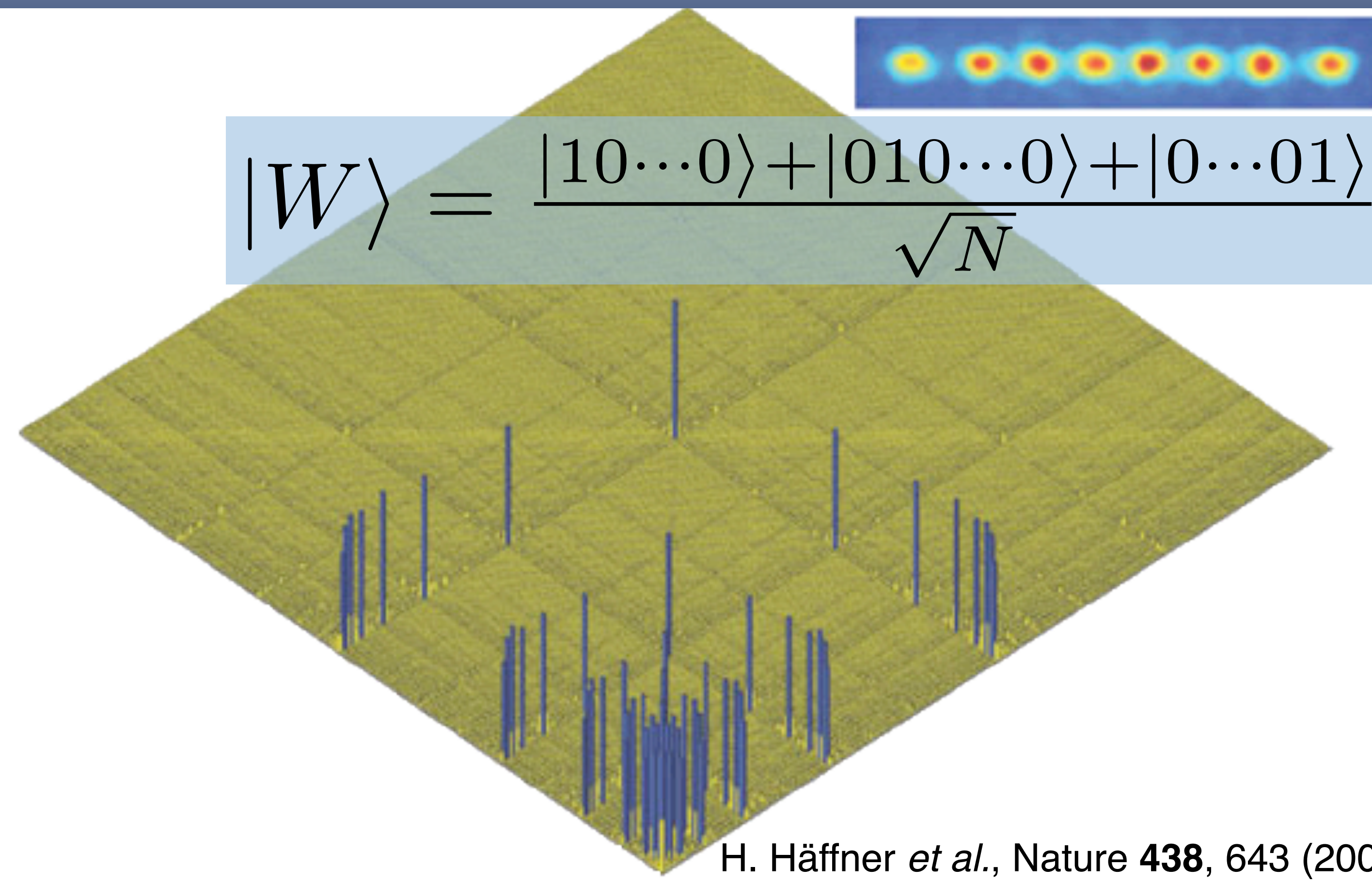
M. Cramer and M.B. Plenio, arXiv:1002.3780

$$p_k = \langle W|\hat{P}_k|W\rangle + r$$

Gaussian, zero mean

100 realizations for each N

$$|W\rangle = \frac{|10\cdots0\rangle + |010\cdots0\rangle + |0\cdots01\rangle}{\sqrt{N}}$$



