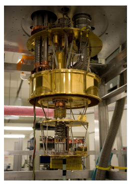
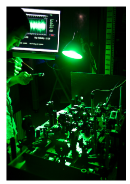
IQC

University of Waterloo



Raymond Laflamme
Institute for Quantum computing
University of Waterloo
www.iqc.ca
laflamme@iqc.ca





Plan

Lecture 1. Quantum Error Correction

Lecture 2. Experimental Quantum Error Correction

Lecture 3. Characterising noise and benchmarking

Plan of Lecture 1

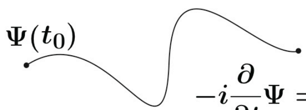
1. Why Quantum Error Correction
2. Classical Error Correction
3. Quantum Error Correction
   - Noise
   - Quantum Error Correcting Codes
   - Accuracy Threshold Theorem

NMR QIP at IQC

J. Chamillard, C. Ryan, B. Coish, M. Ditty, C. Madaiah, G. Pasante, J. Zhang, A. de Souza, M. Laforest, D, Gangloff, O. Moussa, U. Sinha,

thanks also to
J. Baugh and J. Emerson at IQC
and
D. Cory's group at MIT

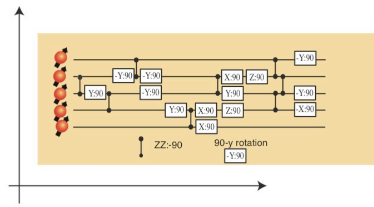
Successes of Quantum Information Science



$$-i\frac{\partial}{\partial t}\Psi = H\Psi$$

- Discovery of the power of quantum mechanics for information processing

  -new language for quantum mechanics



- Discovery of how to control quantum systems
- Proof-of-concepts experiments

Why QEC?

We are not perfect (!):
If a computation takes k gates, the error per gate is p and these errors are independent from gate to gate the probability of success is $(1-p)^k$ an exponential decaying probability as a function of the number of gates.

Origin of the noise:

- **Imprecise control on the system:**

$$U = e^{-i\frac{\pi+\delta}{4}Z}$$

- **The system is not totally isolated**

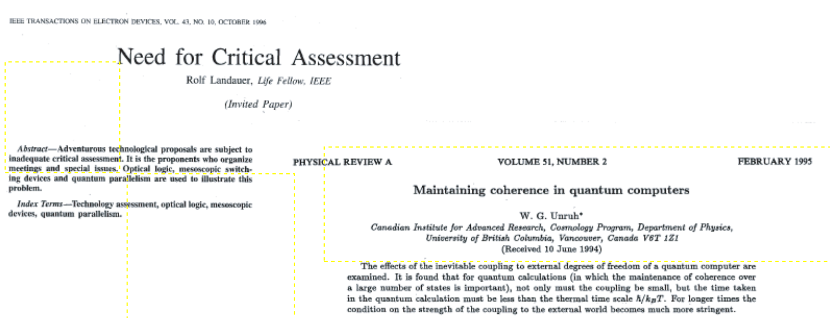
$$U = e^{-i\epsilon Z_1 Z_2}$$

- **We are losing our qubits**

$$|\Psi\rangle \rightarrow \text{nothing}$$

- ...

The Death of Q Computers... (1995)



IEEE TRANSACTIONS ON ELECTRON DEVICES, VOL. 43, NO. 10, OCTOBER 1996

Need for Critical Assessment

Rolf Landauer, *Life Fellow, IEEE*

(Invited Paper)

Abstract—Adventurous technological proposals are subject to inadequate critical assessment. It is the proponents who organize meetings and special issues. Optical logic, mesoscopic switching devices and quantum parallelism are used to illustrate this problem.

Index Terms—Technology assessment, optical logic, mesoscopic devices, quantum parallelism.

PHYSICAL REVIEW A VOLUME 51, NUMBER 2 FEBRUARY 1995

Maintaining coherence in quantum computers

W. G. Unruh*

Canadian Institute for Advanced Research, Cosmology Program, Department of Physics, University of British Columbia, Vancouver, Canada V6T 1Z1

(Received 10 June 1994)

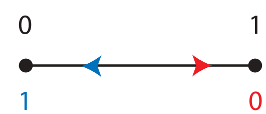
The effects of the inevitable coupling to external degrees of freedom of a quantum computer are examined. It is found that for quantum calculations (in which the maintenance of coherence over a large number of states is important), not only must the coupling be small, but the time taken in the quantum calculation must be less than the thermal time scale $\hbar/k_B T$. For longer times the condition on the strength of the coupling to the external world becomes much more stringent.

Independent error model

Let's assume we have a symmetric error model, independent from one bit to another

With probability (1-p): $0 \Rightarrow 0 \quad ; \quad 1 \Rightarrow 1$
With probability p: $0 \Rightarrow 1 \quad ; \quad 1 \Rightarrow 0$

Space of 1 classical bit



1 bit: classical errors

Error model (lossless)

With probability (1-p): $0 \Rightarrow 0$
With probability (1-q): $1 \Rightarrow 1$
With probability p: $0 \Rightarrow 1$
With probability q: $1 \Rightarrow 0$

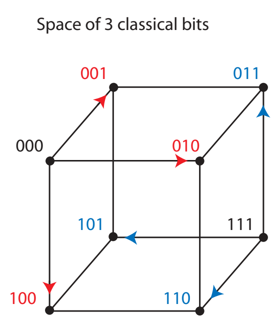
Note: if we have more than one bit we have to learn about correlations between the errors

Independent error model

Thus if we take 3 bits and encode 0 into 000 and 1 into 111 we will have

$$000 \longrightarrow \begin{cases} 000 & (1-p)^3 \\ \left.\begin{array}{l} 001 \\ 010 \\ 100 \end{array}\right\} & p(1-p)^2 \\ \left.\begin{array}{l} 011 \\ 110 \\ 101 \end{array}\right\} & p^2(1-p) \\ 111 & p^3 \end{cases}$$

and an analogous effect on 111.
Let's make the assumption that $p << 1$ and thus we can neglect the second order term in p. Then under the influence of the noise we have the following effect:



Note that the messages and their corresponding corrupted versions do not overlap, i.e. the 000 with corrupted version $001, 010, 10$ does not overlap with 111 or $110, 101, 011$. Thus it is possible to "undo" the effect of the noise by resetting the bits to the one obtain by taking a majority vote of the 3 bits at end. This resets $000, 001, 010, 100$ to 000 and $111, 110, 101, 011$ to 111. If we include the errors which occur to order p^2, we would not be able to correct them.

Independent error model

We can identify the following elements in this error correction operations:

- the noise model
- an encoding
- an error correction operation

Sometimes the encoding is thought as copying the information: if this would be essential it would be impossible in the quantum world because of the no-cloning theorem.
The error correction operation could be thought as measuring the bits and taking majority, this again would not be helpful if it would be essential as it would destroy the quantum information.

Quantum Error Correction

Error models

Generic 1 qubit error

A generic qubit has the state

$$|\Psi\rangle = \alpha|0\rangle + \beta|1\rangle$$

but qubits might not be isolated (and now we know that there can be information hidden in quantum correlation between systems) so the most general evolution which include an environment (with state $|\epsilon\rangle$) takes the form

$$\begin{aligned} |0\rangle|\epsilon\rangle &\longrightarrow |0\rangle|\epsilon_0^0\rangle + |1\rangle|\epsilon_0^1\rangle \\ |1\rangle|\epsilon\rangle &\longrightarrow |0\rangle|\epsilon_1^0\rangle + |1\rangle|\epsilon_1^1\rangle \end{aligned}$$

Quantum Error Correction

and thus

$$\begin{aligned} (\alpha|0\rangle + & \beta|1\rangle)|\epsilon\rangle \longrightarrow \\ & (\alpha|0\rangle + \beta|1\rangle)\frac{1}{2}(|\epsilon_0^0\rangle + |\epsilon_1^1\rangle) \quad (\Rightarrow \mathbb{1}|\Psi\rangle) \\ & +(\alpha|0\rangle - \beta|1\rangle)\frac{1}{2}(|\epsilon_0^0\rangle - |\epsilon_1^1\rangle) \quad (\Rightarrow \mathrm{Z}|\Psi\rangle) \\ & +(\alpha|1\rangle + \beta|0\rangle)\frac{1}{2}(|\epsilon_0^1\rangle + |\epsilon_1^0\rangle) \quad (\Rightarrow \mathrm{X}|\Psi\rangle) \\ & +(\alpha|1\rangle - \beta|0\rangle)\frac{1}{2}(|\epsilon_0^1\rangle - |\epsilon_1^0\rangle) \quad (\Rightarrow \mathrm{i}\mathrm{Y}|\Psi\rangle) \end{aligned}$$

The effect of the noise is to apply the error operators $\mathbb{1}, \mathrm{X}, \mathrm{Y}, \mathrm{Z}$ to the state $|\Psi\rangle$ depending on what the state of the environment is.

