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The quantised Hall resistance as a resistance standard

Blaise Jeanneret

metas

The quantised Hall resistance (QHR) as a resistance standard

B. Jeanneret

Federal Office of Metrology (METAS)

Outline

- Introduction
- Physical properties of the QHR
- Cryogenic Current Comparator
- Universality of the QHR
- QHR and the SI
- Applications
- AC-QHE
- Conclusions

1 ppb = 1 part in 10^9

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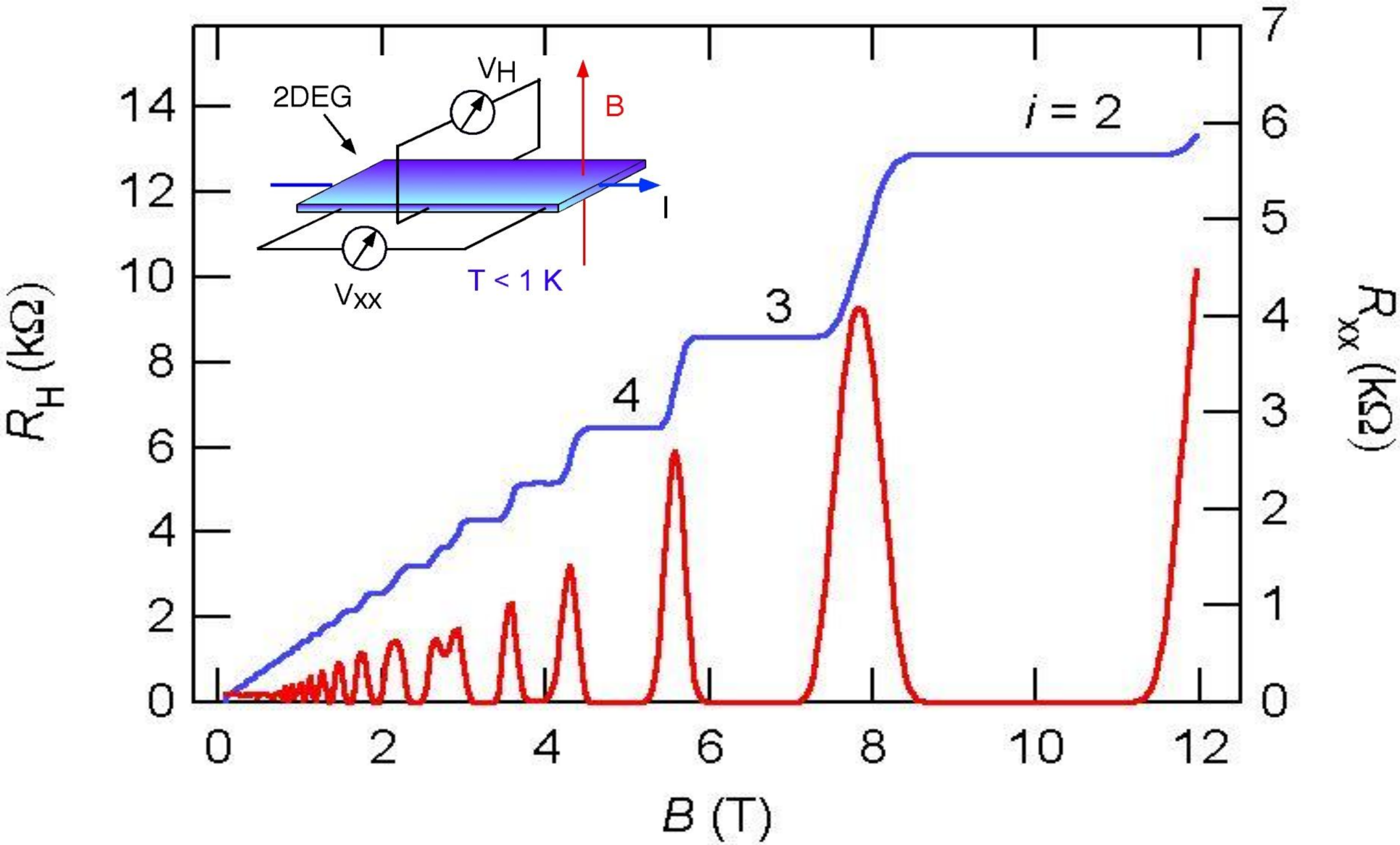
Introduction

$R_K = h/e^2 = 25\,812.807\ldots\ \Omega$



- Ideal systems: T = 0 K, I = 0 A
- No dissipation: $R_{xx} = 0$

$$R_H(i) = h/ie^2$$

$R_H(i)$ is a universal quantity

Localization theory,

Edge state model

- Real experiment: T > 0.3 K, I = 40 µA
Non-ideal samples
- Dissipation: $R_{xx} > 0$

$$R_H(i, R_{xx} \longrightarrow 0) = h/ie^2 \; ??$$

Is $R_H(i)$ a universal quantity?

Independent of device material, mobility, carrier density, plateau index, contact properties.....?

Few quantitative theoretical models available

→ empirical approach,

→ precision measurements

Temperature dependence

- Thermal activation:

$1\,K \leq T \leq 10\,K$

electrons thermally activated

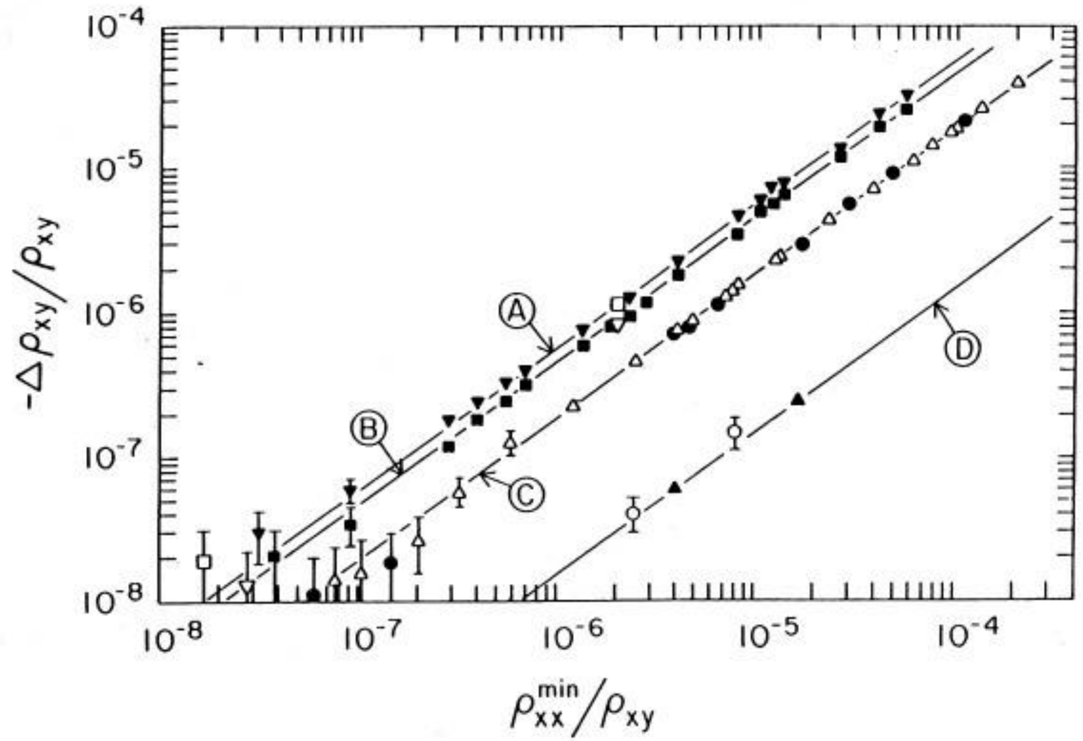
to the nearest extended states

$$\sigma_{xx}(T) = \sigma_{xx}^{0} \cdot e^{-\Delta/kT}$$

$$\Delta = E_F - E_{LL}$$

$$\delta\sigma_{xy}(T) = \sigma_{xy}(T) - \frac{ie^2}{h}$$

$$\boxed{\delta\rho_{xy}(T) = s\,\rho_{xx}(T)}$$



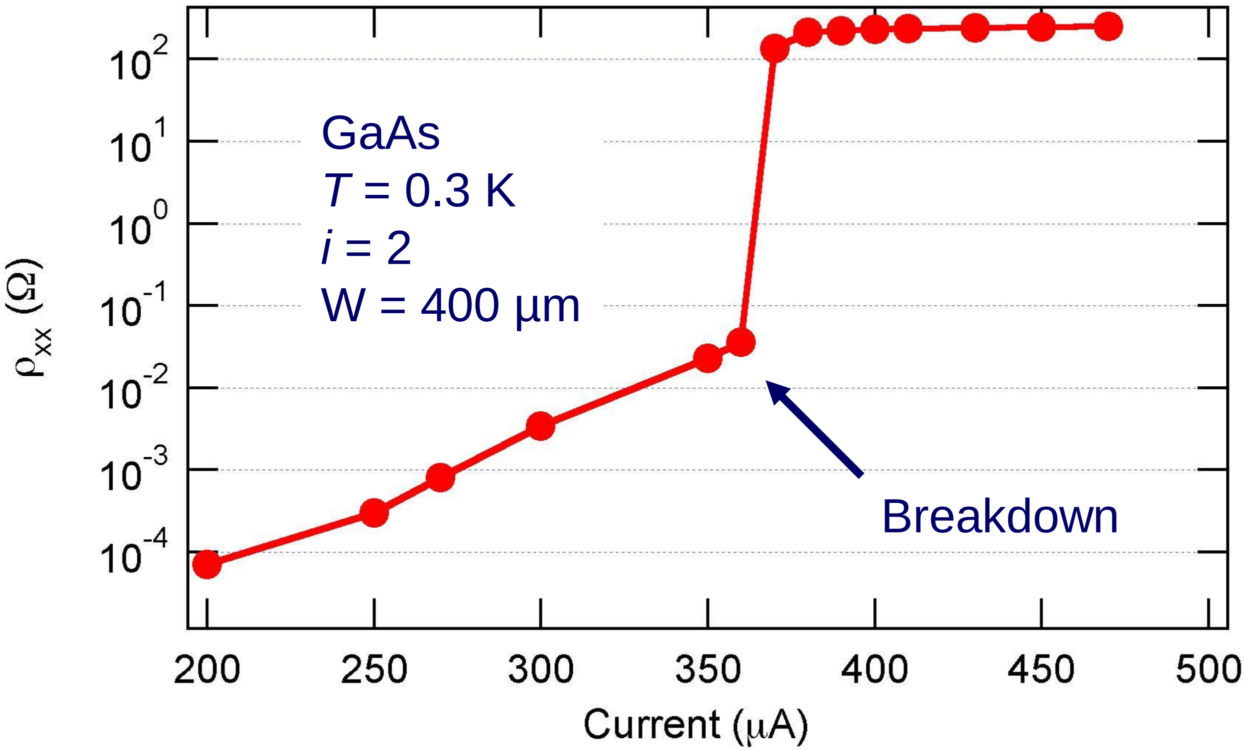
- Cage et al. 1984:

1.2 K < T < 4.2 K,

2 GaAs samples, *i* = 4, *I* = 25 µA

-0.01 < *s* < -0.51

Current dependence



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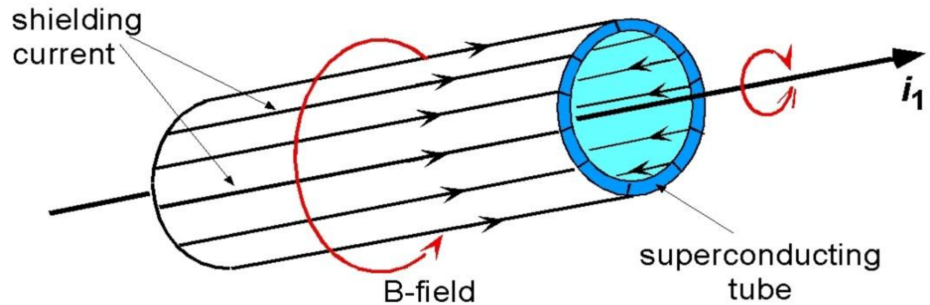
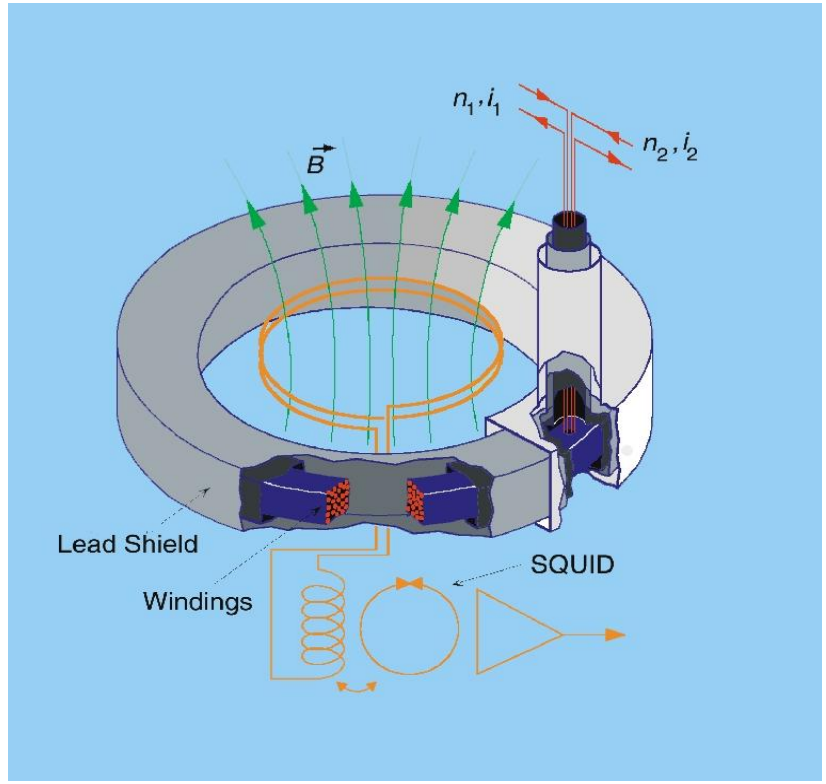
The cryogenic current comparator (CCC): Principles

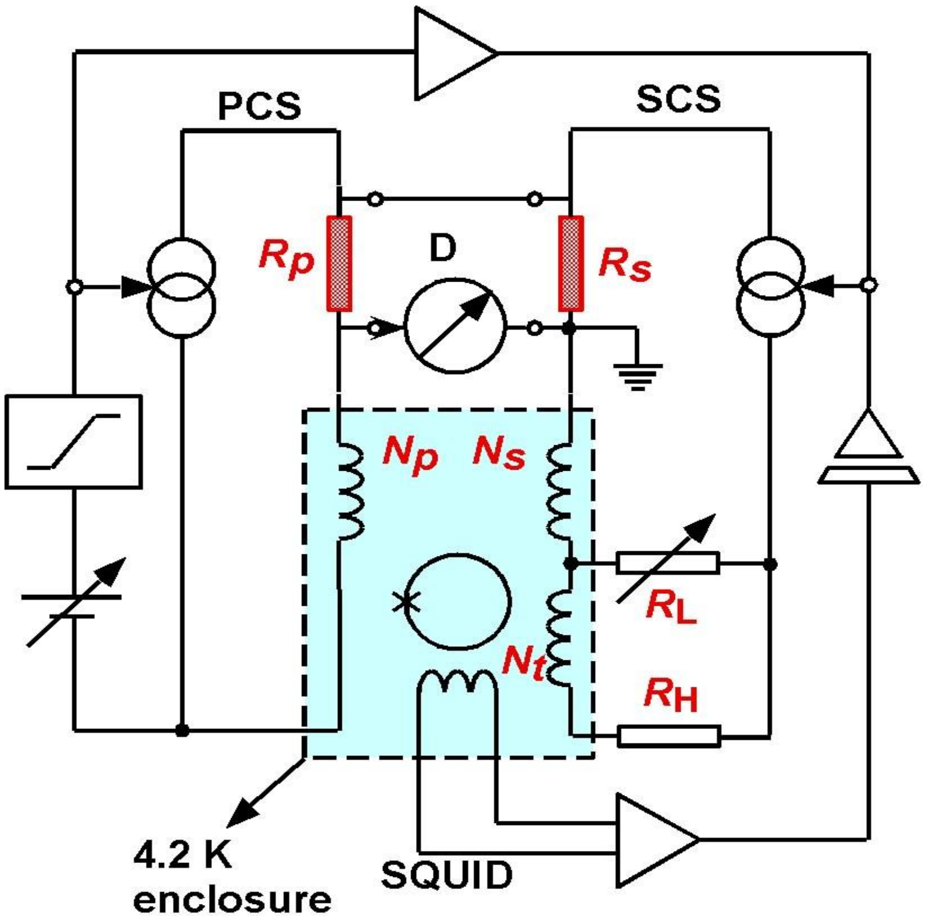
Meissner effect:





Harvey 1972

The CCC bridge:



SQUID:

$$N_P \cdot I_P = N_S \cdot I_S \cdot (1+d)$$

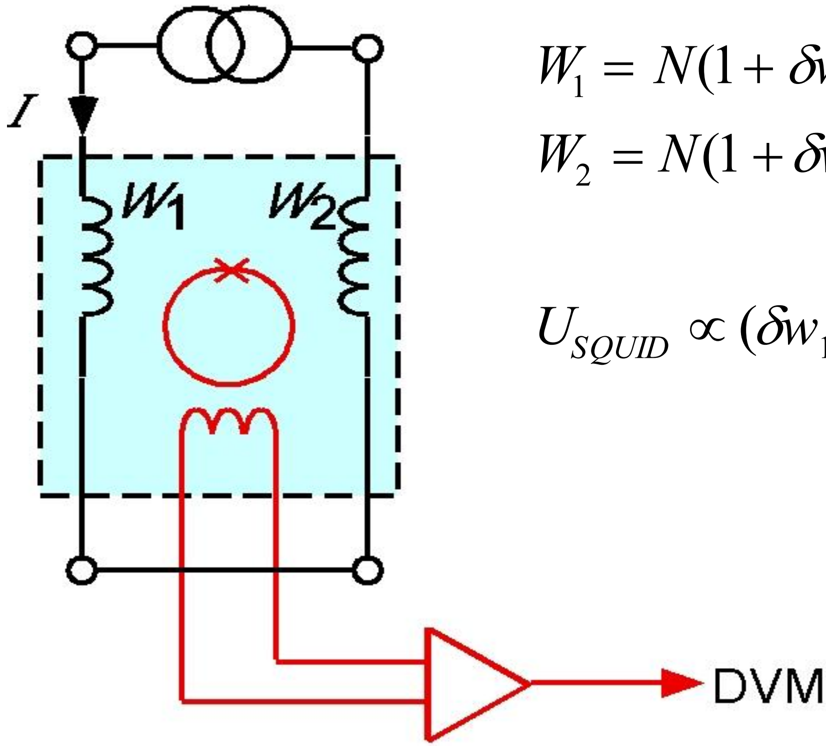
$$\text{with} \quad d = \frac{N_t}{N_S} \cdot \frac{R_L}{R_L + R_H}$$

Detector:

$$U_m = R_S \cdot I_S - R_P \cdot I_P$$

$$\frac{R_P}{R_S} = \frac{N_P}{N_S} \cdot \frac{1}{1+d} \cdot \frac{1}{1 + U_m / U}$$

Ratio accuracy:



$$W_1 = N(1 + \delta w_1)$$

$$W_2 = N(1 + \delta w_2)$$

$$U_{SQUID} \propto (\delta w_1 - \delta w_2)$$

Windings in a binary series:

1, 1, 2, 4, 8, 10, 16, 32, 32, 64, 100, 128, 256, 512, 1000, 1097, 2065, 4130

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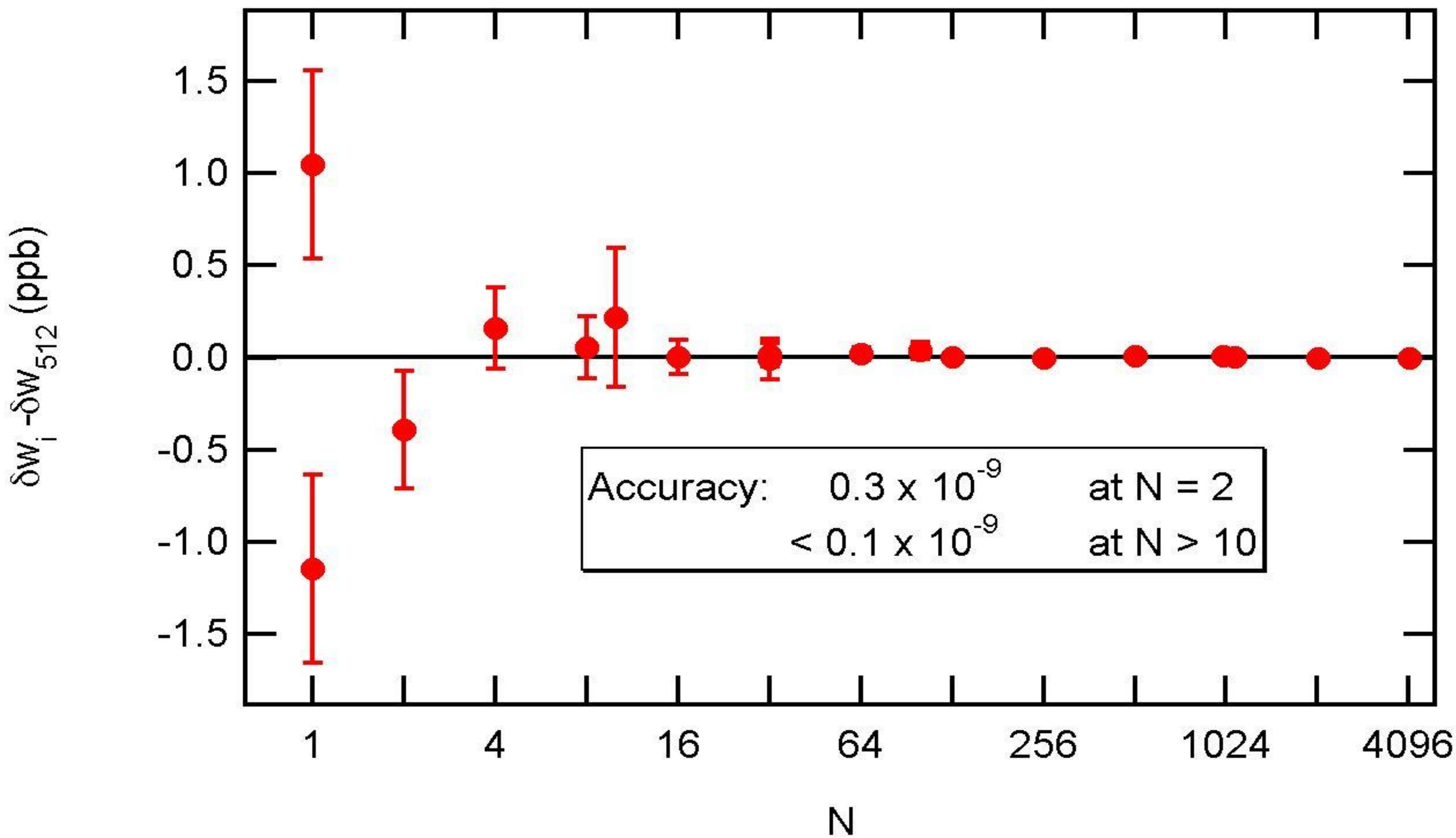
Ratio accuracy



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Performances:

Ratio	rms-noise $[\mathrm{nV}/\sqrt{0.25\ \mathrm{Hz}}]$ Johnson	Detekt.	SQUID	Σ_{ideal}	Σ_{meas}
10 kΩ : 100 Ω	6.4	3.3	0.8	**7.2**	**7.2**
R_H(4) : 100 Ω	0.8	1.4	1.0	**1.9**	**2.5**
100 Ω : 1 Ω	0.6	0.4	0.3	**0.8**	**0.9**

For a typical comparison:

$R_P = R_H(2)$ $N_P = 2065$

$R_S = 100\ \Omega$ $N_S = 16$

$I_P = 50\ \mu A$

Noise: 7 nV / Hz$^{1/2}$

U_A = 2 nΩ / Ω in 2 min

Universality of the quantum Hall effect

- Width dependence
- Contact resistance
- Device material: MOSFET, GaAs and **GRAPHENE**
- Device mobility
- Plateau index
- ……
- …….