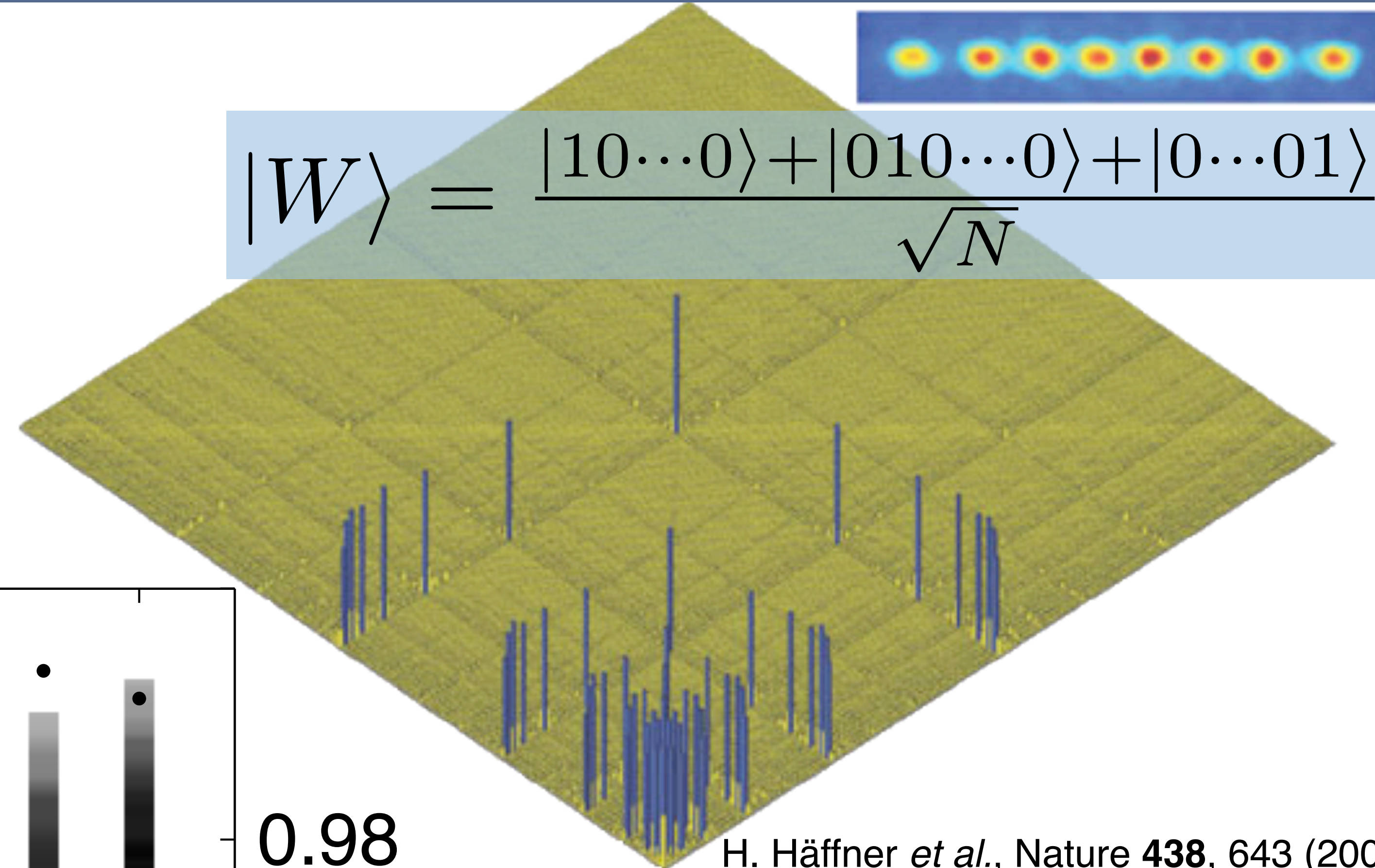
H. Häffner *et al.*, Nature **438**, 643 (2005).