Quantum Error Correction

Note that these four operator form an operator basis in the acting on the 2 dimensional Hilbert space of one qubit. For n qubits we have 4^n possible operators, obtained by the tensor product of each one-qubit operator, i.e.. for two qubits we would have $\mathbb{1}\otimes\mathbb{1}, X\otimes\mathbb{1}, \ldots, X\otimes X, \ldots Z\otimes Z$.

Quantum Error Correction

Phase shift or phase flip

Let's look at some simple examples of noise operators in physical systems such as decoherence:

$$\begin{aligned} |0\rangle|\epsilon\rangle &\longrightarrow |0\rangle|\epsilon_0\rangle = |0\rangle|\epsilon\rangle \\ |1\rangle|\epsilon\rangle &\longrightarrow |1\rangle|\epsilon_1\rangle = \mathrm{e}^{\mathrm{i}\theta}|1\rangle|\epsilon\rangle \end{aligned}$$

Thus

$$(\alpha|0\rangle + \beta|1\rangle)|\epsilon\rangle \longrightarrow (\alpha|0\rangle + \mathrm{e}^{\mathrm{i}\theta}\beta|1\rangle)|\epsilon\rangle$$

and which can be rewritten as

$$(\alpha|0\rangle + e^{i\theta}\beta|1\rangle)|\epsilon\rangle = \frac{1+e^{i\theta}}{2}(\alpha|0\rangle + \beta|1\rangle)|\epsilon\rangle + \frac{1-e^{i\theta}}{2}(\alpha|0\rangle - \beta|1\rangle)|\epsilon\rangle = \frac{1+e^{i\theta}}{2}\mathbb{1}(\alpha|0\rangle + \beta|1\rangle)|\epsilon\rangle + \frac{1-e^{i\theta}}{2}Z(\alpha|0\rangle + \beta|1\rangle)|\epsilon\rangle$$

Here we have a certain amplitude ($\frac{1+e^{i\theta}}{2}$) of nothing happening ($\mathbb{1}$) and ($\frac{1+e^{i\theta}}{2}$) of a Z error happening.

Quantum Error Correction

Krauss operators

Let's suppose that a system and its environment start in a separable state and for simplicity that they are both in pure states ($|\Psi\rangle = |\Psi_s\rangle \otimes |\Psi_e\rangle$).

$$\rho_s = Tr_e U|\Psi_e\rangle \otimes |\Psi_s\rangle\langle\Psi_s| \otimes \langle \Psi_e|U^\dagger = \sum_i \underbrace{\langle\Psi_e^i|U|\Psi_e\rangle}_{\mathcal{E}_i} \otimes |\Psi_s\rangle\langle\Psi_s| \otimes \underbrace{\langle \Psi_e|U^\dagger|\Psi_e^i\rangle}_{\mathcal{E}_i^\dagger}$$

The set of operators $\{\mathcal{E}_i\}$ are called Krauss operators. They are not unique, as we can use another basis for the trace over the environment, but up to this freedom they are uniquely defined. The unitarity of the whole

system-environment implies that

$$\sum_i \mathcal{E}_i^\dagger A \mathcal{E}_i = \mathbb{1}$$

The $\{\mathcal{E}_i\}$ described the non-unitary evolution (when we look only at the first system and the initial state factorizes) and describe the noise influencing the device which we want to use for quantum information processing.

Quantum Error Correcting Codes

An error correcting code is a triple $(\mathcal{C}, \mathcal{E}, \mathcal{R})$such that $E(\mathcal{C}, \mathcal{E}, \mathcal{R}) = 0$. This implies that, on the code $\mathcal{C}$,

$$\mathcal{R}_r\mathcal{E}_a = \lambda_{ra}\mathbb{1}.$$

An equivalent definition for a quantum error correcting code in term of properties of a basis state ($|i_L\rangle$) of the code $\mathcal{C}$

$$\langle i_L|\mathcal{E}_a^\dagger\mathcal{E}_b|j_L\rangle = \delta_{ij}c_{ab}$$

• If $i \neq j$ $\rightarrow$ basis states are mapped to orthogonal states

• If $i = j$ $\rightarrow$ that coherence is preserved (relative length of the basis vectors).

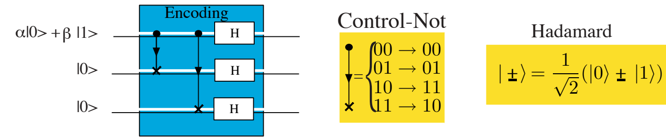
Note if $\{\mathcal{E}_a\}$ is a correctable set of errors, any other set obtained from a linear combination of the these errors also form a correctable set.

The 3-qubit phase error QEC code

Let's look at slightly more complex quantum error correction code, we mention before the model of decoherence. Lets assume that the error are independent from one bit to another. For one qubit the error model is $\{\sqrt{1-p}\mathbb{1}, \sqrt{p}Z\}$. For 3 qubits we get the quantum operation defined by the Krauss operators

$$\{\mathcal{E}_a\} = \{(1-p)^{3/2}\mathbb{1},$$
$$(1-p)\sqrt{p}Z_1, (1-p)\sqrt{p}Z_2, (1-p)\sqrt{p}Z_3,$$
$$p\sqrt{1-p}Z_1Z_2, p\sqrt{1-p}Z_2Z_3, p\sqrt{1-p}Z_1Z_3,$$
$$p^{3/2}Z_1Z_2Z_3\}.$$

Unfortunately we cannot find a code which protects for all these errors but if $p << 1$, the dominant error term is the one error term Z_i term and we can neglect the other ones (as $p^2 << p$).
To protect for at most one Z error, we can get use the encoding from the following quantum circuit



which transform the state into

$$\begin{aligned}(\alpha|0\rangle + \beta|1\rangle)|0\rangle|0\rangle &\rightarrow (\alpha|0\rangle|0\rangle + \beta|1\rangle|1\rangle)|0\rangle \\ &\rightarrow (\alpha|0\rangle|0\rangle|0\rangle + \beta|1\rangle|1\rangle|1\rangle) \\ &\rightarrow (\alpha|+\rangle|+\rangle|+\rangle + \beta|-\rangle|-\rangle|-\rangle).\end{aligned}$$

$$\{\mathcal{E}_a\} \approx \{(1-3p/2)\mathbb{1},$$
$$\sqrt{p}Z_1, \sqrt{p}Z_2, \sqrt{p}Z_3, + \text{higher order in p}$$

and remember that

$$\rho_f = \sum_a \mathcal{E}_a|\Psi\rangle\langle\Psi|\mathcal{E}_a^\dagger$$

And thus the state becomes for each operator

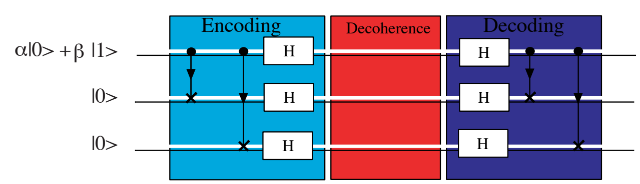
$$(\alpha \ |+\rangle|+\rangle|+\rangle + \beta|-\rangle|-\rangle|-\rangle) \rightarrow$$
$$(\alpha|+\rangle|+\rangle|+\rangle + \beta|-\rangle|-\rangle|-\rangle)\text{with prob. (1-3p/2)}$$
$$(\alpha|-\rangle|+\rangle|+\rangle + \beta|+\rangle|-\rangle|-\rangle)\text{with prob. p}$$
$$(\alpha|+\rangle|-\rangle|+\rangle + \beta|-\rangle|+\rangle|-\rangle)\text{with prob. p}$$
$$(\alpha|+\rangle|+\rangle|-\rangle + \beta|-\rangle|-\rangle|+\rangle)\text{with prob. p}$$

