

Theoretical Aspects of Quantum Effects in Biology

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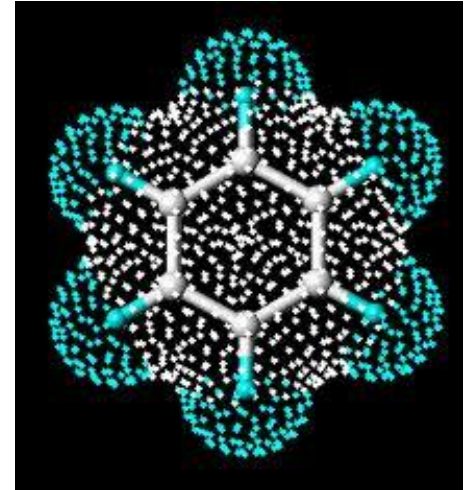
What we are looking for

- Quantum coherence certainly exists at the level of chemical bonds

Electrons are delocalized in a coherent superposition

For us this kind of coherence is of little relevance !

Its in equilibrium , static, short-ranged

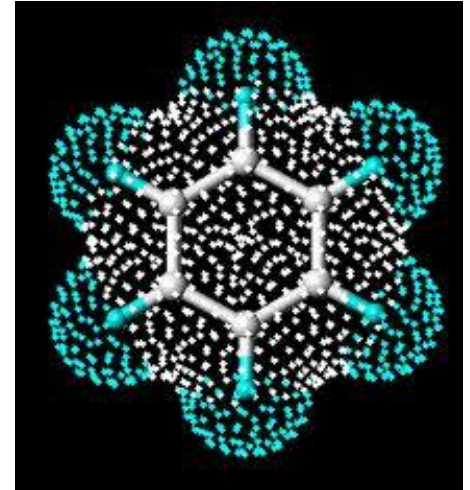


- More interesting is (transport) dynamics because
 - You learn about systems by poking them.
 - Biological processes are necessarily dynamic.
 - Has the potential to explore long range correlations.

What we are looking for

➤ Questions:

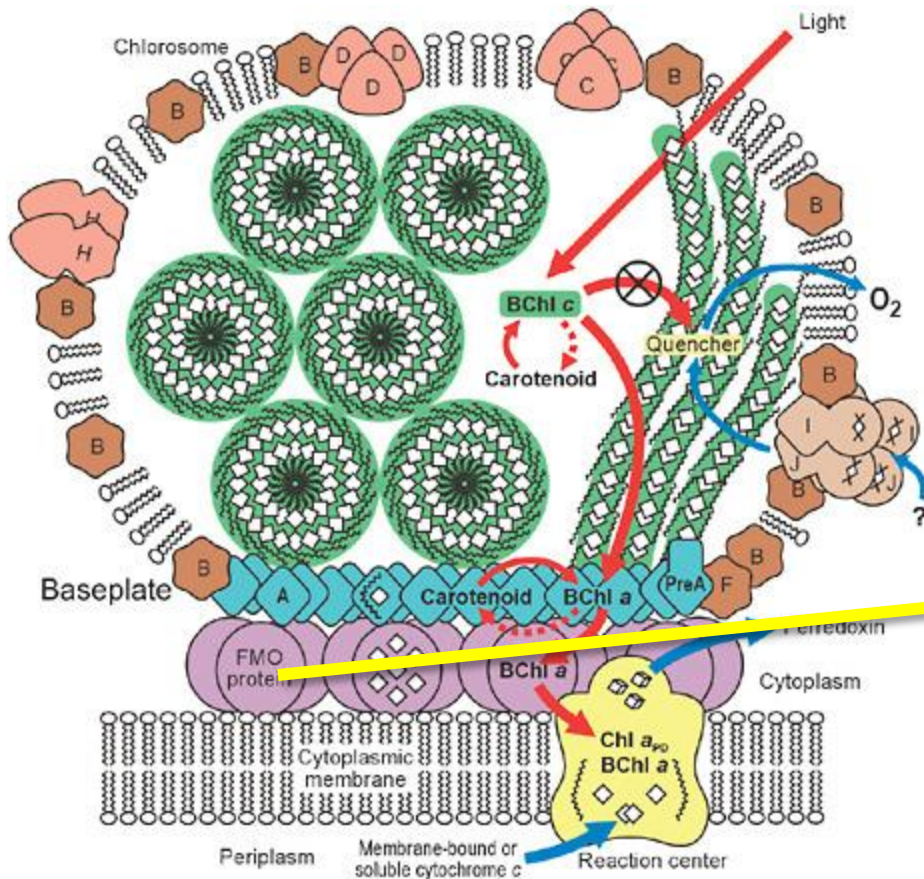
- On what length and time scales do we find coherence ?
- What, if any, is the role of coherence ?
- What, if any, is the role of the environment ?
- Can one quantify coherence and quantum character ?
- How do I verify theoretical hypotheses experimentally ?



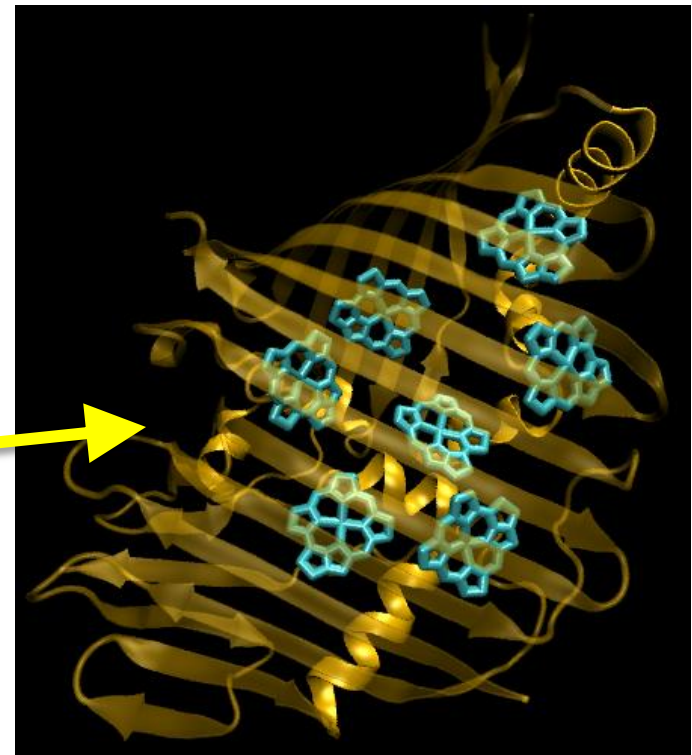


Excitation Transport in Noisy Environments

Photosynthetic Apparatus



Fenna-Matthews-Olsen Complex (FMO)