M. Cramer and M.B. Plenio, arXiv:1002.3780

$$p_k = \langle W | \hat{P}_k | W \rangle + r$$

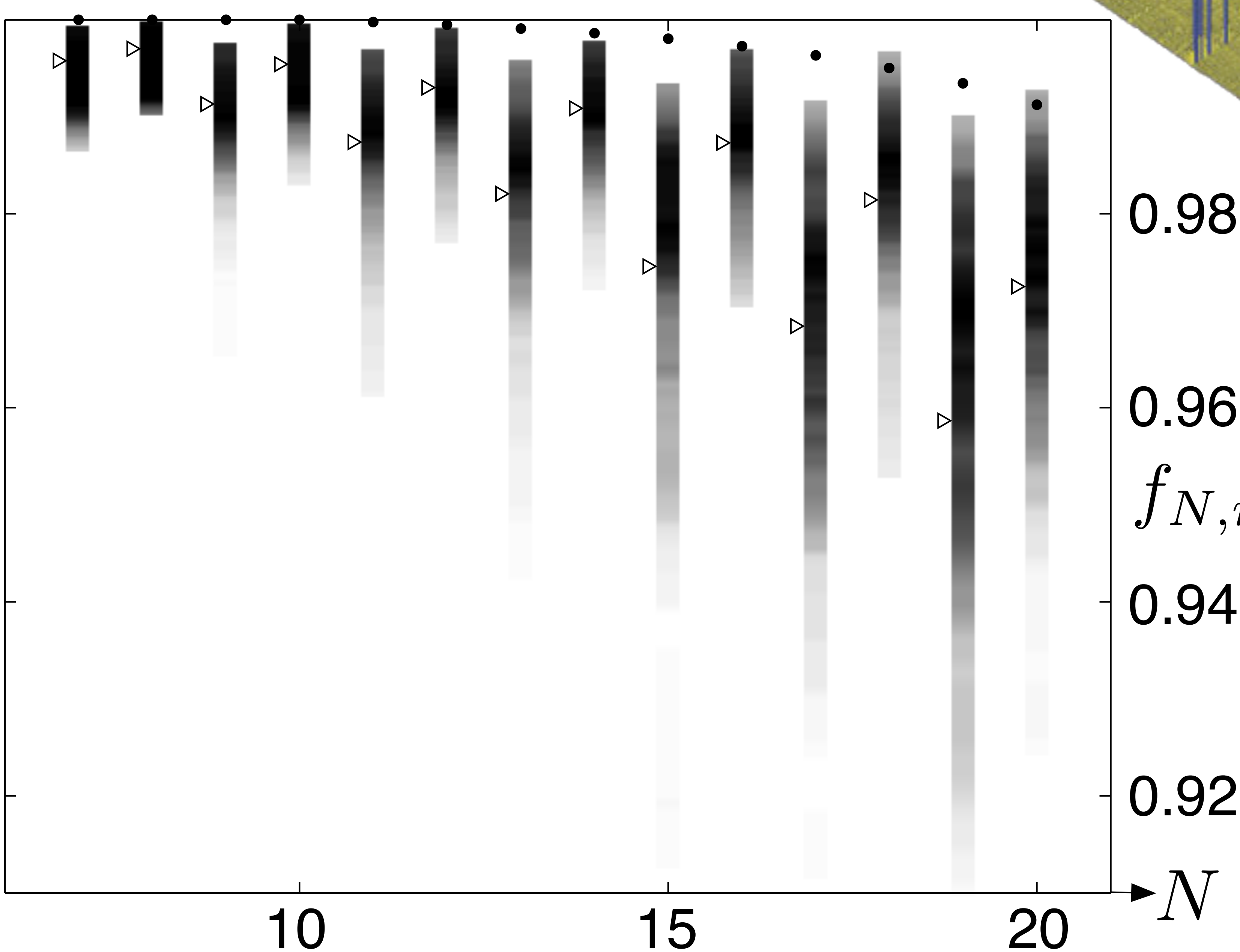
Gaussian, zero mean

100 realizations for each N



$$|W\rangle = \frac{|10\cdots0\rangle + |010\cdots0\rangle + |0\cdots01\rangle}{\sqrt{N}}$$

H. Häffner *et al.*, Nature **438**, 643 (2005).



$$f_{N,n} = |\langle W | X_n \rangle|^2$$

$$n = 4000$$

$\sigma = 0.01$ (odd N)

$\sigma = 0.005$ (even N)

M. Cramer and M.B. Plenio, arXiv:1002.3780

- Efficient ($\mathrm{poly}(N)$) scheme to reconstruct states from few measurements ($\sim N$)
- Extensive numerics suggest convergence. In fact, equipped with candidate MPS, fidelity can under certain conditions be bounded. S.T. Flammia, D. Gross, S.D. Bartlett, R. Somma, arXiv:1002.3839
- Generalization to higher dimensions: Analogues of MPS, e.g., MERA, PEPS,....
- Mixed states: Purification, e.g., Gibbs state minimizes $\mathrm{tr}[\hat{\varrho}\hat{H}] - TS(\hat{\varrho})$ and entropy density $\lim_{k\to\infty} S(\hat{\varrho}_i)/k$ exists