Note: the initial state and its corrupted version are orthogonal and have kept relative coherence

After this circuit we get the states

$$(\alpha|0\rangle + \beta|1\rangle)|0\rangle|0\rangle)\text{with prob. (1-3p/2)}$$
$$(\alpha|1\rangle + \beta|0\rangle)|1\rangle|1\rangle)\text{with prob. p}$$
$$(\alpha|0\rangle + \beta|1\rangle)|1\rangle|0\rangle\text{with prob. p}$$
$$(\alpha|0\rangle + \beta|1\rangle)|0\rangle|1\rangle\text{with prob. p}$$

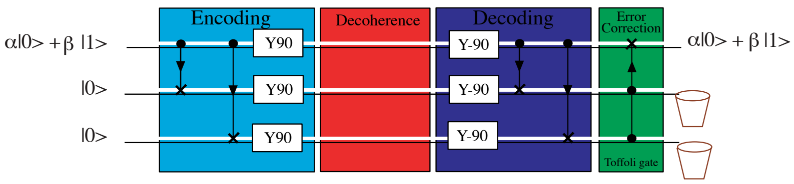
The last two qubits identify which error has occurred. It is called the syndrome.



To get the original state on the first qubit we just need to flip the first bit if and only if the two ancilla bits are in the state $|1\rangle$ (that is called a Toffoli gate) and get

$$(\alpha|0\rangle + \beta|1\rangle)|0\rangle|0\rangle)\text{with prob. (1-3p/2)}$$
$$(\alpha|0\rangle + \beta|1\rangle)|1\rangle|1\rangle)\text{with prob. p}$$
$$(\alpha|0\rangle + \beta|1\rangle)|1\rangle|0\rangle\text{with prob. p}$$
$$(\alpha|0\rangle + \beta|1\rangle)|0\rangle|1\rangle\text{with prob. p}$$

Thus the whole circuit is given by:



Noiseless subsystems

Knill, R L, and Viola. PRL, 84:25252528, 2000

If the noise operators have a symmetry it might possible to find a set of observable that commutes with it, that defines noiseless subsystems with which we could process quantum information

$$[\mathcal{O}_i, \mathcal{E}_a] = 0$$

e.g. if noise is described by the operators $\{1\!\!1, Z_1 Z_2\}$, then a qubit of information can be defined through the operators

$$1\!\!1_E = 1\!\!1_1 1\!\!1_2, X_E = Z_1, Y_E = -Y_1 X_2, Z_E = X_1 X_2.$$

In the above case the, the eigenstates of the operators define the subspace $|01\rangle \pm |10\rangle$ and it is called decoherence free subspace.

Zanardi, Rasetti, PRL 79, 3306, 1997

Duan, Guo, PRL79, 1953, 1997

Lidar, Chuang, Whaley, PRL 81, 2594, 1998

Fault tolerant QEC

We have seen how we can take quantum information and encoded it in a new state so that is more robust against corruption. This is a big step towards having robust quantum information processing. But there some of the questions remaining:

- How do we find codes?
- How do we protect information during information processing?
- How do we encode so that a given algorithm with "N" gates is performed robustly?
- ...

Encoded operations and error propagation

Everything we have done now has assumed that we wanted to keep a state intact, but in quantum computation we need to manipulate states, i.e we need to make transformation

$$\alpha|0_L\rangle + \beta|1_L\rangle \rightarrow \alpha'|0_L\rangle + \beta'|1_L\rangle$$
$$\alpha|+++\rangle + \beta|---\rangle \rightarrow \alpha'|+++\rangle + \beta'|---\rangle$$

We could decode, then do an operation on the qubit and reencode, but this would leave the qubit unprotected from noise. So we need to do gates in such a way that they remain protected.

A crucial element for understanding how to implement gates in a fault tolerant way on encoded states is to see how errors propagates through a circuit. In particular there are gates organized in such a way that one error will propagate to more than one error. These are bad as, if we use 1 error correcting codes, these gates will destroy the advantage of error correction.

A useful set of operations are the normalizer operations. They are operation which preserve the Pauli operators.
Example are given by The Pauli matrices themselves:

$$X \rightarrow ZXZ = -X$$

but these one either give you the same operator or minus this operator. More interesting is the Hadamard gate

$$H = H^\dagger = \begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix} \qquad (1)$$

$$X \rightarrow HXH = Z \text{ and } Z \rightarrow HZH = X$$

Even more interesting are the controlled-gate is also in the normalizer

$$\begin{aligned} X\mathbb{1} &\rightarrow \text{CNOT}\, X\mathbb{1}\, \text{CNOT} = XX \\ Z\mathbb{1} &\rightarrow \text{CNOT}\, Z\mathbb{1}\, \text{CNOT} = Z\mathbb{1} \\ \mathbb{1}X &\rightarrow \text{CNOT}\, \mathbb{1}X\, \text{CNOT} = \mathbb{1}X \\ \mathbb{1}Z &\rightarrow \text{CNOT}\, \mathbb{1}Z\, \text{CNOT} = ZZ \end{aligned}$$

The normalizer can be generated by by the gates

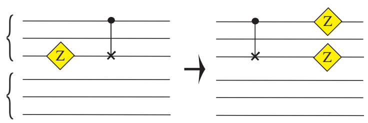
$$\text{Hadamard:} H = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix}$$

$$\text{Phase gate:} P = \begin{pmatrix} 1 & 0 \\ 0 & i \end{pmatrix} \text{ and}$$

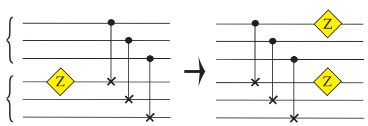
$$\text{CNOT} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{pmatrix}$$

Transversal gates

Bad error:



Good Error:



This latter gate is called transversal, i.e. one qubit of an encoded quite affect acts on at most one qubit of another encoded qubit. Stabilizer operations can be implemented through transversal gates.

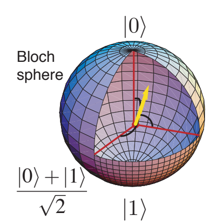
Fault tolerant QEC

Other gates

The normalizer gates are not a universal set of gates when they act on the Pauli matrices. So we need to be able to make other gates. Adding the preparation of the magic state

$$\rho = \frac{1}{2}(\mathbb{1} + \frac{1}{\sqrt{3}}(X + Y + Z))$$

is sufficient to make a universal set of gates. To insure that we are fault tolerant we should show that we can reliably check that we got the magic state or find a way to reduce decrease its imperfection appropriately.



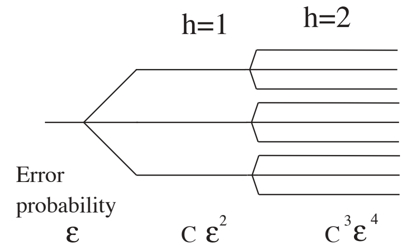
Error correcting codes and fault tolerant operations

The elements of the previous page show that we can we can protect and manipulate information in fault tolerant way: i.e. that if the bare error rate is ϵ, the error rate after the information has been encoded becomes $c\epsilon^2$ (where we have assumed that a one error correcting code has been used.

Thus we can increase the number of operation we do reliably from $1/\epsilon$ to $1/c\epsilon^2$.

The next question is how to increase this number of operations so we can do an algorithm with a larger number of gates?

The idea is to increase the number of error which can be corrected. This can be done in different ways: either by looking at better codes or another possibility is to use concatenation. This idea of the latter methods is to reencode encoded qubits in a hierarchical way. The advantage of this method is that it is possible to arrange the gates so that the error model is the same at all level of the hierarchy.