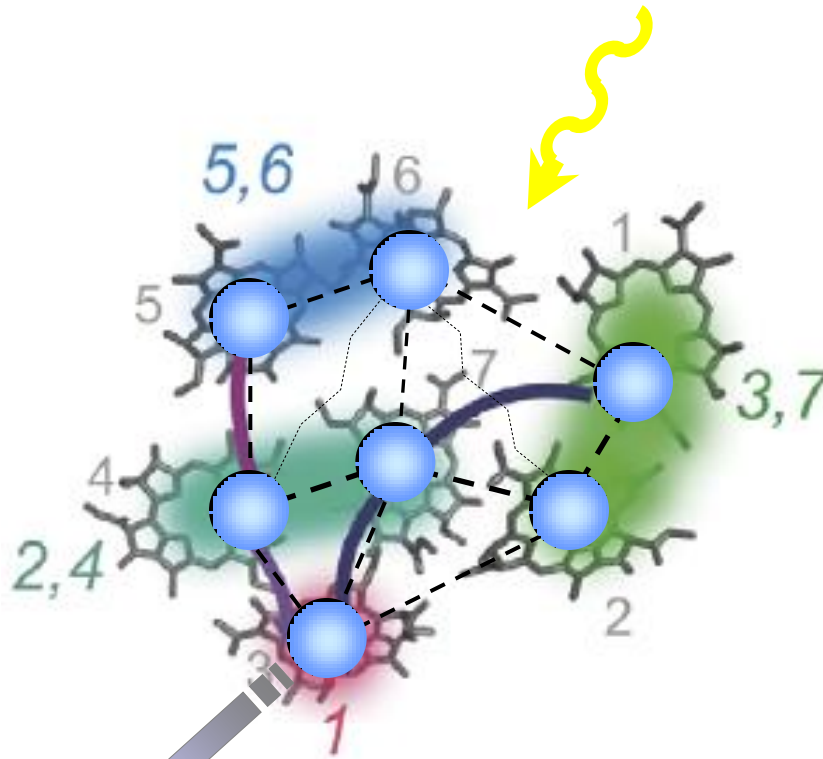
Lossless transport ~ 5 ps



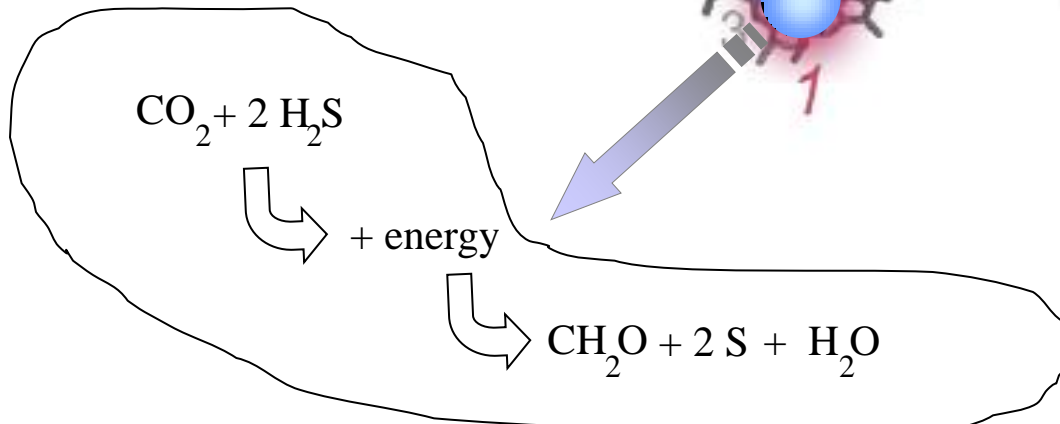
Exciton Transport in Photosynthesis



Green sulphur Bacteria



Reaction Centre

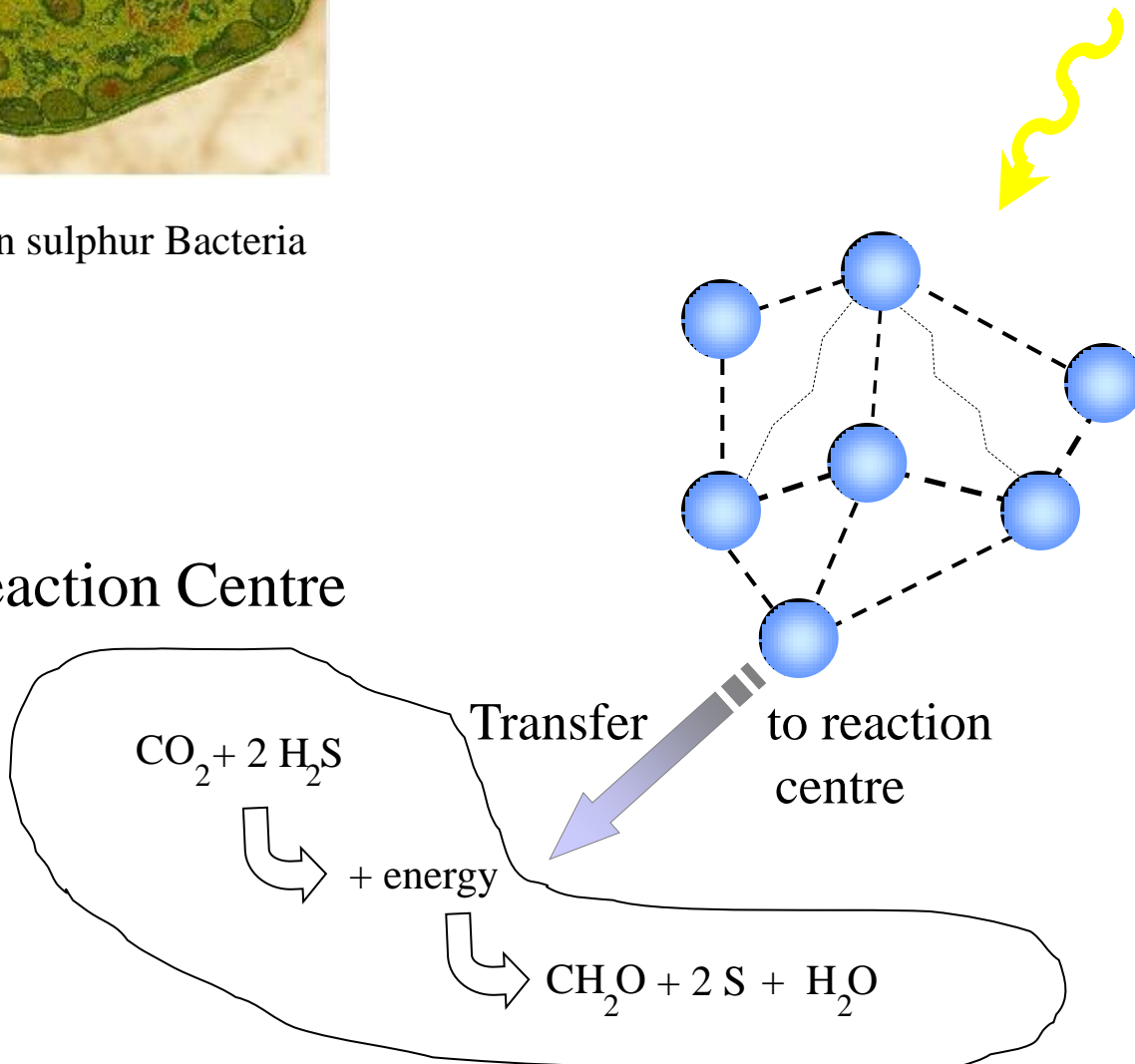


Excitation Transport



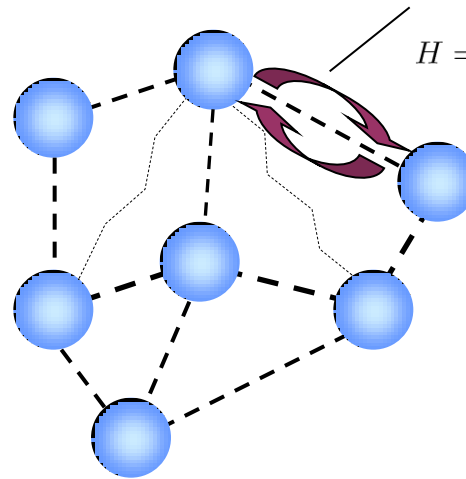
Green sulphur Bacteria

Reaction Centre



$$\frac{d\rho}{dt} = -\frac{i}{\hbar} [H, \rho] + \mathcal{L}$$

Exchange of excitation

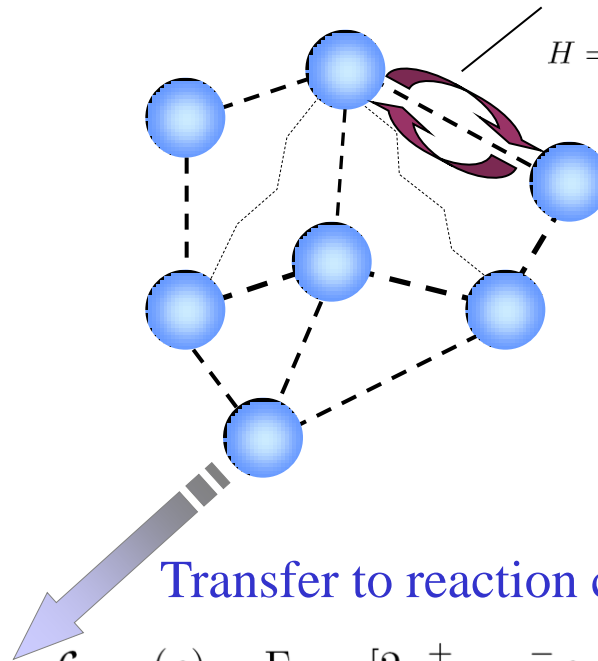


$$H = \sum_{j=1}^N \hbar\omega_j \sigma_j^+ \sigma_j^- + \sum_{j \neq l} \hbar v_{j,l} (\sigma_j^- \sigma_l^+ + \sigma_j^+ \sigma_l^-)$$

$$\frac{d\rho}{dt} = -\frac{i}{\hbar} [H, \rho] + \mathcal{L}$$

Exchange of excitation

$$H = \sum_{j=1}^N \hbar\omega_j \sigma_j^+ \sigma_j^- + \sum_{j \neq l} \hbar v_{j,l} (\sigma_j^- \sigma_l^+ + \sigma_j^+ \sigma_l^-)$$



Transfer to reaction centre

$$\mathcal{L}_{sink}(\rho) = \Gamma_{N+1} [2\sigma_{N+1}^+ \sigma_k^- \rho \sigma_k^+ \sigma_{N+1}^- - \{\sigma_k^+ \sigma_{N+1}^- \sigma_{N+1}^+ \sigma_k^-, \rho\}]$$

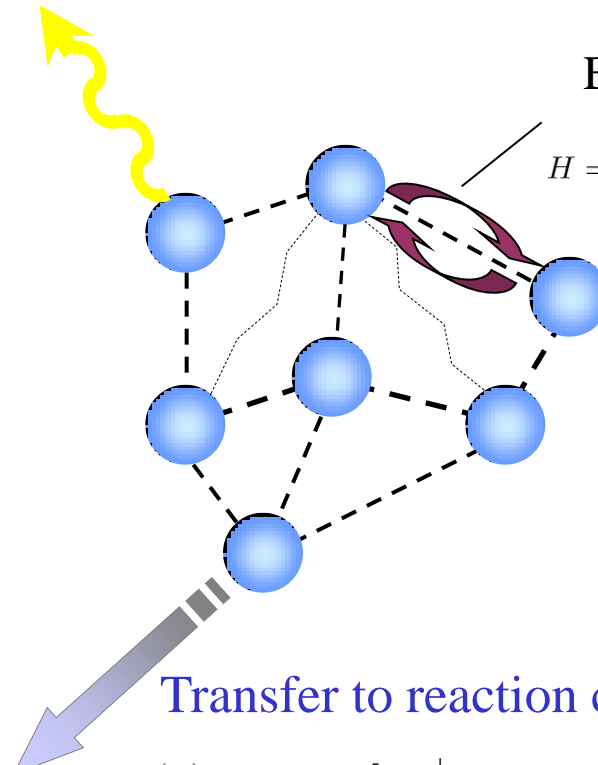
$$\mathcal{L}_{diss}(\rho) = \sum_{j=1}^N \Gamma_j [-\{\sigma_j^+ \sigma_j^-, \rho\} + 2\sigma_j^- \rho \sigma_j^+]$$

Loss of excitation

$$\frac{d\rho}{dt} = -\frac{i}{\hbar} [H, \rho] + \mathcal{L}$$

Exchange of excitation

$$H = \sum_{j=1}^N \hbar\omega_j \sigma_j^+ \sigma_j^- + \sum_{j \neq l} \hbar v_{j,l} (\sigma_j^- \sigma_l^+ + \sigma_j^+ \sigma_l^-)$$



$$\mathcal{L}_{sink}(\rho) = \Gamma_{N+1} [2\sigma_{N+1}^+ \sigma_k^- \rho \sigma_k^+ \sigma_{N+1}^- - \{\sigma_k^+ \sigma_{N+1}^- \sigma_{N+1}^+ \sigma_k^-, \rho\}]$$

$$\mathcal{L}_{diss}(\rho) = \sum_{j=1}^N \Gamma_j [-\{\sigma_j^+ \sigma_j^-, \rho\} + 2\sigma_j^- \rho \sigma_j^+]$$

Loss of excitation

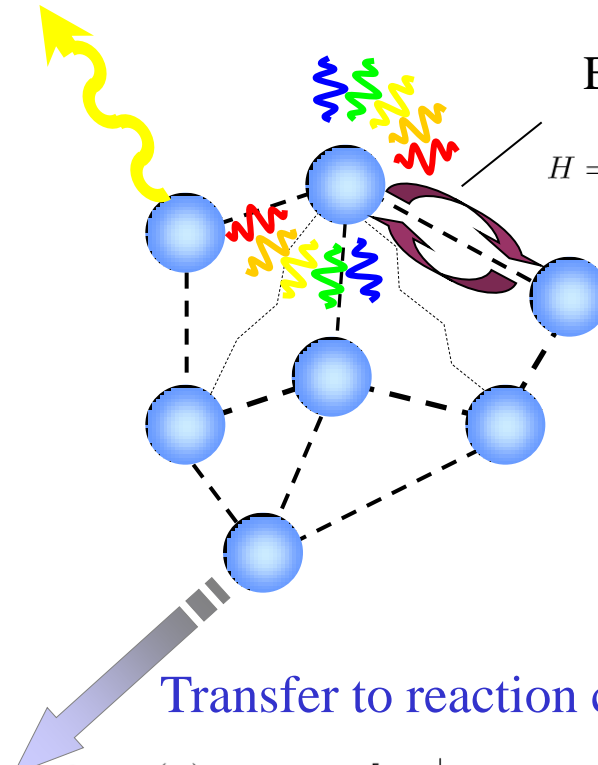
$$\frac{d\rho}{dt} = -\frac{i}{\hbar} [H, \rho] + \mathcal{L}$$

$$\mathcal{L}_{deph}(\rho) = \sum_{j=1}^N \gamma_j [-\{\sigma_j^+ \sigma_j^-, \rho\} + 2\sigma_j^+ \sigma_j^- \rho \sigma_j^+ \sigma_j^-]$$

Dephasing

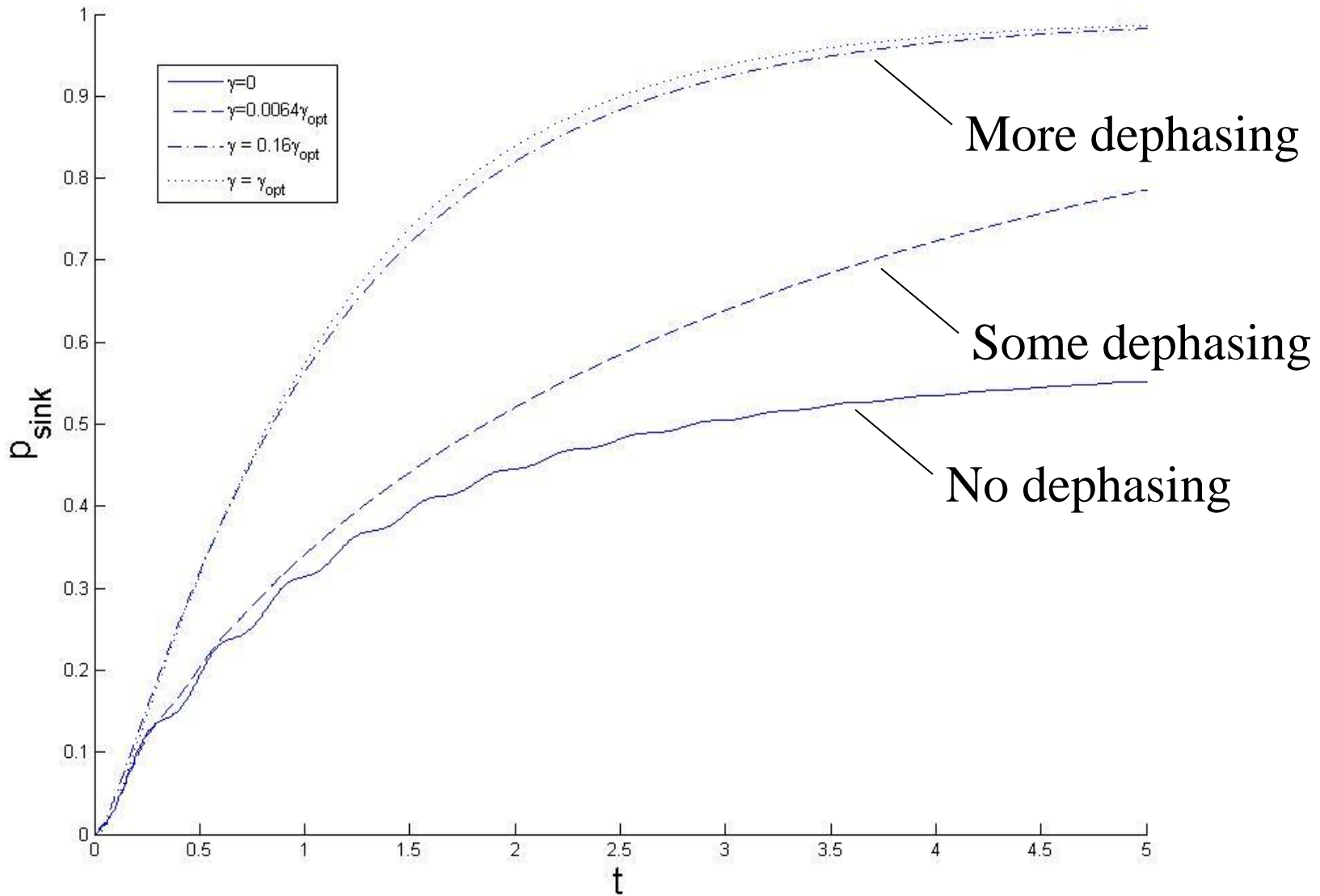
Exchange of excitation

$$H = \sum_{j=1}^N \hbar\omega_j \sigma_j^+ \sigma_j^- + \sum_{j \neq l} \hbar v_{j,l} (\sigma_j^- \sigma_l^+ + \sigma_j^+ \sigma_l^-)$$