Accuracy threshold theorem:

In presence of noise, a quantum computation can be as long as required with any desired accuracy using only a polynomial increase of resources as long as the noise level is below a threshold value:

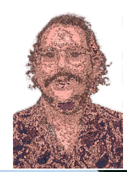
$$P_{error} < P_{threshold}$$

The threshold can be estimated calculated to be around 10^{-2} with the following assumptions:

- Operations can be done in parallel
- Errors are independent from one qubit to qubit
- Any two qubits can interact in one operation
- There are no lost of qubits
- Classical computing comes for free
- There is a supply of fresh qubits on demand at no cost

Proofs bring the threshold to $10^{-3,-4}$

Accuracy threshold theorem



A quantum computation can be as long as required with any desired accuracy as long as the noise level is below a threshold value

$P < 10^{-6,-5,-4,\ldots,-1?}$

Knill et al.; Science, 279, 342, 1998
Kitaev, Russ. Math Survey 1997
Aharonov & Ben Or, ACM press
Preskill, PRSL, 454, 257, 1998

Significance:

-imperfections and imprecisions are not fundamental objections to quantum computation
-it gives criteria for scalability
-its requirements are a guide for experimentalists
-it is a benchmark to compare different technologies

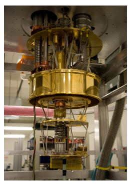
Ingredients for FTQEC

- **Parallel operations**
- **Good quantum control**
- **Ability to extract entropy**
- **Knowledge of the noise**
  - No lost of qubits
  - Independent or quasi independent errors
  - Depolarising model
  - Memory and gate errors
  - ...

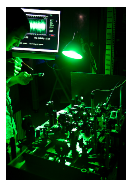
IQC



Raymond Laflamme
Institute for Quantum computing
University of Waterloo
www.iqc.ca
laflamme@iqc.ca







Plan

Lecture 1. Quantum Error Correction

Lecture 2. Experimental Quantum Error Correction

Lecture 3. Characterising noise and benchmarking

Plan of Lecture 2

1. Assumptions of accuracy threshold theorem

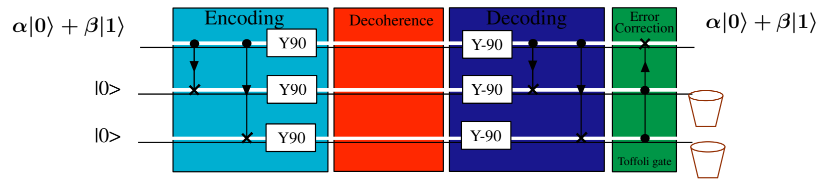
2. Demonstration of Control
   - 3 qubit QEC
   - 5 qubit code
   - DFS and neutron interferometry
   - Magic state distillation

3. Pumping out entropy

Ingredients for FTQEC

- Parallel operations
- Good quantum control
- Ability to extract entropy
- Knowledge of the noise
  - No lost of qubits
  - Independent or quasi independent errors
  - Depolarising model
  - Memory and gate errors
  - . . .

3 qubit code for phase errors



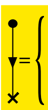
$\alpha|0\rangle + \beta|1\rangle$

$\alpha|+++\rangle + \beta|---\rangle$
$\alpha|-++\rangle + \beta|+--\rangle$
$\alpha|+-+\rangle + \beta|-+-\rangle$
$\alpha|++-\rangle + \beta|--+\rangle$

Errors: $+ \rightarrow -$
$- \rightarrow +$

$(\alpha|0\rangle + \beta|1\rangle)|00\rangle$
$(\alpha|1\rangle + \beta|0\rangle)|11\rangle$
$(\alpha|0\rangle + \beta|1\rangle)|01\rangle$
$(\alpha|0\rangle + \beta|1\rangle)|10\rangle$

Control-Not



$$\begin{cases} 00 \rightarrow 00 \\ 01 \rightarrow 01 \\ 10 \rightarrow 11 \\ 11 \rightarrow 10 \end{cases}$$

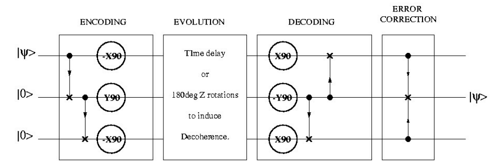
$$|\pm\rangle = \frac{1}{\sqrt{2}}(|0\rangle \pm |1\rangle)$$

$$(\alpha|0\rangle + \beta|1\rangle) \otimes \begin{cases} 00 \\ 11 \\ 01 \\ 10 \end{cases}$$

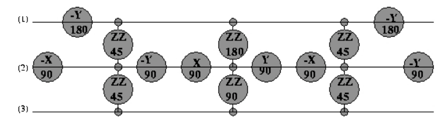
$\sim 1 - 3\gamma^2$

Phase QEC NMR circuit

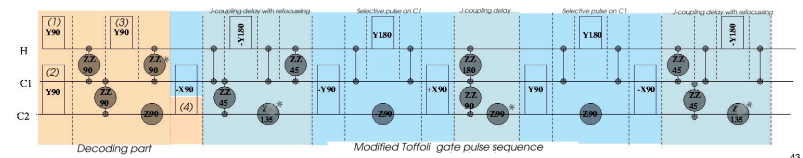
NMR implementation of the decoding and error correction:



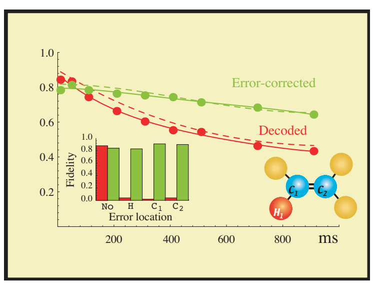
Toffoli gate:



and the full decoding and Toffoli, including some optimization



Experimental results



The demonstration of quantum error correction is in the shape of the green curve which does not have the first order error,i.e. with no error correcting (red curve) the fidelity goes as $e^{-t/T_2} \approx 1 - t/T_2$ and if we have error correction as $\approx 1 - c(t/T_2)^2 + \dots$ The green curve is much flatter than the red one.

Experimental Quantum Error Correction:
D. G. Cory, M. D. Price, W. Maas, E. Knill, R. Laflamme, W. H. Zurek,T. F. Havel and S. S. Somaroo, PRL 81, 2152, 1998

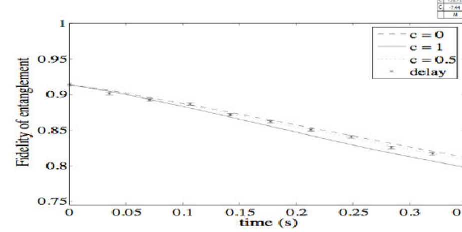
The 3 qubit QEC: noise correlation

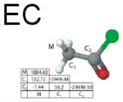
It is possible to use quantum error correction to learn about properties of the noise, e.g. properties of correlations between phase errors on different qubits.

Noise correlation through QEC

M. Laforest et. al., PRA 75, 012331, 2007

Results





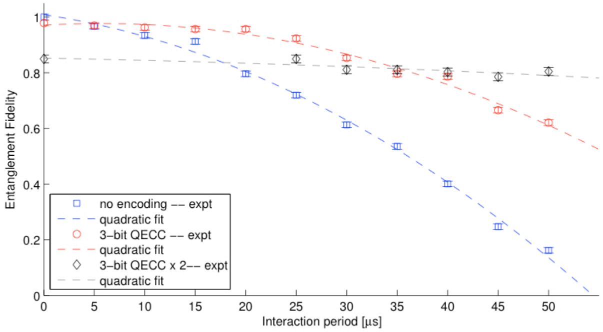
Acetyl Chloride

-QEC technique: c=0.5±0.2
-NMR technique: c=0.3±0.2

-c=0 and c=1 curve are within 3% of each other
-Shows that Gaussian noise is a good noise model for NMR

2 rounds of the 3 qubit QEC

O. Moussa PhD Thesis April 2010

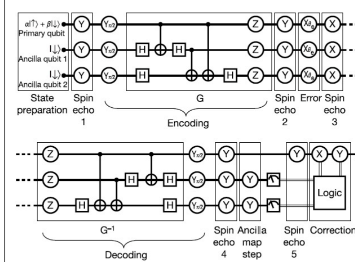


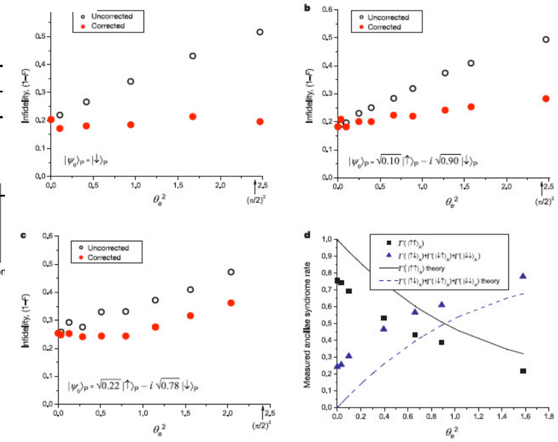
3 bit code in an ion trap

Realization of quantum error correction

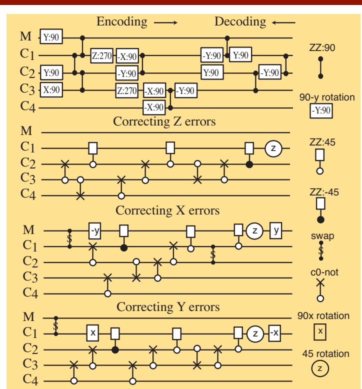
J. Chiaverini[1], D. Leibfried[1], T. Schaetz[1*], M. D. Barrett[1*], R. B. Blakestad[1], J. Britton[1], W. M. Itano[1], J. D. Jost[1], E. Knill[2], C. Langer[1], R. Ozeri[1] & D. J. Wineland[1]

[1]*Time and Frequency Division,* [2]*Mathematical and Computational Sciences Division, NIST, Boulder, Colorado 80305, USA*



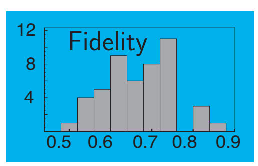


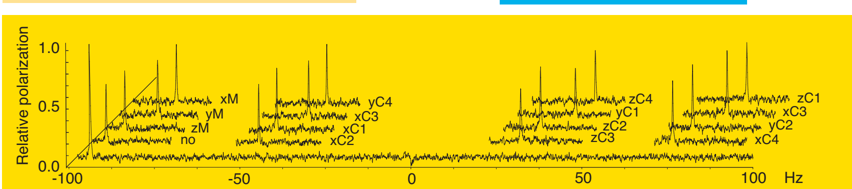
5 bit quantum error correcting code



Implementation of the 5 bit code with the stabilizer $Z^2Y^3Y^4X^5$, $Z^1Y^2Y^3X^4$, $Y^2Z^3Z^4Z^5$ and $X^1Z^2X^3Z^4$, including decoding and error correction for a basis of 1 qubit errors [1].

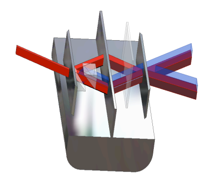
Knill, Laflamme, Martinez, Negrevergne, PRL 404,308,2000





DFS in neutron interferometry

D.Cory, D. Pushin, private communication



$$|01\rangle \rightarrow \frac{1}{\sqrt{2}}|01\rangle + |10\rangle \rightarrow \alpha|01\rangle + \beta|10\rangle$$

or in "logical" terms:

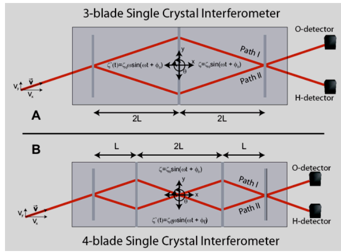
$$|0_L\rangle \rightarrow \frac{1}{\sqrt{2}}|0_L\rangle + |1_L\rangle \rightarrow \alpha|0_L\rangle + \beta|1_L\rangle$$

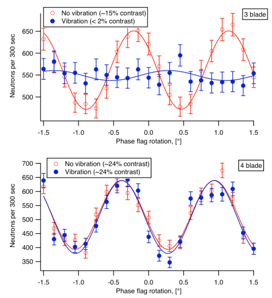
The dominant noise is a phase shift due to rotation in the vertical axis, i.e. $e^{i\theta Z}$

DFS in neutron interferometry

D.Cory, D. Pushin, private communication

In the 4-blade case we have path 1 and path 2 canceling each other phase gain/loss and this is similar to 2 qubit system subject to the noise $Z_1 Z_2$ which has a DFS $\{|01_L\rangle, |10_L\rangle\}$.

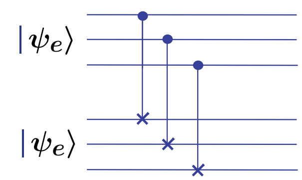




Magic state distillation

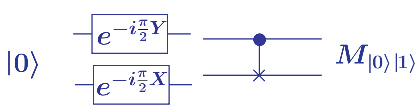
In brief a quantum computer needs to:
Prepare a state, compute, measure

For imperfect devices, we need to use fault tolerance. For state preparation and measurement, need only to repeat, but for the computation it is more complicated. Simplify by using transversal gates



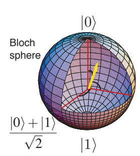
Magic state distillation

Other possibility is to use only generators of the Clifford group, with state preparation and measuremen in the computational basis:



and include the preparation of

$$|\pi/8\rangle, or\ \rho = \frac{1}{2}\Big(\mathbb{1} + \frac{1}{\sqrt{3}}(X+Y+Z)\Big)$$



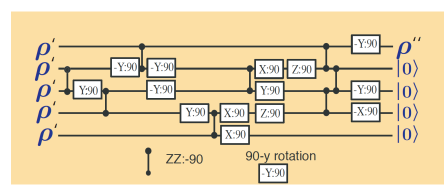
Magic state distillation

Kitaev and Bravyi Phys. Rev. A 71 (2005) 022316

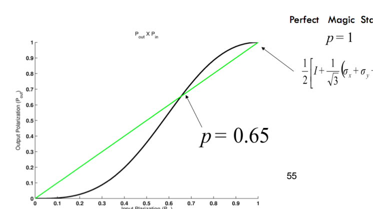
If ρ has imperfection such as

$$\rho' = \frac{1}{2}\mathbb{1} + \frac{p'}{\sqrt{3}}(X+Y+Z)$$

we can use the decoding of 5 bit code to purify the state



i.e., if p' is near enough 1, $p'' > p'$

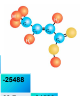


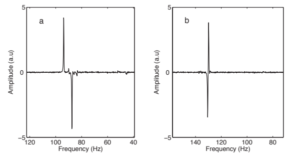
Magic state distillation

Use crotonic acid

A.Souza, J. Zhnag, C. Ryan & R.L. in preparation

	M	H_1	H_2	C_1	C_2	C_3	C_4	Spin	T_2(s)
M	-1309							M	0.84
H_1	6.9	-4864						H_1	0.85
H_2	-1.7	15.5	-4086					H_2	0.84
C_1	127.5	3.8	6.2	-2990				C_1	1.27
C_2	-7.1	156.0	-0.7	41.6	-25488			C_2	1.17
C_3	6.6	-1.8	162.9	1.6	69.7	-21586		C_3	1.19
C_4	-0.9	6.5	3.3	7.1	1.4	72.4	-29398	C_4	1.13

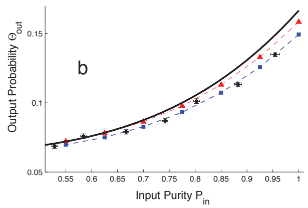


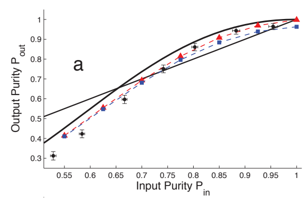


Prepare the state $|0\rangle\langle 0|_{H_1} \otimes Z_{H_2} \otimes |00000\rangle\langle 00000|_{MC_1C_2C_3C_4}$

Distill and get (for the 5 qubits)

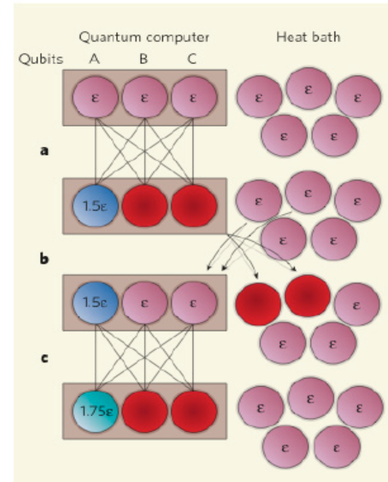
$$\theta_1\rho_1|00000\rangle\langle 00000| + \theta_2\rho_2|00001\rangle\langle 00001| + \dots$$