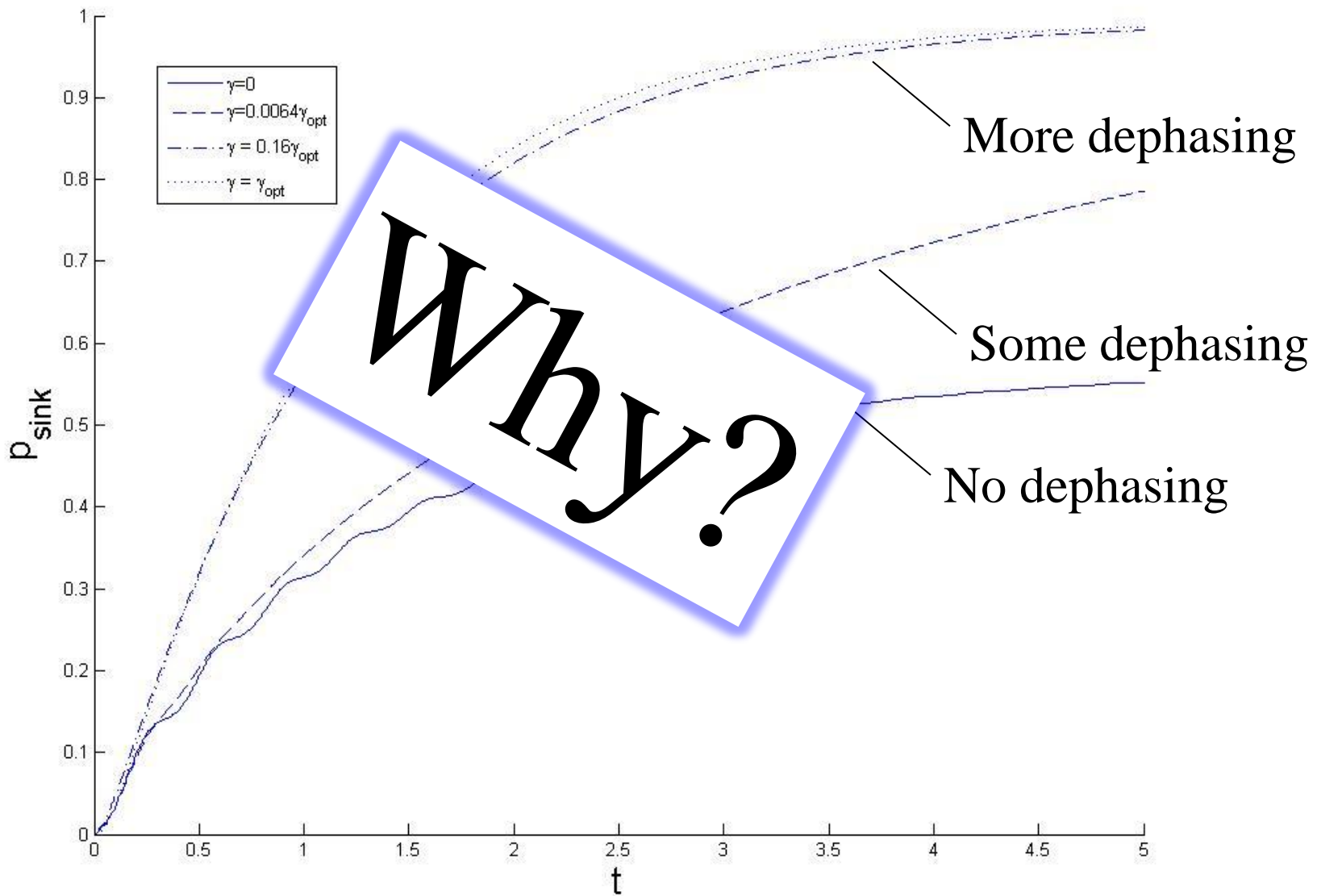


$$\mathcal{L}_{sink}(\rho) = \Gamma_{N+1} [2\sigma_{N+1}^+ \sigma_k^- \rho \sigma_k^+ \sigma_{N+1}^- - \{\sigma_k^+ \sigma_{N+1}^- \sigma_{N+1}^+ \sigma_k^-, \rho\}]$$

Noise Assisted Transport



Noise Assisted Transport





Deconstruct the FMO Hamiltonian

Excitonic states that are excited have small overlap with sink.

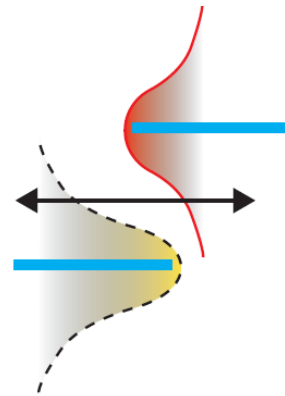
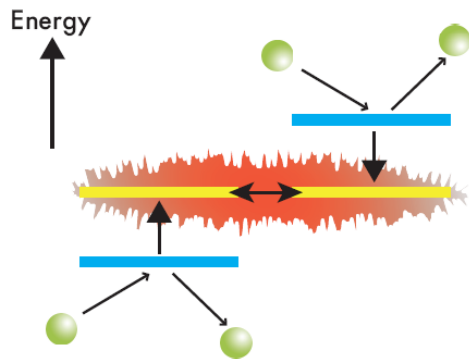
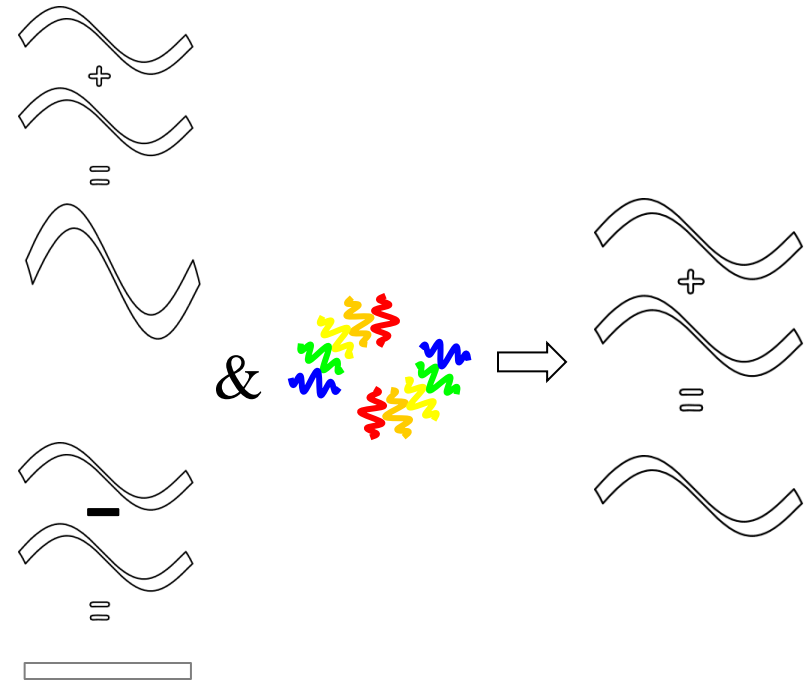
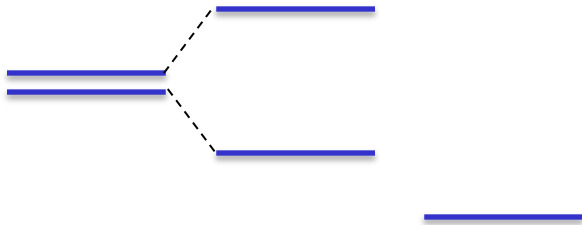


Deconstruct the FMO Hamiltonian

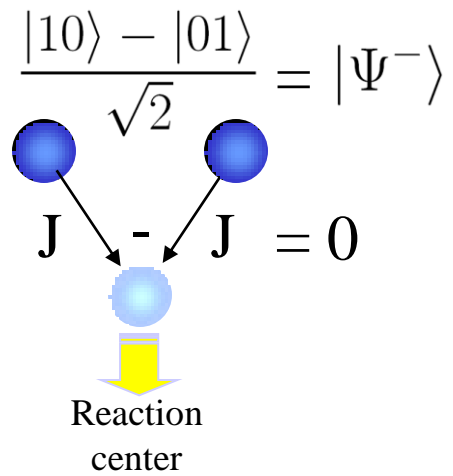
Excitonic states that are excited have small overlap with sink.

$$H = \begin{pmatrix} 215 & -104.1 & 5.1 & -4.3 & 4.7 & -15.1 & -7.8 \\ -104.1 & 220.0 & 32.6 & 7.1 & 5.4 & 8.3 & 0.8 \\ 5.1 & 32.6 & 0.0 & -46.8 & 1.0 & -8.1 & 5.1 \\ -4.3 & 7.1 & -46.8 & 125.0 & -70.7 & -14.7 & -61.5 \\ 4.7 & 5.4 & 1.0 & -70.7 & 450.0 & 89.7 & -2.5 \\ -15.1 & 8.3 & -8.1 & -14.7 & 89.7 & 330.0 & 32.7 \\ -7.8 & 0.8 & 5.1 & -61.5 & -2.5 & 32.7 & 280.0 \end{pmatrix}$$

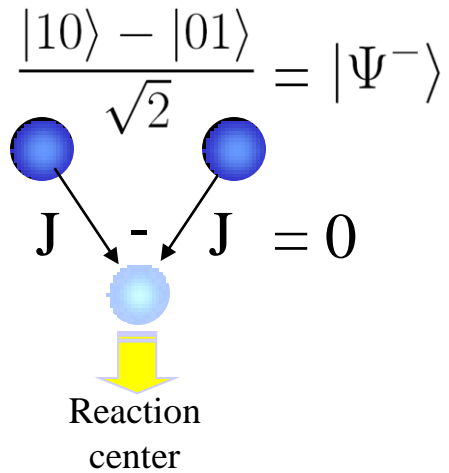
Deconstructing Coherence and Decoherence



Destructive Interference and Invariant States



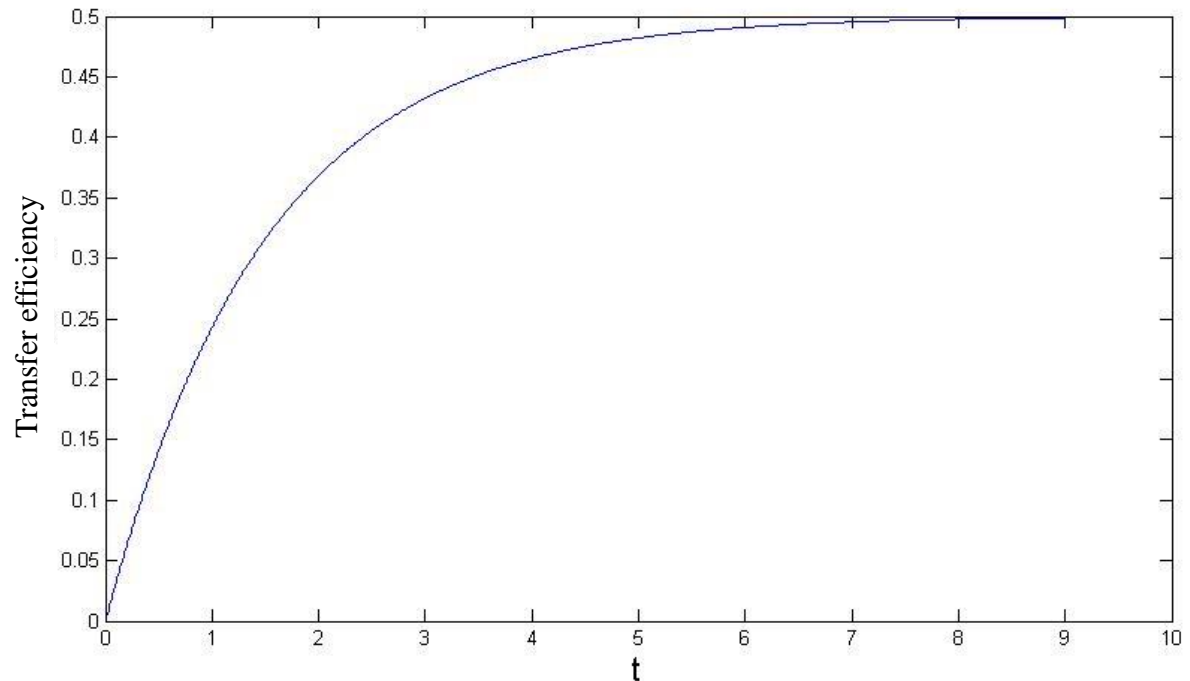
Destructive Interference and Invariant States



$$|01\rangle = \frac{1}{\sqrt{2}} \left[\frac{|01\rangle - |10\rangle}{\sqrt{2}} + \frac{|01\rangle + |10\rangle}{\sqrt{2}} \right]$$