Input	Output
0.95	0.96
0.88	0.94
0.80	0.86
0.73	0.74
0.67	0.59

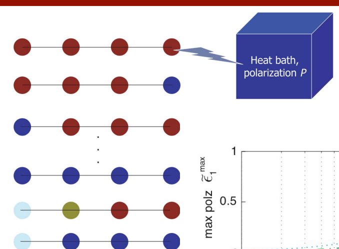
Algorithmic cooling with a heat bath

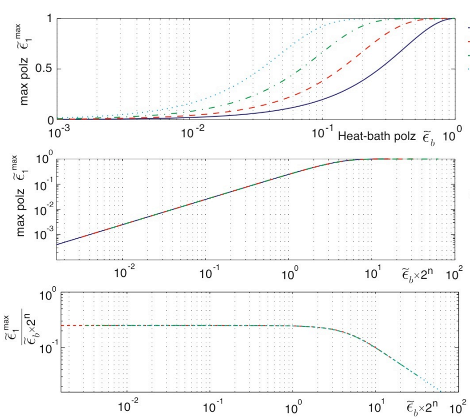


Schulman and Vazirani. Proceedings of the 31th Annual ACM Symposium on the Theory of Computation (STOC), pages 322–329, 1998.

Schulman, Mor and Weinstein, PRL94, 2005

Algorithmic cooling with heat bath





$\tilde{\epsilon}_b \gg 2^{-n}$

$\rightarrow \tilde{\epsilon}_1^{max} = 1$

$\tilde{\epsilon}_b < 2^{-n}$

$\rightarrow \tilde{\epsilon}_1^{max} = \tilde{\epsilon}_b \times 2^{n-2}$

Algorithmic cooling

We have seen that we can cool a subset of spins by swapping states. For excample, with 3 spins, implementing a gate that swaps $|011\rangle \leftrightarrow |100\rangle$ will increase the order of the first spin at the expense of the last two. We could concatenate this process to reach polarization of order 1.

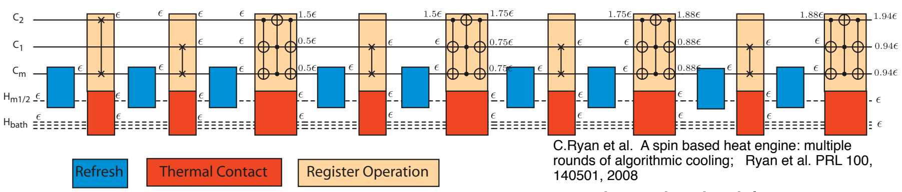
$$\rho \sim e^{-\beta H} \sim \frac{1}{2^n}(\mathbb{1} - \beta\omega(Z_1 + Z_2 + Z_3) + \dots)$$

$$\rho^d_{\text{thermal}} \approx \frac{\beta\omega}{8}\begin{pmatrix} 3 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & -3 \end{pmatrix} \iff \rho^d_{\text{pol}} \approx \frac{\beta\omega}{8}\begin{pmatrix} 3 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & -3 \end{pmatrix}$$

$$\bar{\rho}^d_{\text{pol}} = \text{Tr}_{2,3}\rho^d_{\text{pol}} \approx \frac{3}{4}\beta\omega\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

We could concatenate this process to reach polarization of O(1), but this would take a lot of ressources ($\sim 1/\beta^2$).

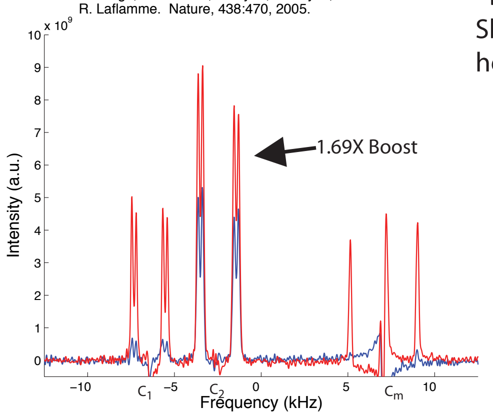
Multiple Rounds of Algorithmic Cooling



C.Ryan et al. A spin based heat engine: multiple rounds of algorithmic cooling; Ryan et al. PRL 100, 140501, 2008

J. Baugh, O. Moussa, C. Ryan, A. Nayak, and R. Laflamme. Nature, 438:470, 2005.

- By using heat-bath able to surpass Shannon/Soresnsen bound of 1.5X heat-bath polarization



Polarization Boost w.r.t. heat-bath

Compression Step	C_2	C_1	C_m
1	1.39	0.47	0.49
2	1.56	0.68	0.71
3	1.64	0.76	0.79
4	1.69	0.79	0.84

IQC

Raymond Laflamme
Institute for Quantum computing
University of Waterloo
www.iqc.ca
laflamme@iqc.ca

Plan

Lecture 1. Quantum Error Correction

Lecture 2. Experimental Quantum Error Correction

Lecture 3. Characterising noise and benchmarking

Plan of Lecture 3

1. Characterising noise

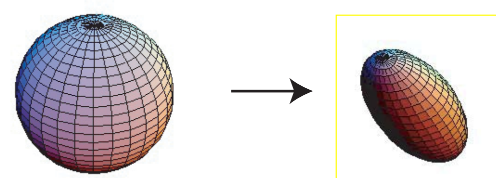
2. Benchmarking gates

Characterising noise in q. systems

Process tomography:

$$\rho_f = \sum_k A_k \rho_i A_k^\dagger = \sum_{kl} \chi_{kl} P_k \rho_i P_l$$

For one quibt, 12 parameters are required as described by the evolution of the Bloch sphere:



For n qubits, we need to provide $4^{2n} - 4^n$ numbers to do so.

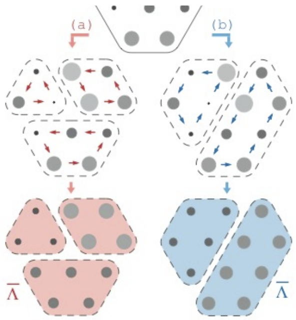
Coarse graining

- We are not interested in all the elements that describe the full noise superopeartor but only a coarse graining of them.
- If we are interested in implementing quantum error corrrection, we can ask what is the probability to get one, or two, or k qubit error, independent of the location and independent of the type of error $\sigma_{x,y,z}$. The question is can we do this efficiently?
- Coarse graining is equivalent to implement a symmetry.

Emerson, Silva, Moussa, Ryan, Laforest, Baugh, Cory, Laflamme, Science 317, 1893, 2007

$$\Lambda(\rho) = \sum_k^{D^2} A_k \rho A_k^\dagger$$

$$\rho_f = \sum_k p_{\{k\}} P_{\{k\}} \rho P_{\{k\}}^\dagger$$



Schematic illustration of coarse-graining.

Coarse graining

1) to coarse error type average over $SU(2)^{\otimes n}$

$$\rho_f = \sum_k \int d\mu(U) U^\dagger A_k U \rho_i U^\dagger A_k^\dagger U$$

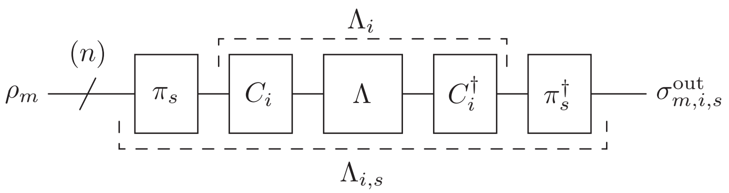
This is an example of a 2-design, and the integral can be replaced by a sum

$$\rho_f = \sum_k \sum_\alpha C_\alpha^\dagger A_k C_\alpha \rho_i C_\alpha^\dagger A_k^\dagger C_\alpha$$

where C_α belongs to the Clifford group $\sim \mathcal{SP}$ with $\mathcal{P} = \{\mathbb{1}, X, Y, Z\}$, $\mathcal{S} = \{e^{-i\frac{\pi}{4}X}, e^{-i\frac{\pi}{4}Y}, e^{-i\frac{\pi}{4}Z}\}$

2) coarse grain the position by symmetrising using permutation π_s

Coarse graining



If we implement all the elements in the Clifford group and permutation, we would have an exponential number of terms , but the sum can be estimated by sampling and using the Chernoff bound.