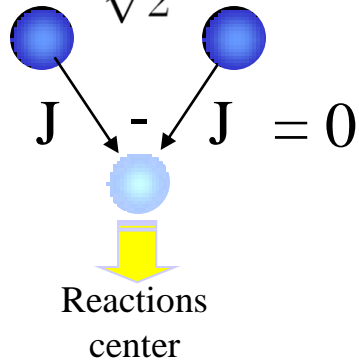


$$\rho = \frac{1}{2} |\psi^-\rangle \langle \psi^-| + \frac{1}{2} |00\rangle \langle 00|$$

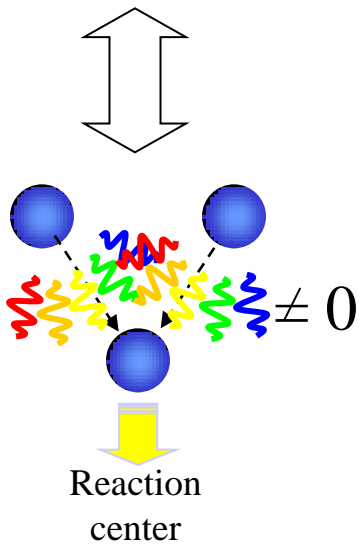


Noise Inhibits Destructive Interference

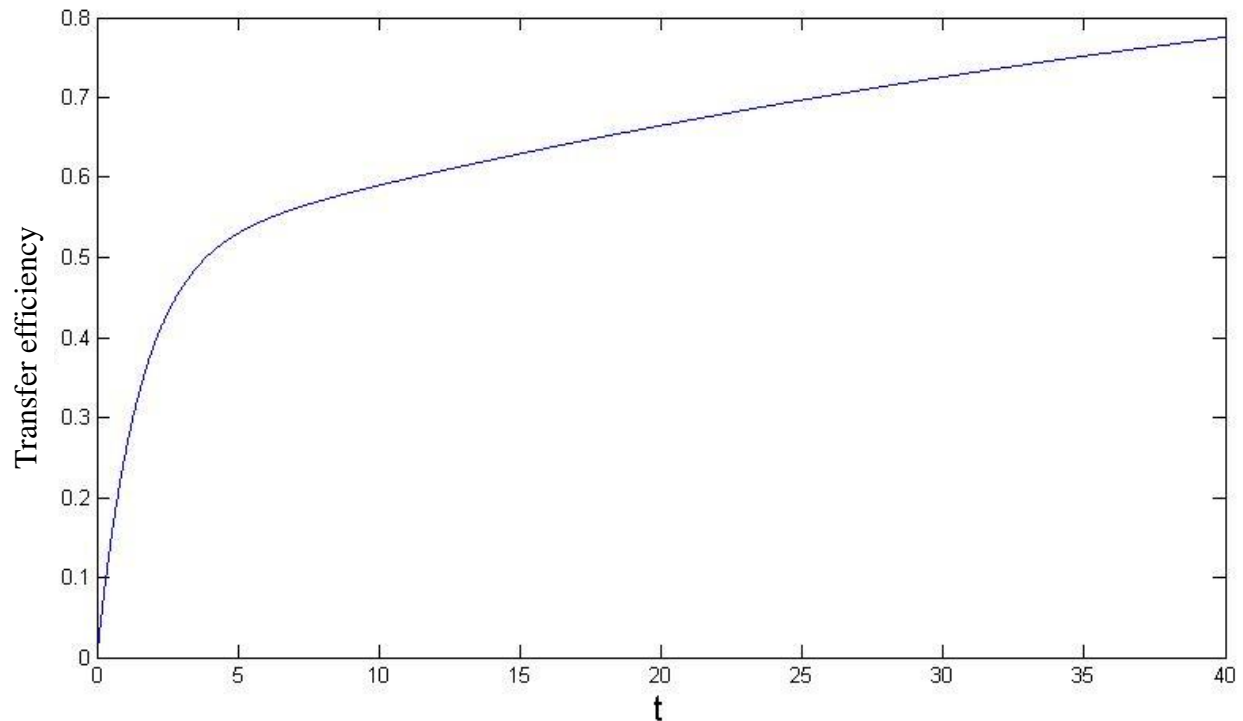
$$\frac{|10\rangle - |01\rangle}{\sqrt{2}} = |\Psi^-\rangle$$



Decoherence inhibits destructive interference !

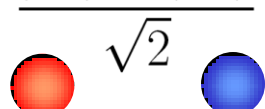


Reduction of destructive interference



Disorder Inhibits Destructive Interference

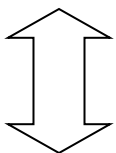
$$\frac{|10\rangle - |01\rangle}{\sqrt{2}}$$



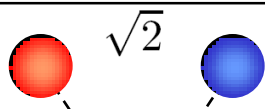
$$J - J = 0$$

Destructive interference

Reaction center



$$\frac{|10\rangle - e^{i\Delta\omega t}|01\rangle}{\sqrt{2}}$$

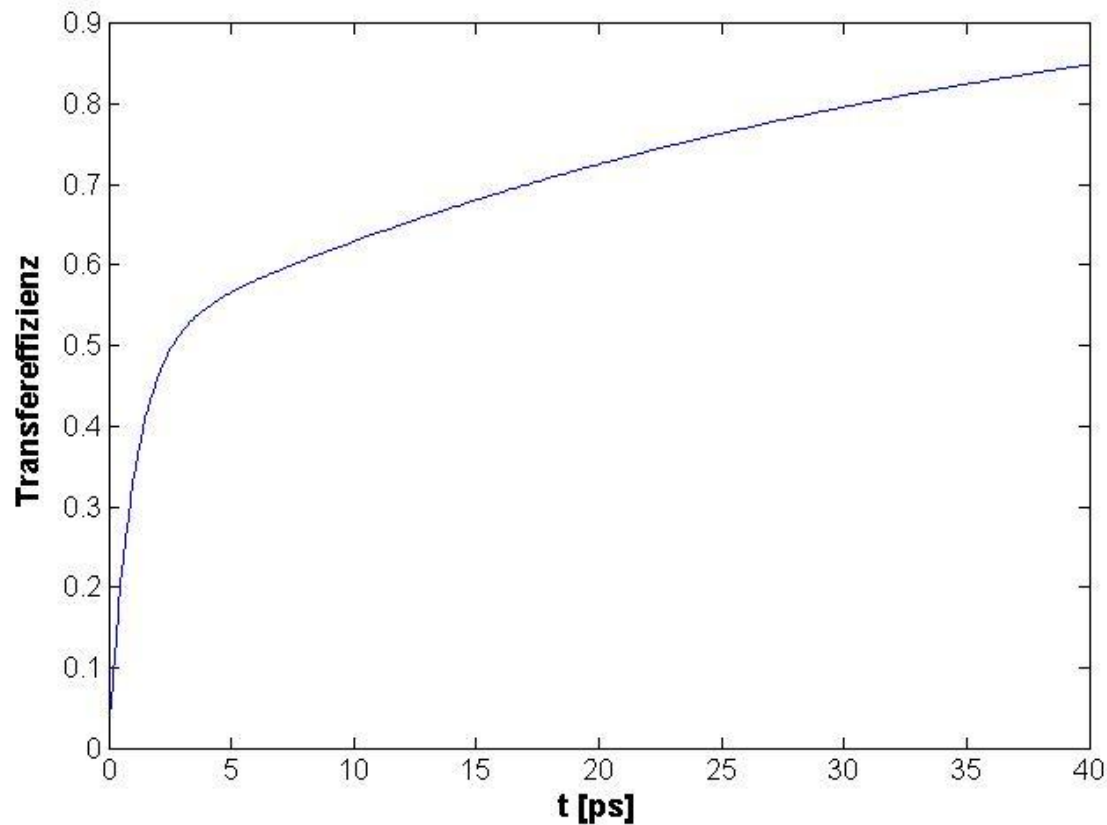


$$J + J \neq 0$$

Reaction center

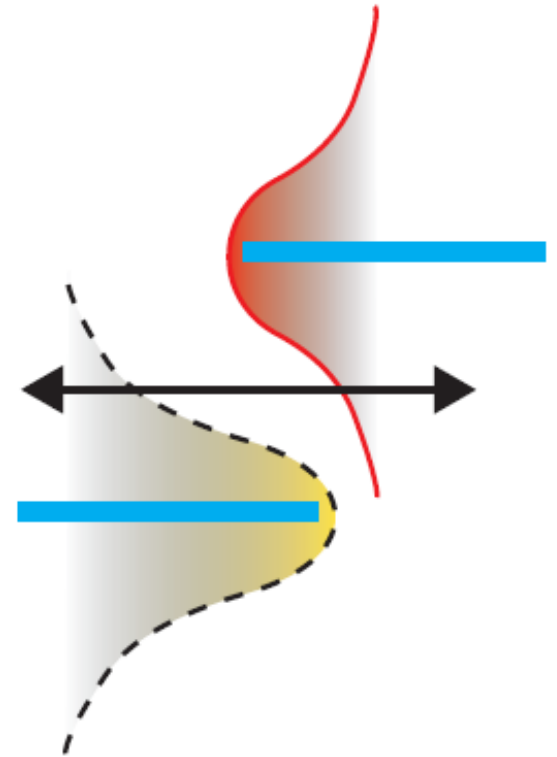
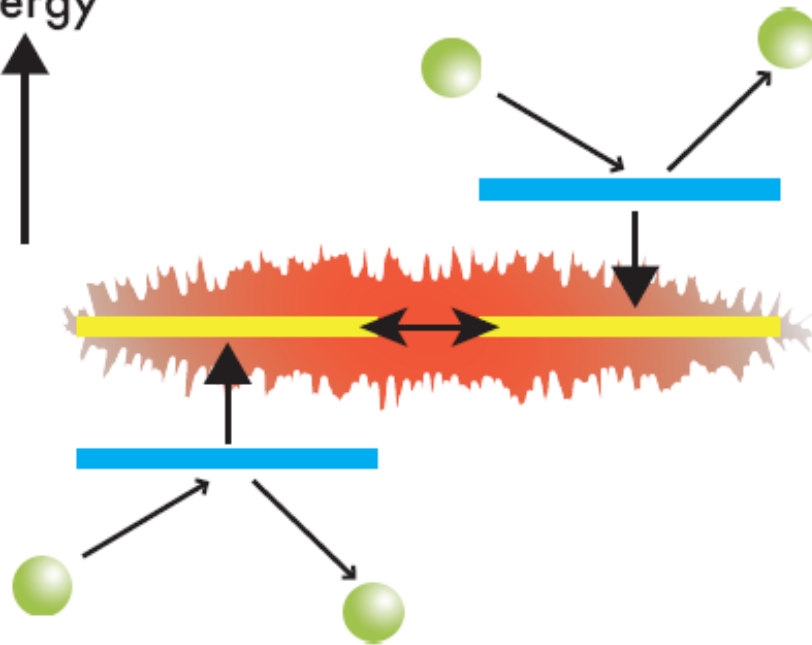
Reduction of destructive interference

Static disorder inhibits destructive interference !

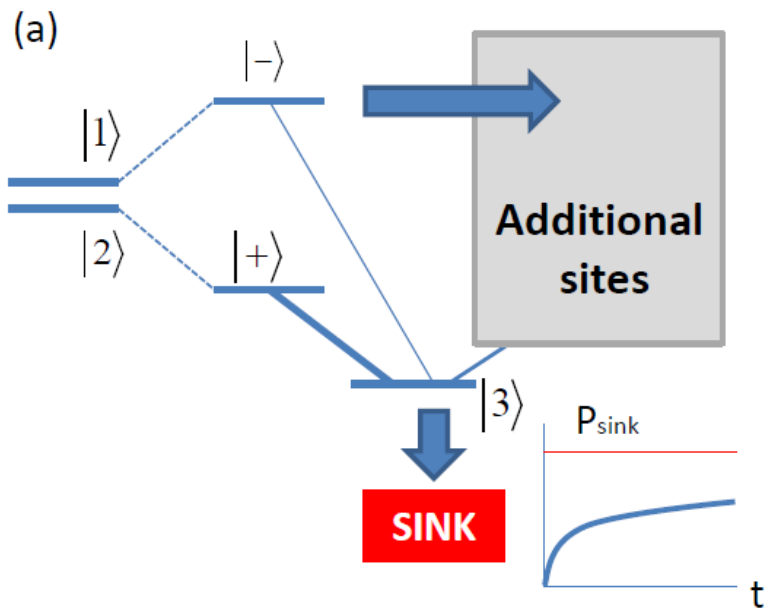


Noise bridges energy gaps

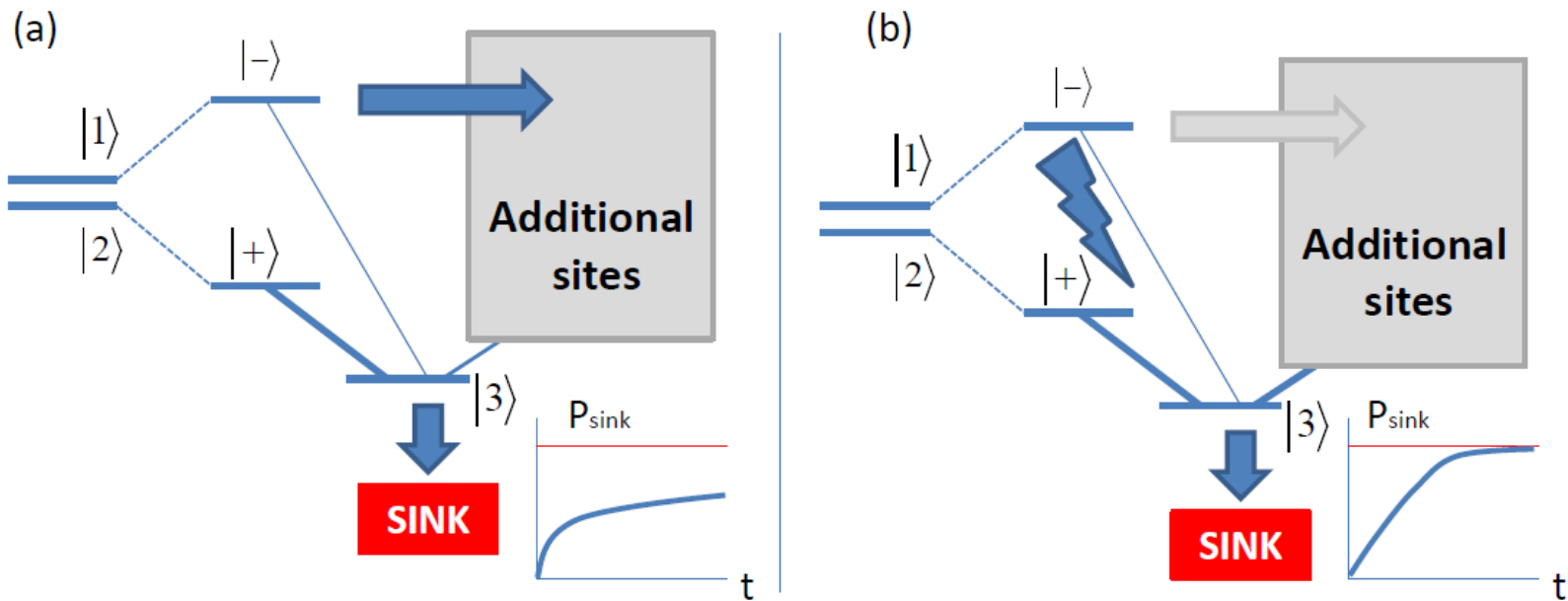
Energy



Coherence shifts resonances



Coherence shifts resonances



Deconstruct the FMO Hamiltonian

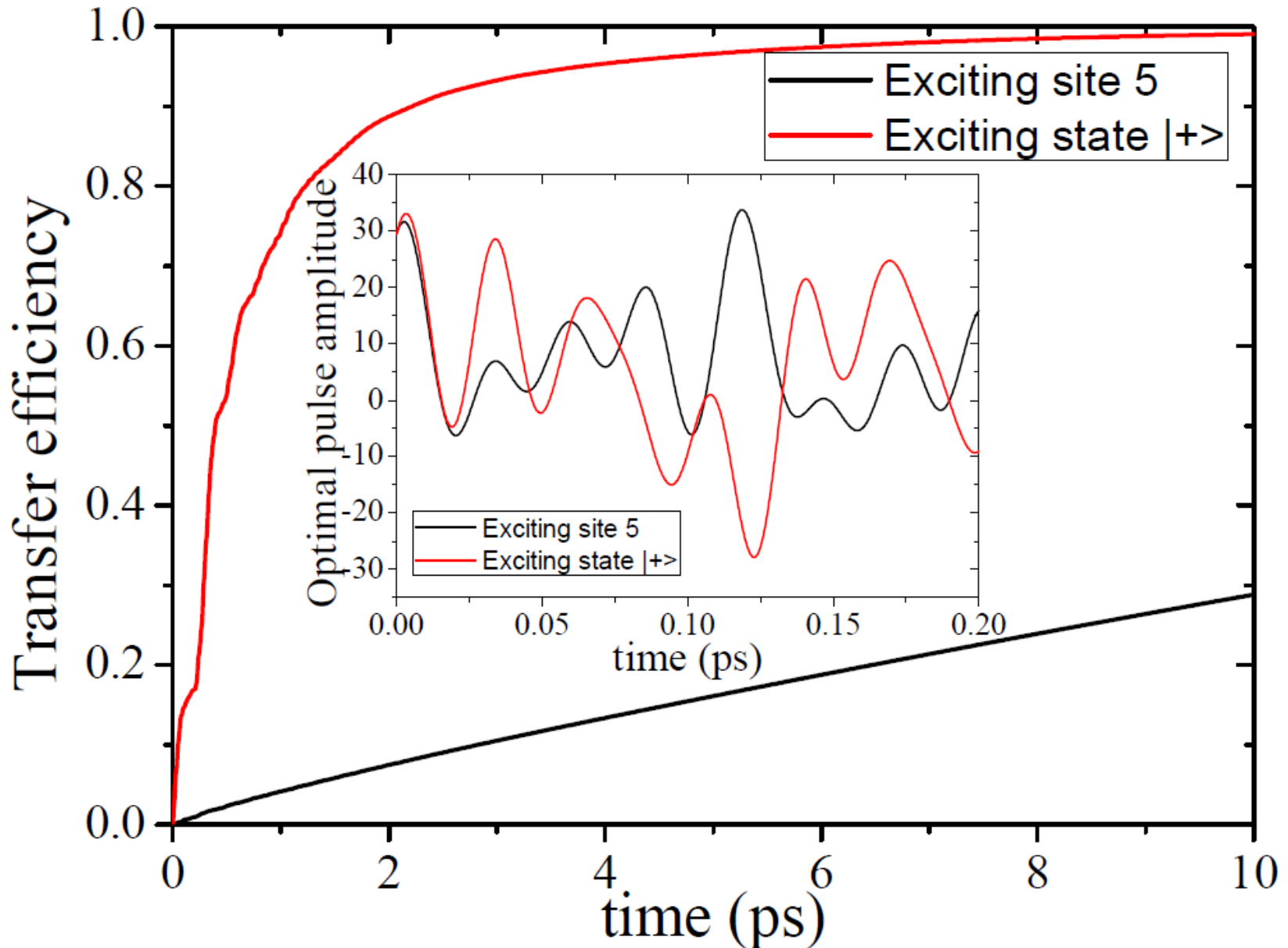
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where we have shifted the zero of energy by 12 230 (all numbers are given in the units of $1.988\,865 \times 10^{-23} \text{ nm} = 1.2414 \times 10^{-4} \text{ eV}$) for all sites corresponding to a wavelength of $\cong 800 \text{ nm}$.

Can test the relevance of structural elements for dynamics by selectively adding noise in computer simulation, but ...

... test in real system would be more convincing

Use optimal control for experimental tests



System – Environment Description

Need description of the dynamics of complex quantum systems in the presence of intermediate noise levels

➤ Master equation approaches (perturbative)
not accurate as interaction is comparable to intersite interaction

➤ Nakajima-Zwanzig (time non-local)
in principle exact but impossible to solve

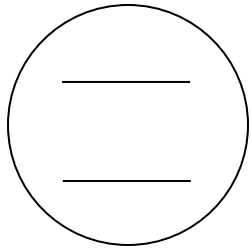
➤ Hierarchy methods
hard for arbitrary spectral densities

Ishizaka & Fleming, J. Chem. Phys. 2009
Ishizaki et al, Phys Chem Chem Phys 2010

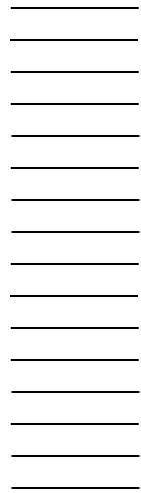
➤ Transformation techniques
Efficient but correlated noise may be challenging

Prior, Chin, Huelga, Plenio, PRL 2010
Chin, Rivas, Huelga, Plenio, J Math Phys 2010

System – Environment Description


 \mathcal{P}

&



Evolution equation of global system

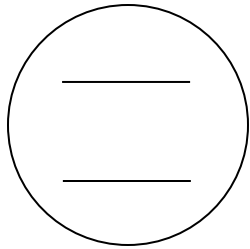
$$\dot{\rho} = L\rho$$

We are interested in dynamics of the system only $\Rightarrow \mathcal{P}\rho$

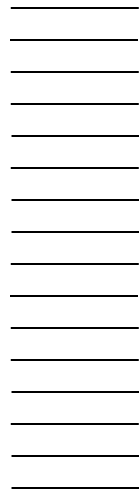
$$\frac{d}{dt} \mathcal{P}\rho = \mathcal{P}L\mathcal{P}\rho + \mathcal{P}L(1 - \mathcal{P})\rho$$

$$\frac{d}{dt} (1 - \mathcal{P})\rho = (1 - \mathcal{P})L(1 - \mathcal{P})\rho + (1 - \mathcal{P})L\mathcal{P}\rho$$

System – Environment Description


 \mathcal{P}

&



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$$\dot{\rho} = L\rho$$

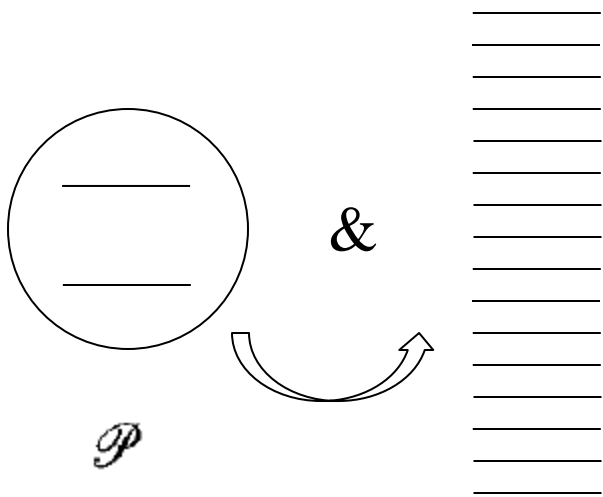
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$$\frac{d}{dt} \mathcal{P}\rho(t) = \mathcal{P}L\mathcal{P}\rho(t) + \int_0^t d\tau \mathcal{P}L e^{(1 - \mathcal{P})L\tau} (1 - \mathcal{P})L\mathcal{P}\rho(t - \tau)$$