(see Emerson et al. arXiv:0707.0685)

In practice, implementing the symmetrisation can be done by starting with the state $|000\ldots\rangle$ and measure the Hamming size (i.e. the number of 1) in the final state.

2 qubit example

- Let assume we have noise with Kraus operators of the form:

$$A_k = \exp[i\frac{\pi}{4}(Z_1 + Z_2)]$$
$$= \{\frac{1}{2}(\mathbb{1} \otimes \mathbb{1} + Z \otimes \mathbb{1} + \mathbb{1} \otimes Z + ZZ\}$$

$$\rho_f = \sum_k A_K \rho_i A_k^\dagger = \sum_{kl} \chi_{kl} P_k \rho_i P_l$$

$\frac{1}{4}\times$

	II	ZI	XI	YI	IZ	IX	IY	ZZ	ZX	ZY	XZ	XX	XY	YZ	YX	YY
II	1	1			1			1								
ZI	1	1			1			1								
XI																
YI																
IZ	1	1			1			1								
IX																
IY																
ZZ	1	1			1			1								
ZX																
ZY																
XZ																
XX																
XY																
YZ																
YX																
YY																

2 qubit example

$$\rho_f = \sum_k A_K \rho_i A_k^\dagger = \sum_{kl} \chi_{kl} P_k \rho_i P_l$$

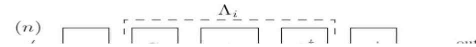
$$\bar{\rho}_f = \sum_{kl} \sum_\alpha \chi_{kl} C_\alpha^\dagger P_k C_\alpha \rho_i C_\alpha^\dagger P_l^\dagger C_\alpha$$

- Summing over the Pauli group gets rid of off-diagonal of χ_{kl}, i.e. $\mathbb{1}\mathbb{1}\mathbb{1}\rho_i\mathbb{1}Z\mathbb{1} + X\mathbb{1}X\rho_iXZX + Y\mathbb{1}Y\rho_iYZY + Z\mathbb{1}Z\rho_iZZZ = 0$
- Summing over the symplectic group equalize the appearance of the operators X, Y and Z, e.g. $\mathcal{S}\{X\}\mathcal{S}^\dagger = \frac{1}{3}\{X, Y, Z\}$
- By inputting a state of the for $|00\rangle$, measuring in the Z basis, counting the number of time states with Hamming weight i appears (this becomes equivalent to summing over the permutation group to homogeneise errors over all qubits), we can estimate p_i (noting that Z errors do not affect that state).

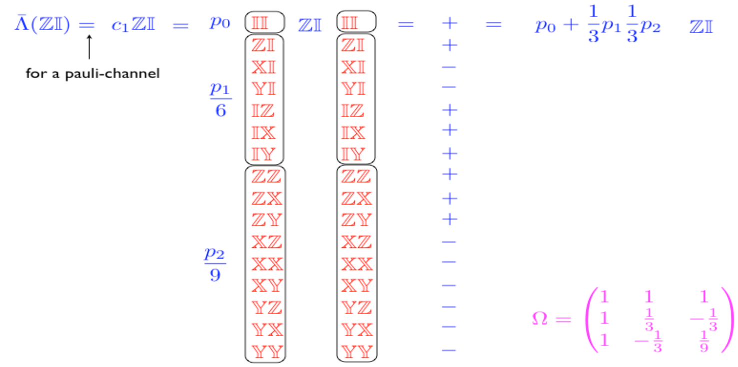
$$\bar{\rho}_f = \sum_j \frac{p_j}{\#j} P_{\{j\}} |00\rangle\langle 00| P_{\{j\}}^\dagger$$

NMR implementation: 2 qubits

It is possible to adapt the pure state protocol to NMR, the idea is to use a series of n initial states of the form



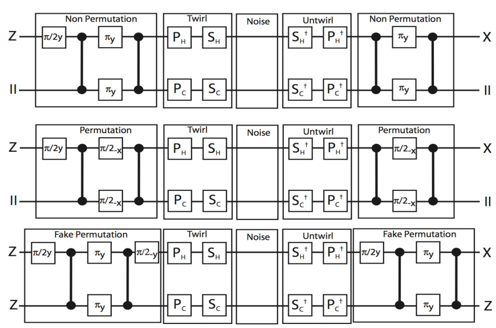
An isomorphism



NMR implementation: 2 qubits

Twirling in liquid state NMR

The Implementation

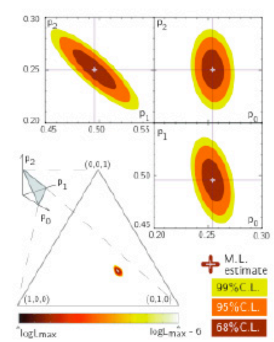


Non-permutation and fake permutation are perform so that all experiment can be compared on the same footing.

NMR implementation: 2 qubits

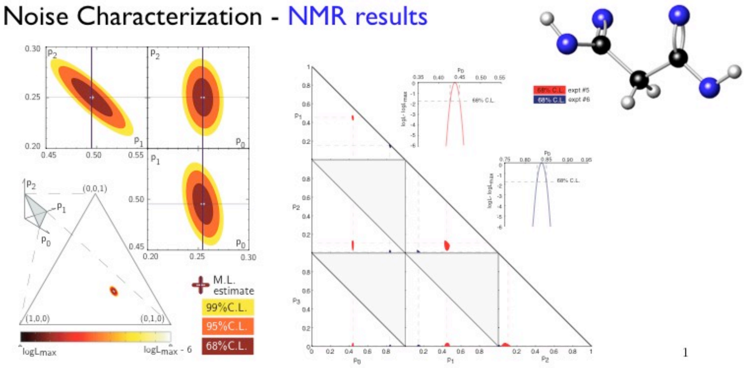
The results

#	Map Description	Noise Operators	k	p_0	p_1	p_2
1	Engineered: $\mathbf{p} = [0, 1, 0]$.	$\frac{1}{\sqrt{2}} Z\mathbb{1} + \mathbb{1}Z$	288	$0.000^{+0.004}$	$0.991^{+0.009}_{-0.015}$	$0.009^{+0.017}_{-0.009}$
2	Engineered: $\mathbf{p} = [0, 0, 1]$.	ZZ	288	$0.001^{+0.006}_{-0.001}$	$0.004^{+0.011}_{-0.004}$	$0.996^{+0.004}_{-0.011}$
3	Engineered: $\mathbf{p} = [\frac{1}{4}, \frac{1}{2}, \frac{1}{4}]$.	$\exp[i\frac{\pi}{4}(Z_1 + Z_2)]$	288	$0.254^{+0.010}_{-0.010}$	$0.495^{+0.021}_{-0.020}$	$0.250^{+0.019}_{-0.019}$



Experimental results

Noise Characterization - NMR results



1

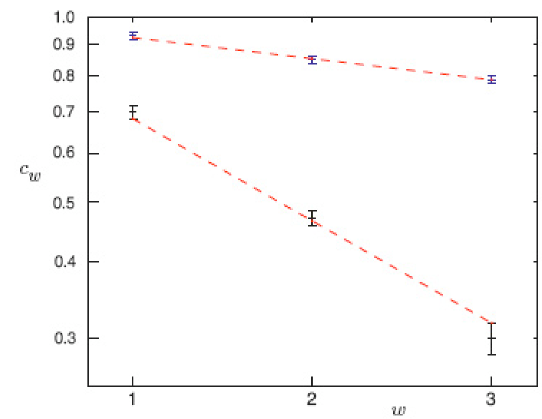
#	Map Description	Krauss operators (A_k)	k_m	p_0	p_1	p_2	p_3
1	Engineered: $\mathbf{p} = [0, 1, 0]$.	$\frac{1}{\sqrt{2}}\{Z_1, Z_2\}$	288	$0.000^{+0.004}$	$0.991^{+0.009}_{-0.015}$	$0.009^{+0.017}_{-0.009}$	-
2	Engineered: $\mathbf{p} = [0, 0, 1]$.	$\{Z_1 Z_2\}$	288	$0.001^{+0.006}_{-0.001}$	$0.004^{+0.011}_{-0.004}$	$0.996^{+0.004}_{-0.011}$	-
3	Engineered: $\mathbf{p} = [1/4, 1/2, 1/4]$.	$\{exp[i\frac{\pi}{4}(Z_1 + Z_2)]\}$	288	$0.254^{+0.010}_{-0.010}$	$0.495^{+0.021}_{-0.020}$	$0.250^{+0.019}_{-0.019}$	-
4	Engineered: $\mathbf{p} = [0, 1, 0, 0]$.	$\frac{1}{\sqrt{3}}\{Z_1, Z_2, Z_3\}$	432	$0.01^{+0.01}_{-0.01}$	$0.99^{+0.01}_{-0.03}$	$0.01^{+0.02}_{-0.01}$	$0.00^{+0.01}$
5	Natural noise (a)	unknown	432	$0.44^{+0.01}_{-0.02}$	$0.45^{+0.03}_{-0.03}$	$0.10^{+0.04}_{-0.08}$	$0.01^{+0.03}_{-0.01}$
6	Natural noise (b)	unknown	432	$0.84^{+0.01}_{-0.01}$	$0.15^{+0.02}_{-0.03}$	$0.01^{+0.03}_{-0.01}$	$0.00^{+0.02}$

TABLE I: Summary of experimental results.