System – Environment Description



Evolution equation of global system

$$\dot{\rho} = L\rho$$

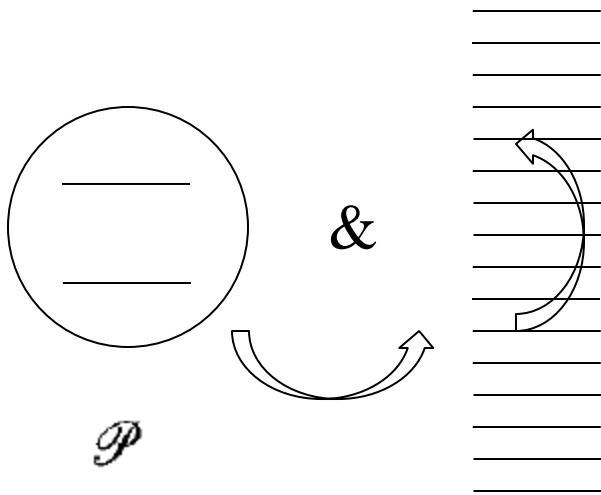
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System – Environment Description



Evolution equation of global system

$$\dot{\rho} = L\rho$$

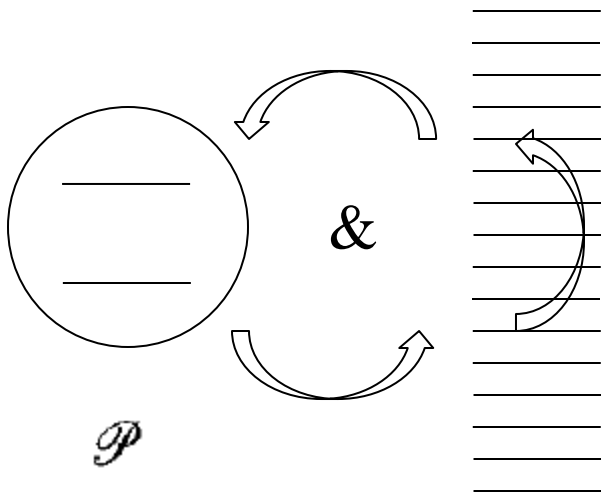
We are interested in dynamics of the system only $\Rightarrow \mathcal{P}\rho$

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System – Environment Description



Evolution equation of global system

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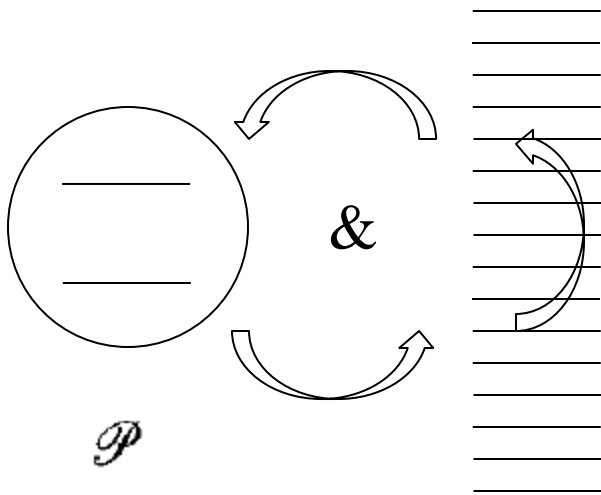
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System – Environment Description



Evolution equation of global system

$$\dot{\rho} = L\rho$$

We are interested in dynamics of the system only $\Rightarrow \mathcal{P}\rho$

$$\frac{d}{dt} \mathcal{P}\rho = \mathcal{P}L\mathcal{P}\rho + \mathcal{P}L(1 - \mathcal{P})\rho$$

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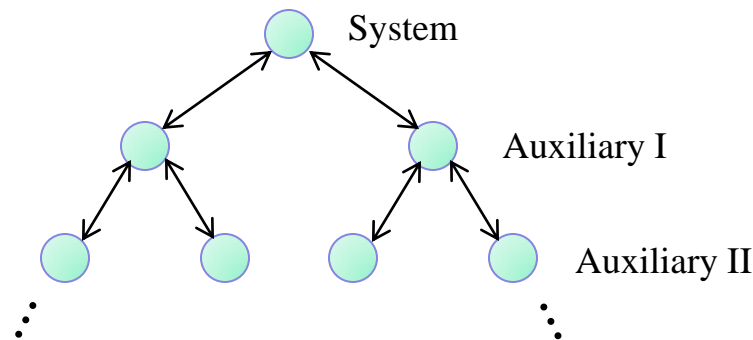
perturbative \Leftarrow Gives Lindblad/Redfield $\rho(t)$

System – Environment Description

Hierarchy method aims to replace integral solution by a set of coupled differential equations with auxiliary operators.

- Has been applied successfully for specific spectral densities (FMO ..)
- Number of elements in hierarchy can grow rapidly
- Coefficients can be hard to compute for general spectral densities
- Error in cut-off not known

Ishizaka & Fleming, J. Chem. Phys. 2009
 Ishizaki et al, Phys Chem Chem Phys 2010

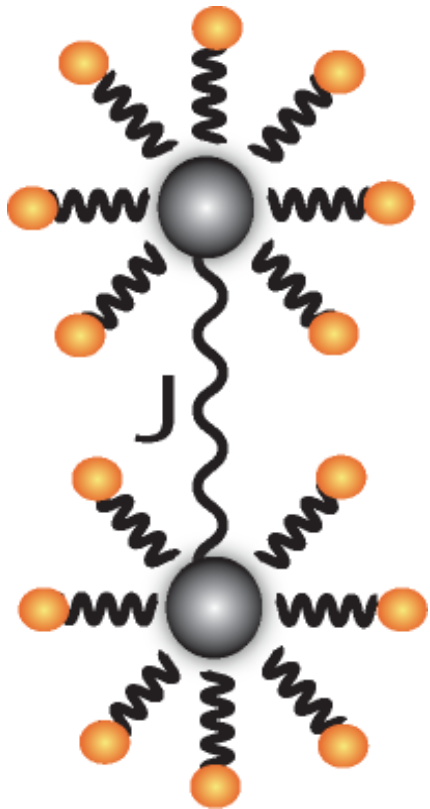


Would like method with controllable and certifiable error that keeps all available information.

Transforming the spin-boson model

Aim: Full **many-body** simulation, treat system and bath on equal footing for arbitrary bath spectral densities

Method of choice for 1-D systems: **T-DMRG** – Numerically exact method for highly-correlated many-body systems in 1D

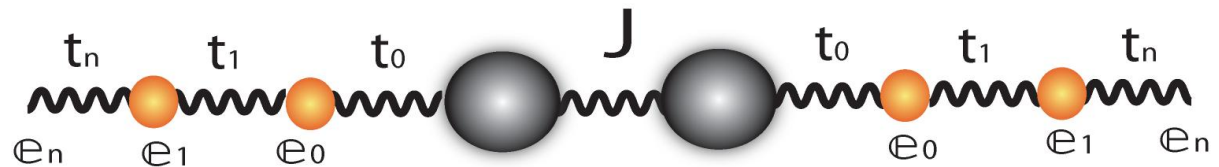
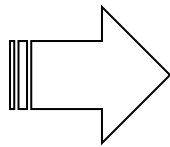
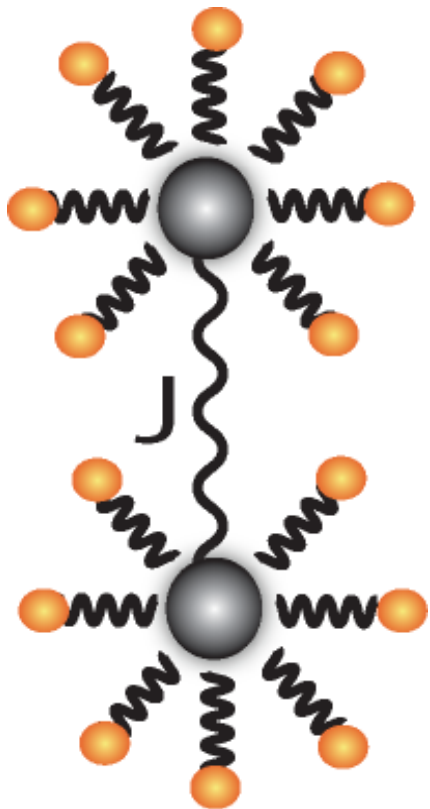


Problem: Geometry of spin-boson model is not T-DMRG friendly

Transforming the spin-boson model

Aim: Full **many-body** simulation, treat system and bath on equal footing for arbitrary bath spectral densities

Method of choice for 1-D systems: **T-DMRG** – Numerically exact method for highly-correlated many-body systems in 1D



Previous implementations – discrete, numerically unstable.

Bulla, Tong & Vojta, Phys. Rev. Lett. 2003;
Bulla, Lee, Tong, & Vojta, Phys. Rev. B 2005.

New – Analytical mapping with orthogonal polynomials

Prior, Chin, Huelga, Plenio, PRL 2010
Chin, Rivas, Huelga, Plenio, J Math Phys 2010

Alternative via (discrete) orthogonal polynomials

$$H_{\text{res}} = \int_0^{x_{\text{max}}} dx g(x) a_x^\dagger a_x$$

usually

$$g(x) = gx$$

$$V = \int_0^{x_{\text{max}}} dx h(x) \hat{A}(a_x + a_x^\dagger)$$

Goal: Find new modes

$$b_n^\dagger = \int_0^{x_{\text{max}}} dx U_n(x) a_x^\dagger$$

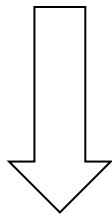
Each $U_n(x)$ can be considered a (orthonormal) polynomial !

such that

$$c_0 \hat{A}(b_0 + b_0^\dagger) + \sum_{n=0}^{\infty} \omega_n b_n^\dagger b_n + t_n b_{n+1}^\dagger b_n + t_n b_n^\dagger b_{n+1}$$

Still need to find the orthogonal polynomials

For each choice of scalar product (hence spectral density) these are uniquely determined and there are recursion relations.



Numerics: **OrthPol** determines these and is numerically stable

W. Gautschi, ACM Trans Math Soft. 1994



Analytics: For many spectral densities we know recursions exactly

$$J(\omega) = 2\pi\alpha\omega_c^{1-s}\omega^s e^{-\frac{\omega}{\omega_c}} = \pi h^2(g^{-1}(\omega)) \frac{dg^{-1}(\omega)}{d\omega}$$

OPs are **Laguerre**
Polynomials

Recurrence coefficients
known **analytically**

$$\omega_n = \omega_c (2n + 1 + s),$$

$$t_n = \omega_c \sqrt{(n + 1)(n + s + 1)}$$



$$J(\omega) = 2\pi\alpha\omega_c^{1-s}\omega^s e^{-\frac{\omega}{\omega_c}} = \pi h^2(g^{-1}(\omega)) \frac{dg^{-1}(\omega)}{d\omega}$$



OPs are **Laguerre**
Polynomials

Formulae show that \rightarrow

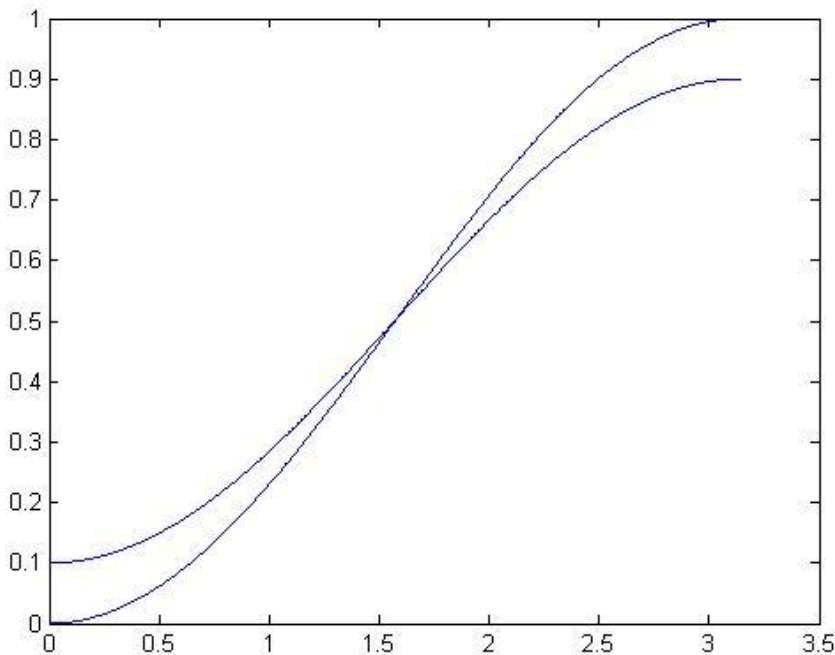
Recurrence coefficients
known **analytically**

$$\lim_{n \rightarrow \infty} \epsilon_n \rightarrow \frac{\omega_c}{2}$$
$$\lim_{n \rightarrow \infty} t_n \rightarrow \frac{\omega_c}{4}$$

Physical origin of asymptotic behaviour



Excitations **propagate away** from system – **Irreversibility**.



Diagonalise asymptotic chain

$$\omega_k = \frac{\omega_c}{2} (1 - \cos(\pi k))$$

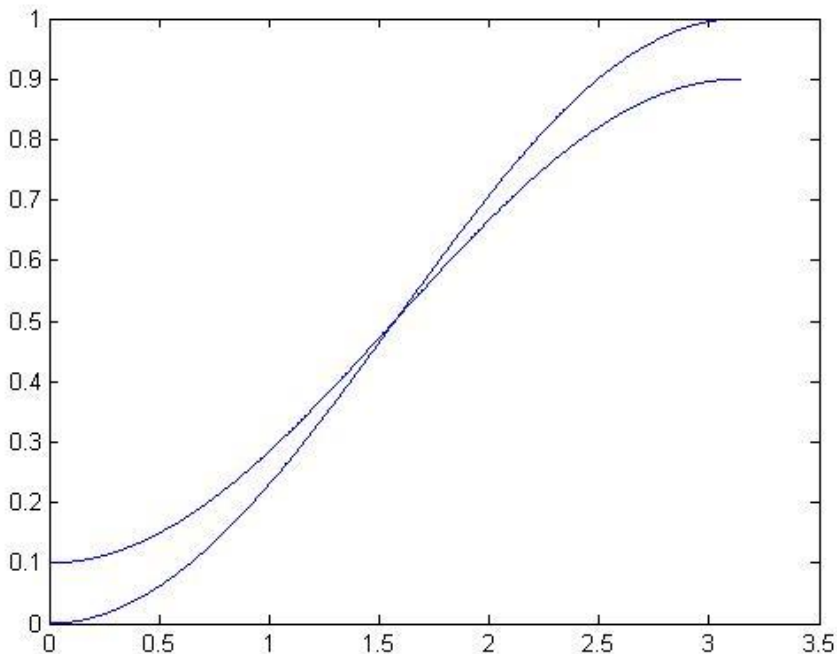
Asymptotic recurrence coefficients give uniform chain with gapless dispersion and bandwidth ω_c

Physical origin of asymptotic behaviour

Encodes spectral density



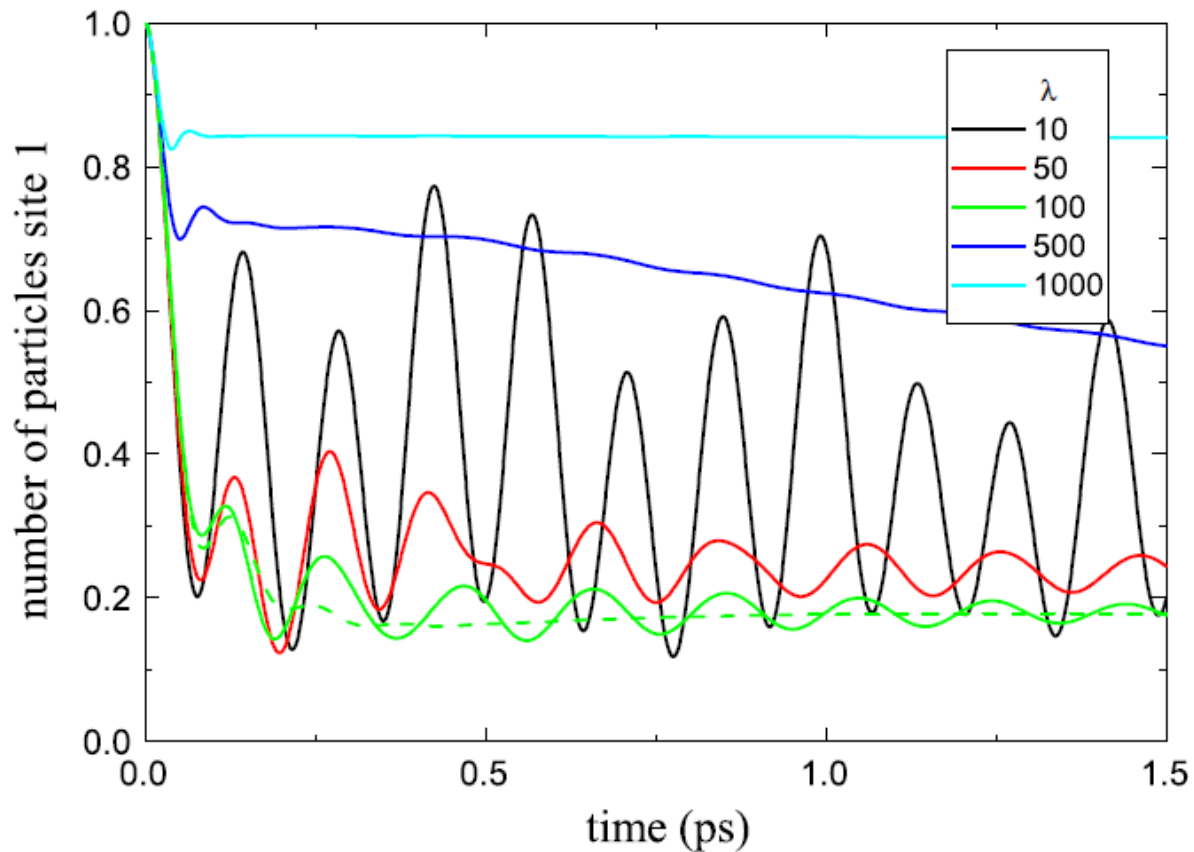
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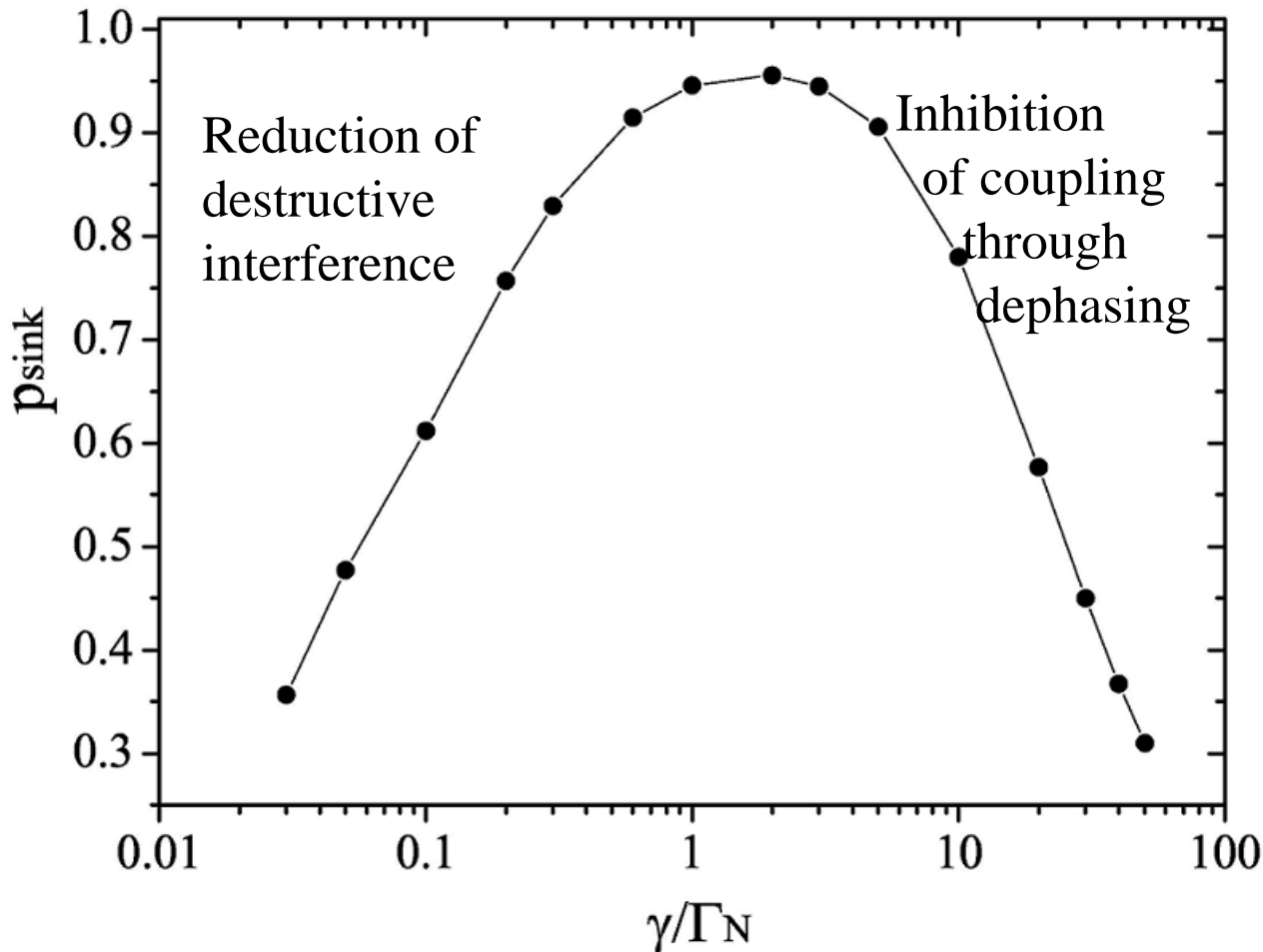
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Asymptotic recurrence coefficients give uniform chain with gapless dispersion and bandwidth ω_c



$$J(\omega) = \frac{2\pi\lambda[1000\omega^5 e^{-(\omega/\omega_1)(1/2)} + 4.3\omega^5 e^{-(\omega/\omega_2)(1/2)}]}{9!(1000\omega_1^5 + 4.3\omega_2^5)} + 4\pi S_H \omega_H^2 \delta(\omega - \omega_H),$$

Dependence on Noise Strength

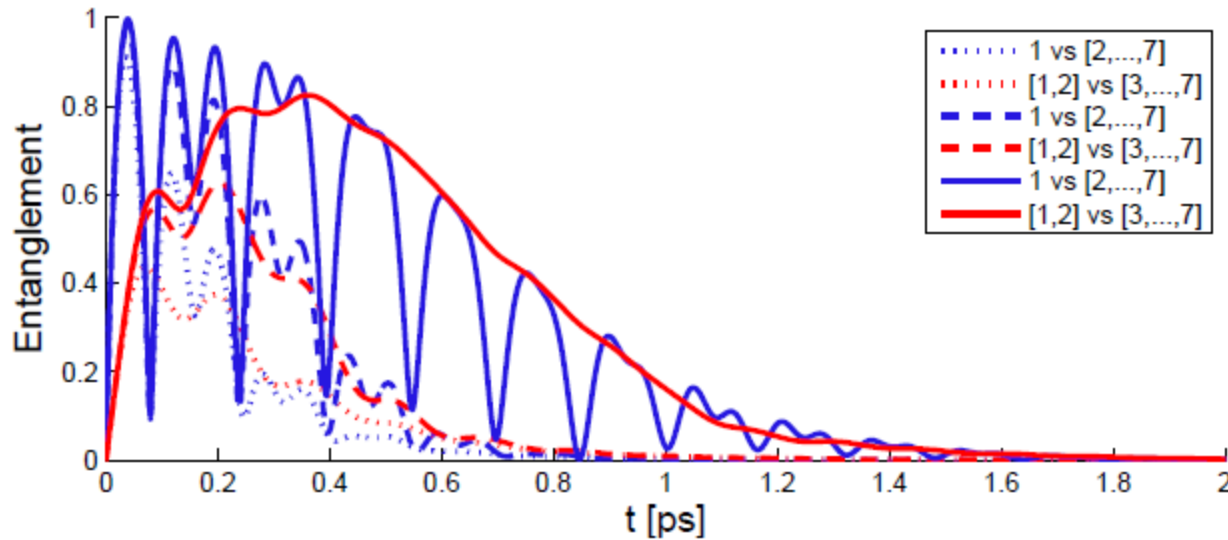


Optimal noise level is such that some coherence survives

System halfway between quantum and classical world !

Classical versus Quantum: Take I

Quantify entanglement/coherence of states and coherence



Agree on subsystems

Define quantity that decreases under local operations

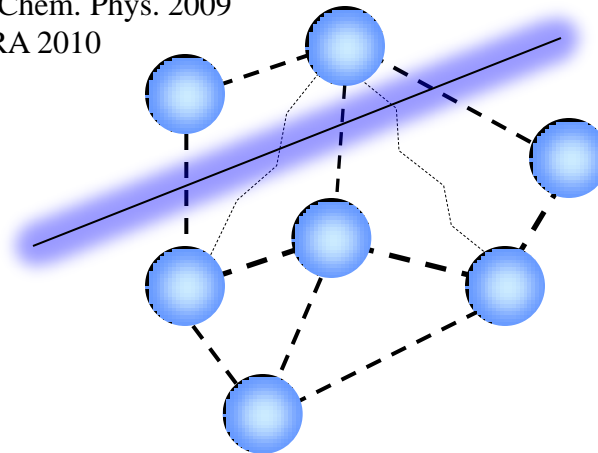
Draw plot & analyze

Is locality requirement natural here ? Why consider entanglement when dynamics is non-local ?