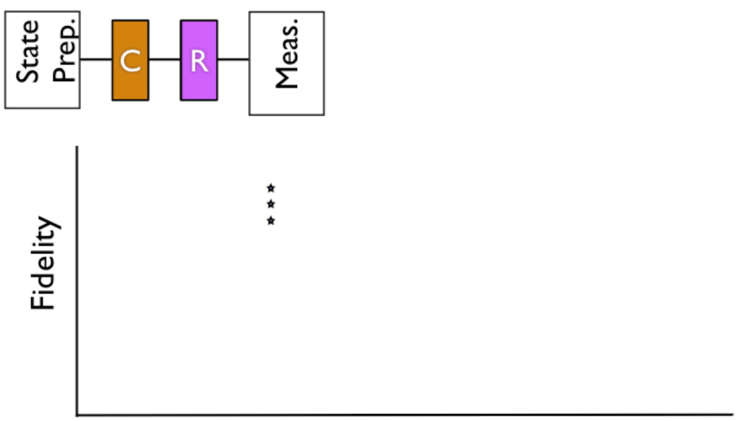
Checking for noise independence

If the noise is independent

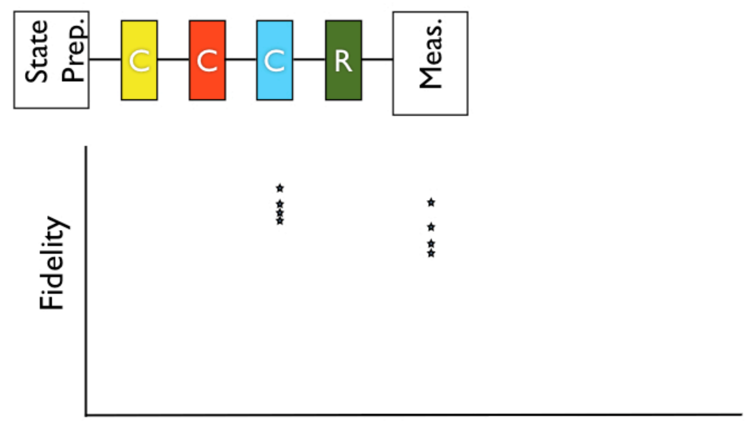
$$c_\omega = (c_1)^\omega$$



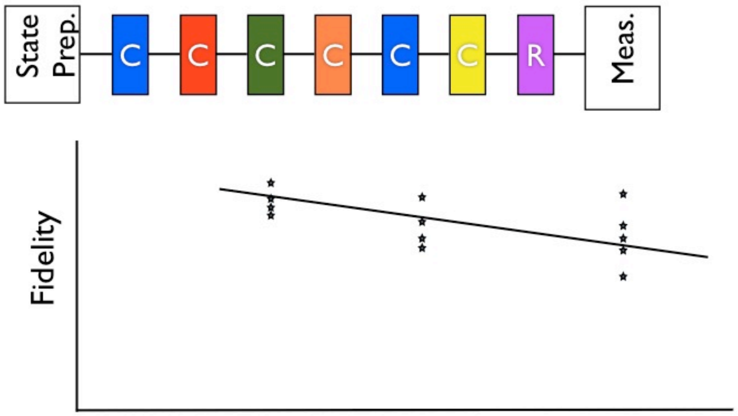
Benchmarking gates



Benchmarking gates

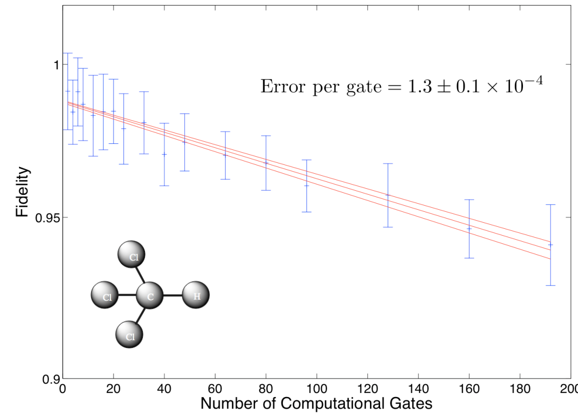


Benchmarking gates



Benchmarking gates

Single qubit Gates



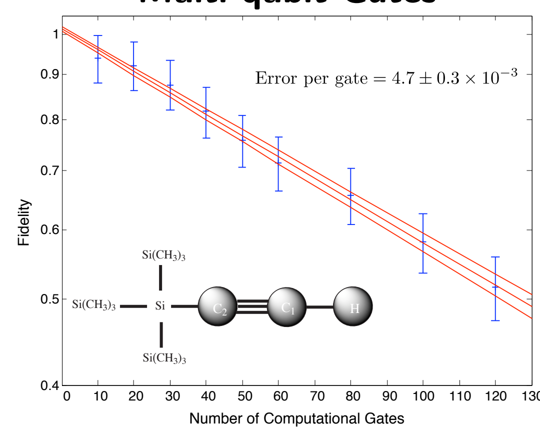
Benchmarking gates

Single qubit Comparison Summary Table

System	Error Rate	Reference
Ion Trap (single)	0.00482	PRA **77** 012307 (2008)
Liquid-State NMR	0.00013	NJP **11** 013034 (2009)
Superconducting	0.011	PRL **102** 090502 (2009)
Ion Trap (crystal)	0.0008	arXiv:0906.0398 (2009)
ESR	0.0007	PRL **95** 200501 (2005)
Neutral Atoms	0.01	arXiv:0811.3634 (2008)

Benchmarking gates

Multi-qubit Gates



Benchmarking gates

Multi-qubit Comparison Summary Table

System	Error/Fidelty	Reference
liquid-state NMR	0.0047	NJP **11** 013034 (2009)
ion-trap (single)	99.3%	Nat. Phys. **4** 463 (2008)
superconducting	91%	Nature **460** 240 (2009)
NV centre	89%	Science **320** 1326 (2008)
Linear Optics	90%	PRL **93** 080502 (2004)
Neutral Atoms	73%	arXiv:0907.5552 (2009)
ESR	95%	Nature **455** 1085 (2008)

Conclusion

- **Control methods are necessary for building robust quantum information processors; quantum error correction is one of them and it is scalable in theory**
- **Ideas and concepts of quantum error correction are being implemented today in the laboratory in a variety of technologies, testing the assumptions of fault tolerant quantum computation**
- **In order to implement quantum error correction as we go towards larger quantum processor we will need to characterise the noise and methods to do that are being developed**

Thanks

PI PERIMETER INSTITUTE FOR THEORETICAL PHYSICS
Ontario MINISTRY OF RESEARCH & INNOVATION
University of Waterloo
NSERC CRSNG
25 years cifar Canadian Institute for Advanced Research
Ontario Innovation Trust
The National Security Agency
ARDA
mitacs
ONTARIO CHALLENGE FUND Putting R&D to Work
Canada Foundation for Innovation Fondation canadienne pour l'innovation
Canada Research Chairs Chaires de recherche du Canada
Communications Security Establishment Centre de la sécurité des télécommunications
Premier's Research Excellence Awards
sgi

BRUKER canada
helios | oceana
IBM