Caruso, Chin, Datta, Huelga, Plenio, J. Chem. Phys. 2009

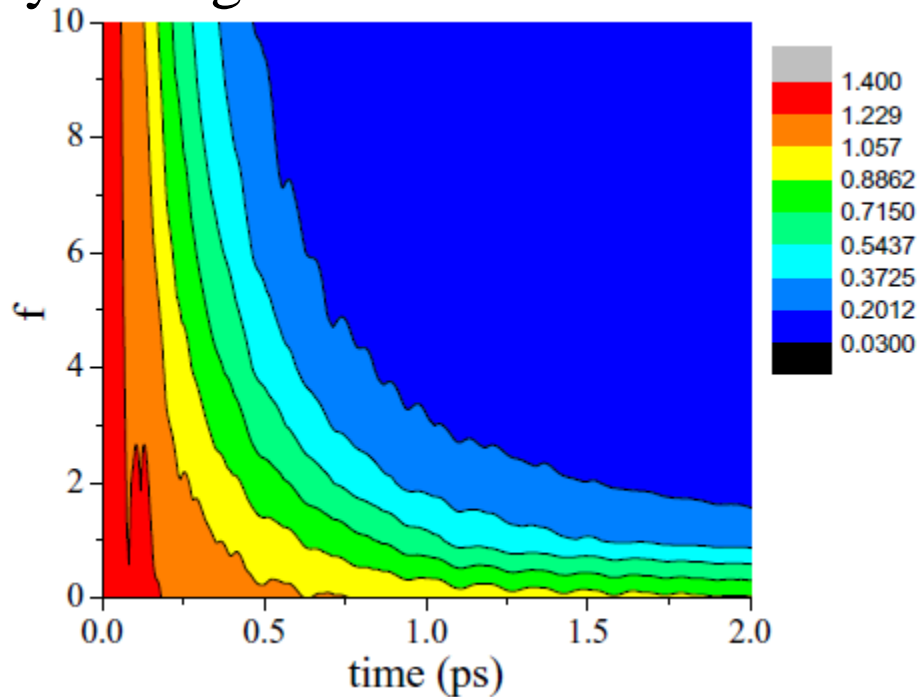
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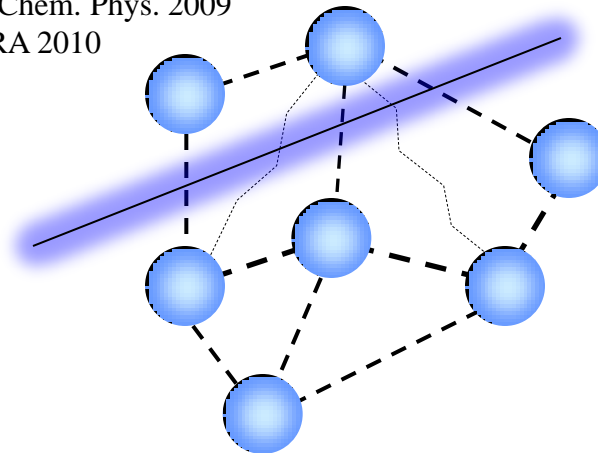


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Tutorial Review:
 Plenio & Virmani, QIC 2007

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Define quantity that decreases under local operations

Draw plot & analyze

Is locality requirement natural here ? Why consider entanglement when dynamics is non-local ?

Consider the power of evolution to generate entanglement.
 → Entangling power

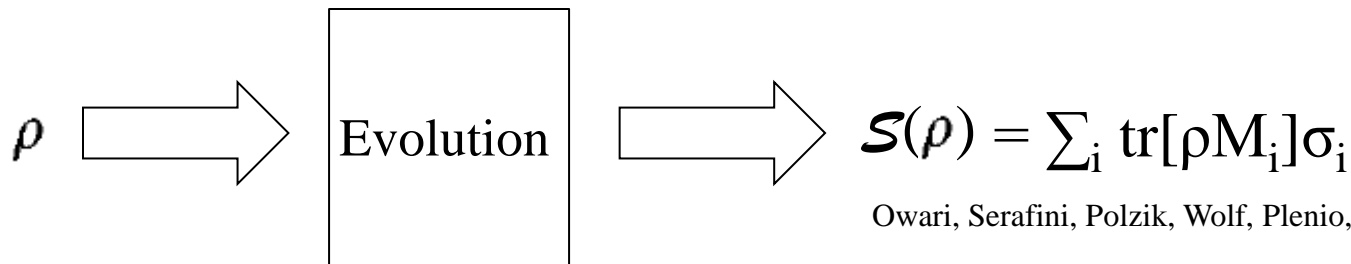
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When is a dynamics classical/quantum ?



Classical versus Quantum: Take II

When is a dynamics classical/quantum ?



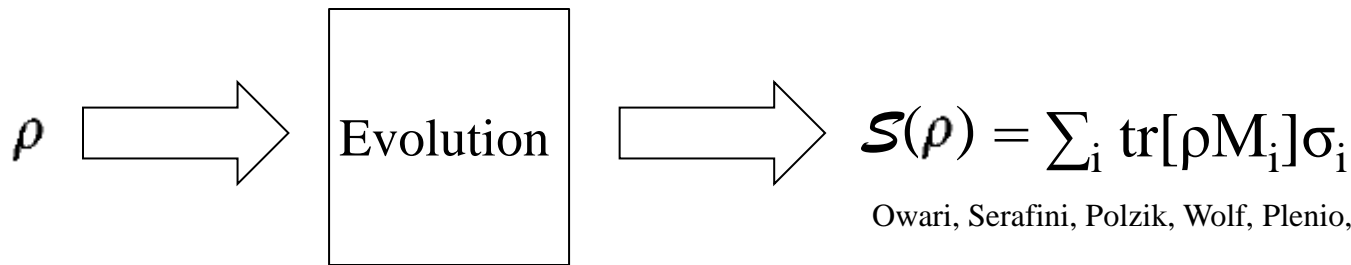
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Is classical if we can replace box my demon that measures and reprepares the state.

Classical states can be perfectly distinguished and reprepared while quantum states cannot.

Classical versus Quantum: Take II

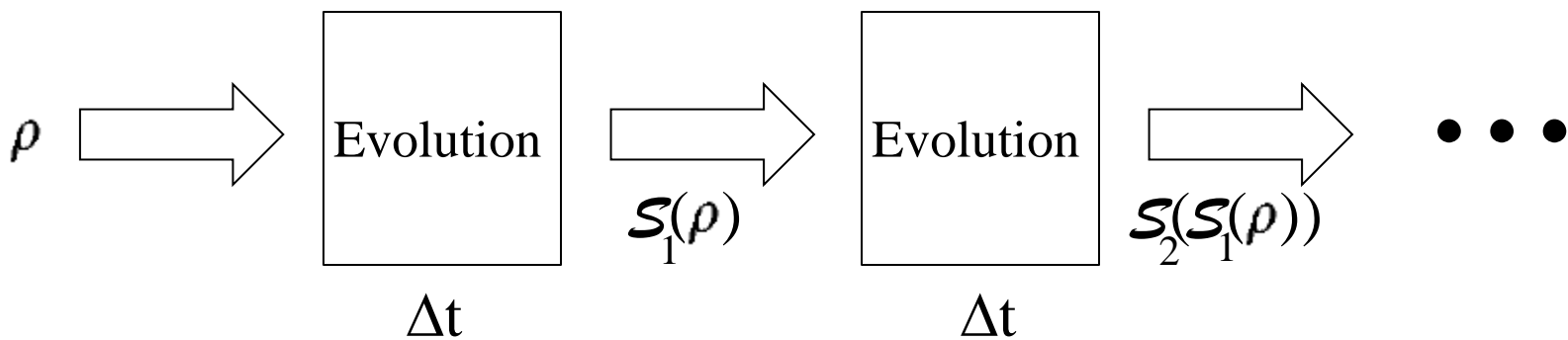
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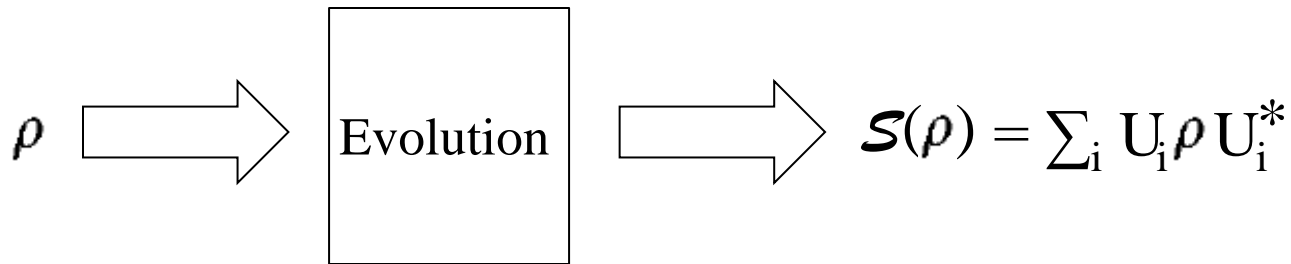
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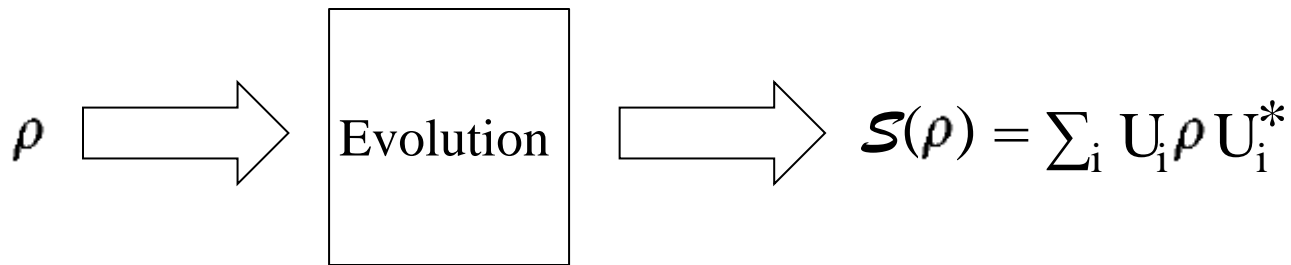


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No quantum correlations are built up with environment and dynamics of environment is classical

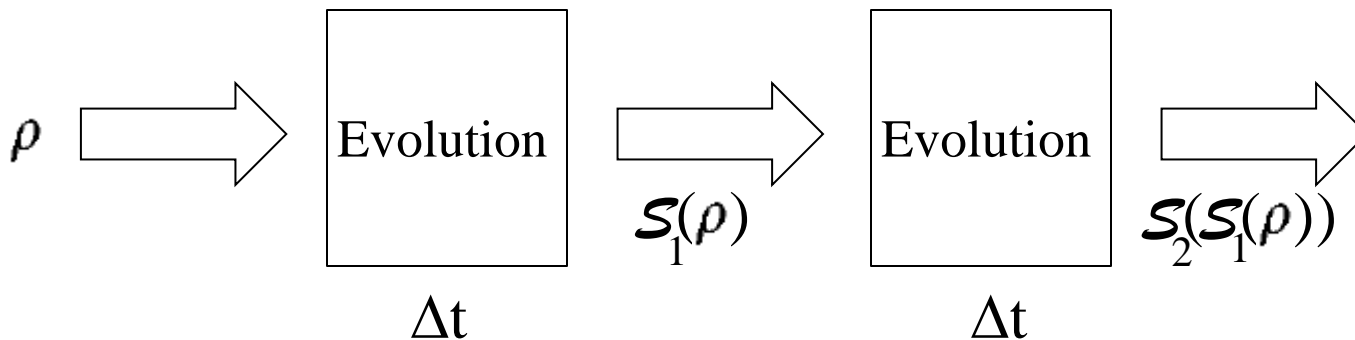
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Optimal performance for intermediate levels of noise, in which master equations or rate equation are inaccurate.

Develop numerical and analytical methods for this regime from QI & condensed matter

For optimal performance system sits between classical and quantum regime

Quantify entanglement, coherence and quantum character of evolution using QI methods

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