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Width dependence

Theoretical model:

$$\frac{\Delta R_H(i)}{R_H(i)} = \alpha\left(\frac{l}{w}\right)^2$$

$$\Delta R_H(i) = R_H(i,w) - R_H(i,w=\infty)$$

$$l = \sqrt{\hbar / eB}$$ magnetic length

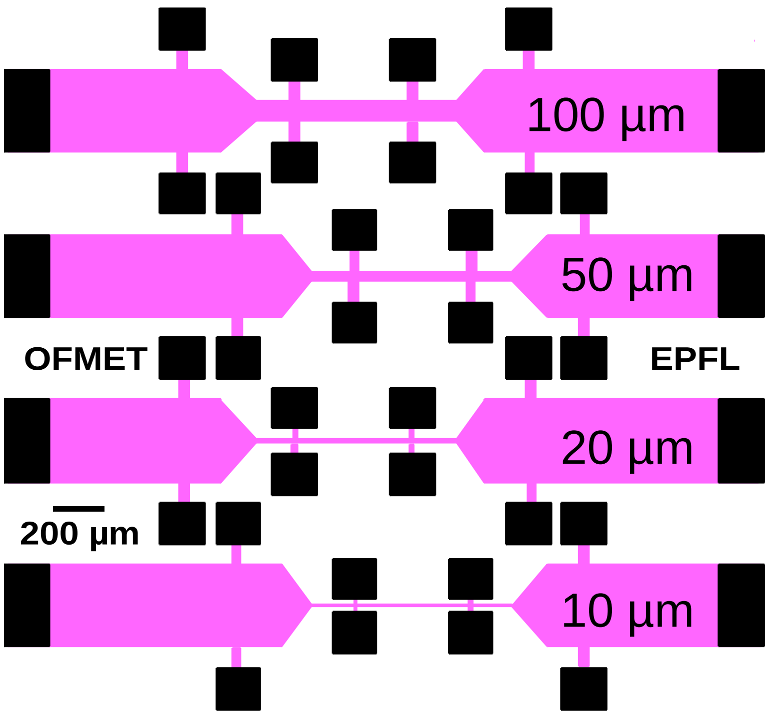
- No theoretical prediction for α

- A. H. MacDonald, P. Streda, 1984
- B. Shapiro, 1986
- W. Brenig and K. Wysokinski, 1986
- R. Johnston and L. Schweitzer, 1988.

Experiment

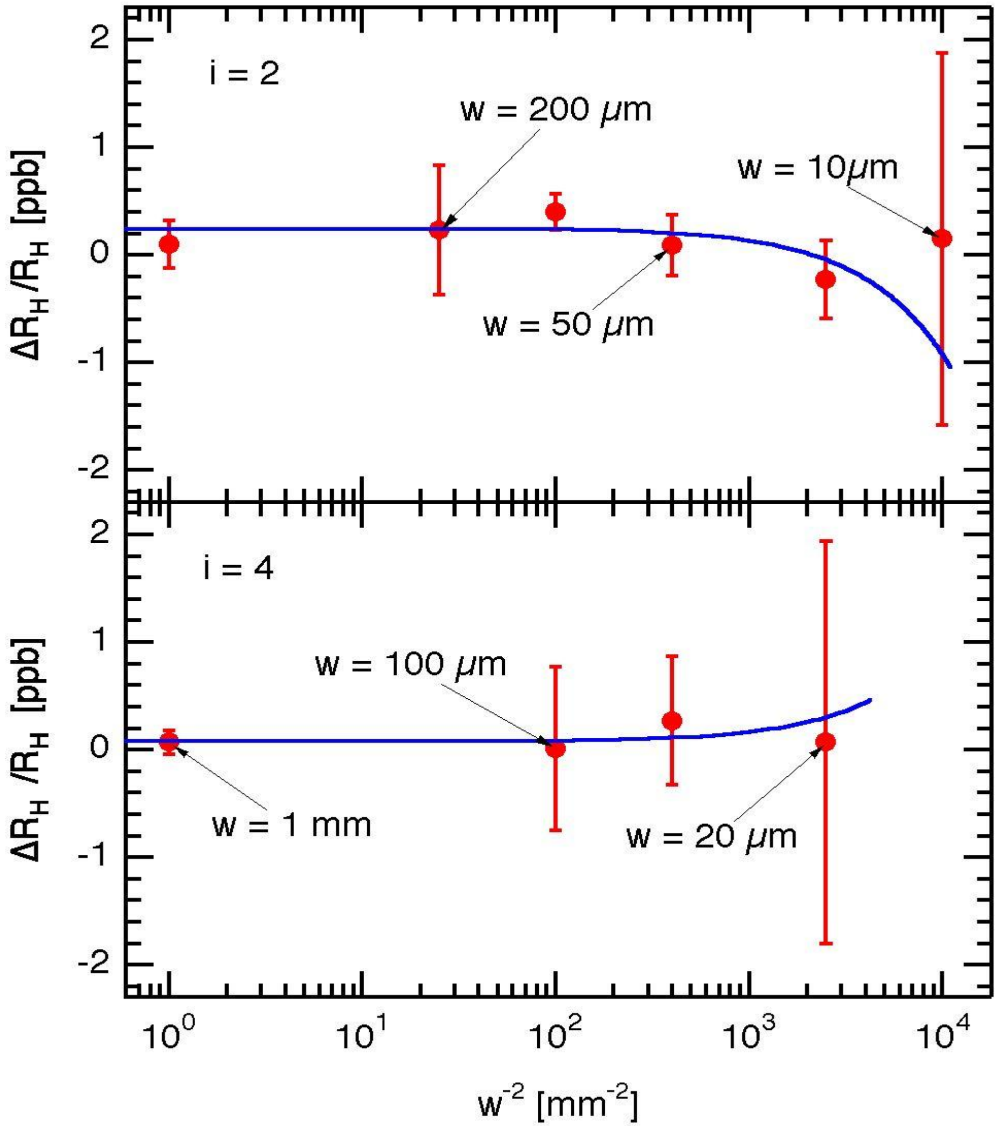
Samples: $\mu = 42\ T^{-1}$

$n = 4.8 \times 10^{15}\ m^{-2}$



Measurement procedure:

- Cooling rate: a couple of hours
- T = 0.3K
- V_{xx} measured before and after measurements
- Trypically R_{xx} < 100 µΩ
- R_H measured on two contact pairs
- Reference sample: 1 mm wide



- No size effect observed within the measurement uncertainty
- Value of α:

$$\alpha_2 = (-1.8 \pm 1.8) \times 10^{-3}$$

$$\alpha_4 = (0.7 \pm 5.0) \times 10^{-3}$$

- Deviation on 500 µm wide samples:

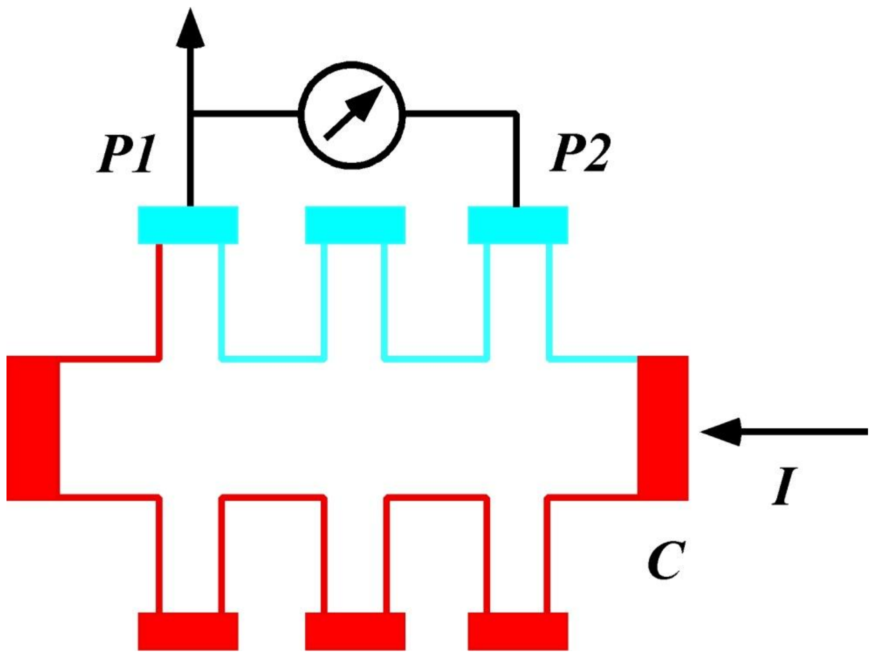
i = 2 < 0.001 ppb

i = 4 < 0.003 ppb

- No influence

Effect of the contact resistance R_c

M. Büttiker, 1992: "...*It is likely, therefore, that in the future, contacts will play an essential role in assessing the accuracy of the QHE*."



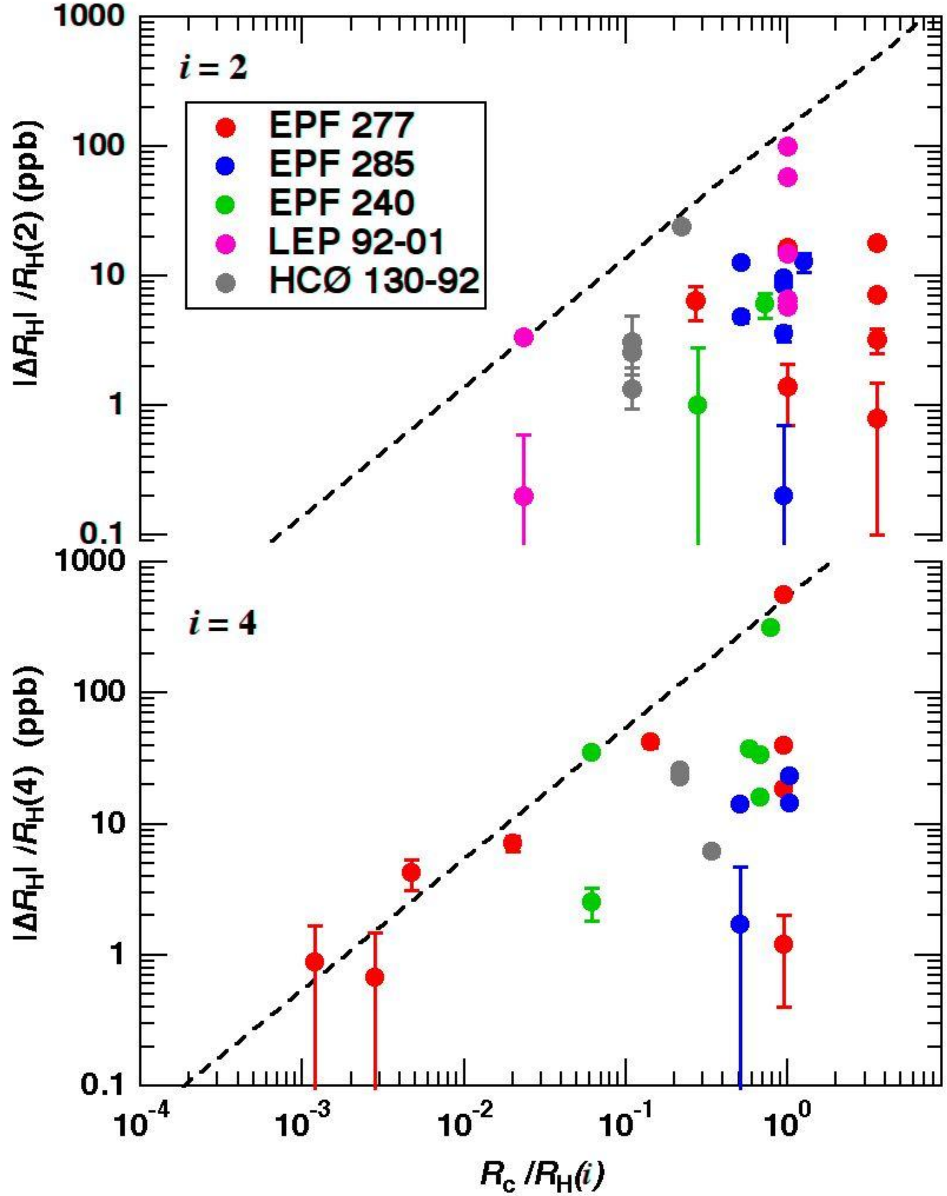
On a perfectly quantised plateau:

$$R_c(P1) = V_{P1\text{-}P2} / I$$

To induce a high value of R_c:

1) Apply a high voltage (10 - 20V) at B = 0

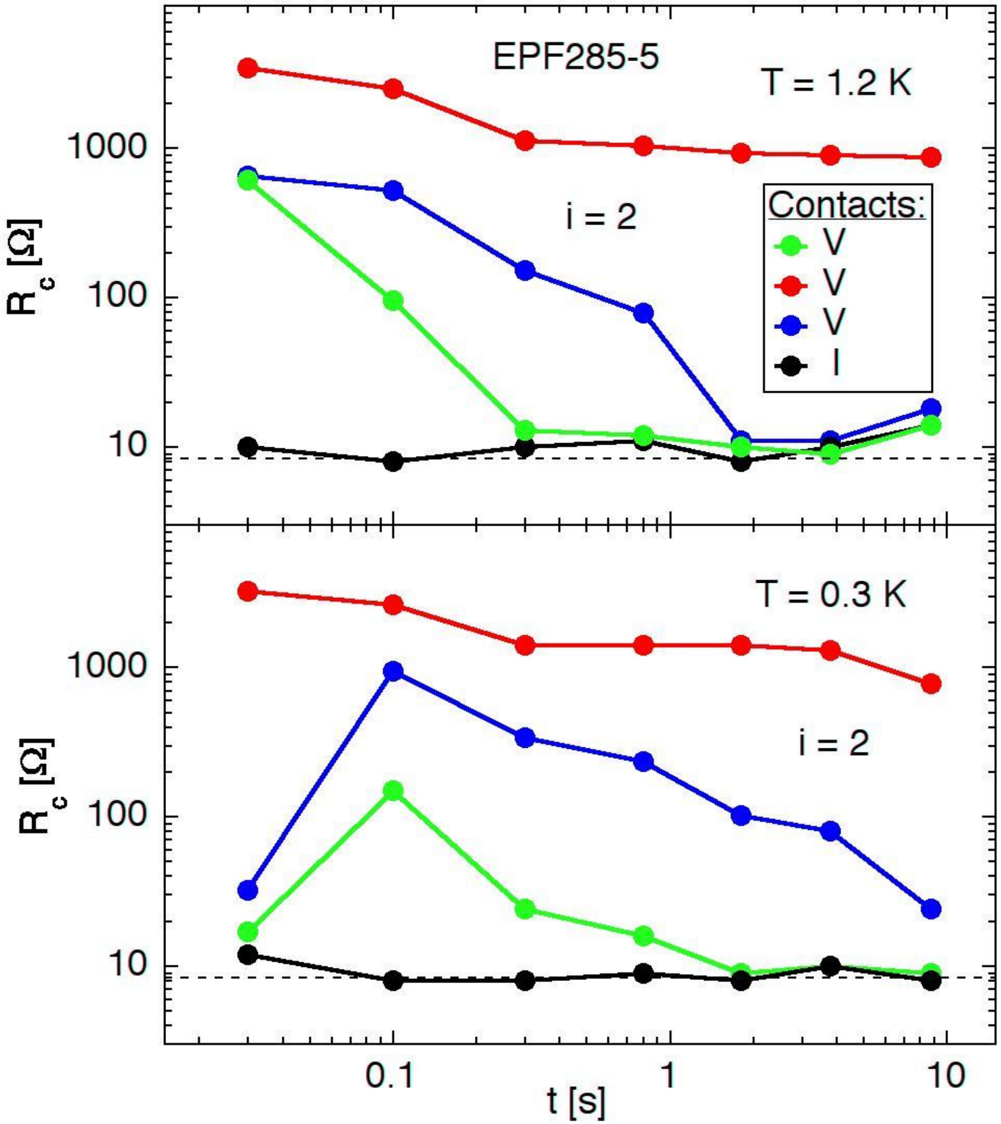
2) Cool the QHR to base T in 2-3 min



GaAs Samples
T = 0.3 K
I = 20 µA

- No simple relation between ΔR_H and R_c
- Deviation of R_H related to finite V_{xx}

$R_c < 10\ \Omega$ ⟶ $\Delta R_H / R_H < 1$ ppb



Infrared illumination:

- Pulses with a 900 nm diode R_c
- Cable resistance 8.5 Ω

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MOSFET-GaAs

- Hartland et al., NPL, 1991:
  Direct comparison of two QHR using a CCC

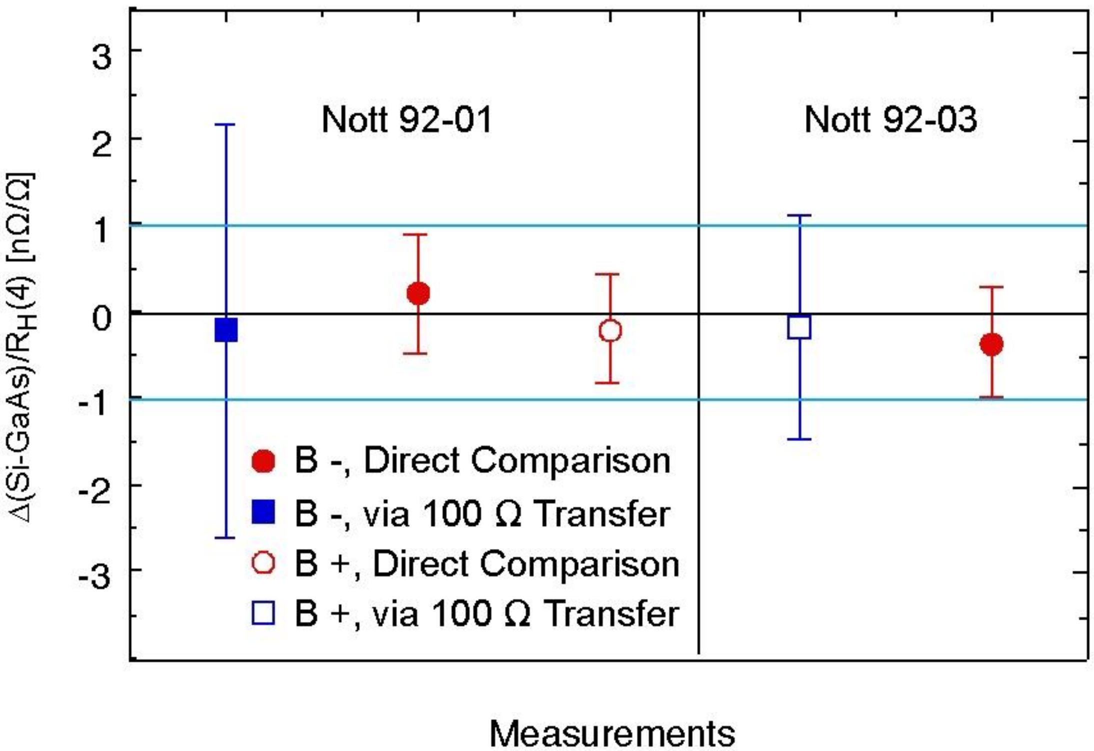
$$\frac{\Delta R_H(\mathrm{MOSFET} - \mathrm{GaAs})}{R_H} \leq 3.5 \times 10^{-10}$$

- Kawaji, Yoshihiro (ETL), vanDegrift (NIST) 1992:
  Deviations in $R_H(4)$ up to 0.3 ppm despite the absence of dissipation.
  MOSFET with small critical current, low gate voltage

- Theoretical model by Heinonen et al.:
  Perfect quantization with dissipation
  (short range elastic scatterers located at the edges)

metas

MOSFET measurements



Jeckelmann et al., OFMET, 1996

$$\frac{\Delta R_H(\mathrm{MOSFET} - \mathrm{GaAs})}{R_H} \leq 2.3 \times 10^{-10}$$

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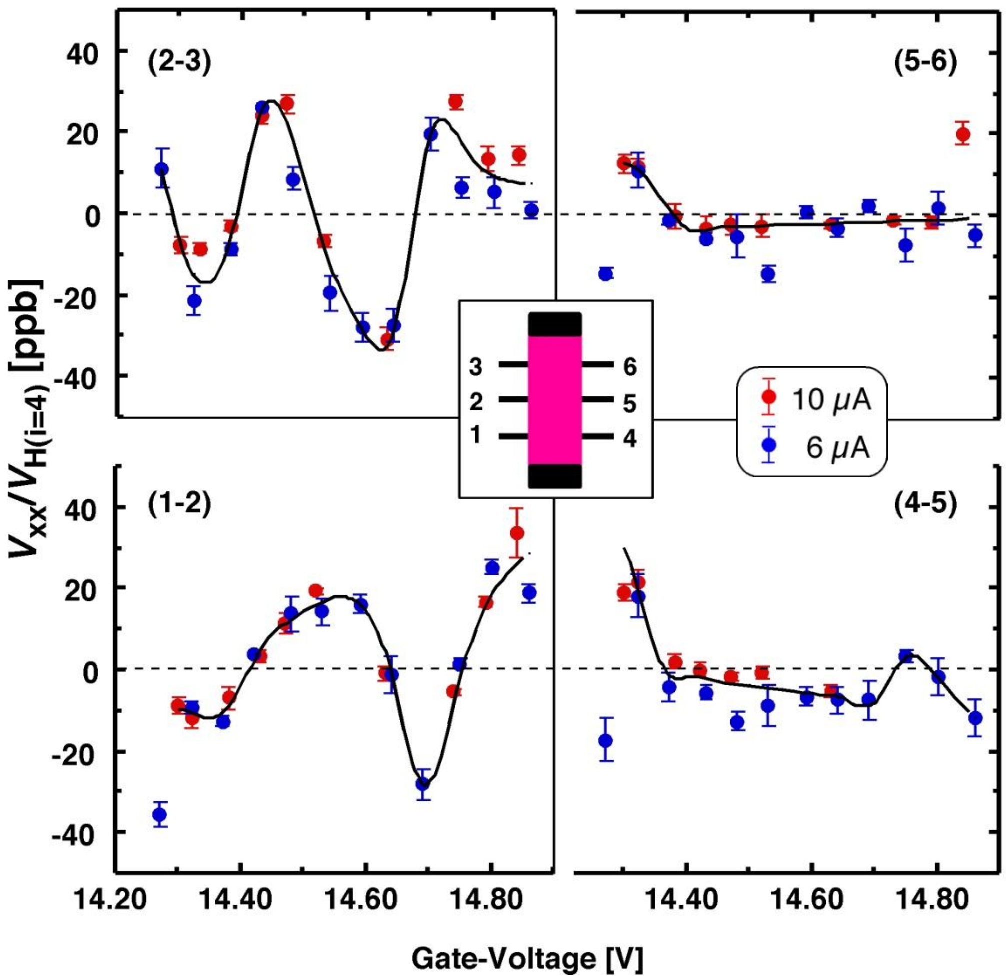
SONY MOSFET



Anomalous results can be explained by:

- contact effects
- asymmetric longitudinal voltages
- $R_{xx} \approx 0$ ⟶ $\Delta R_H = 0$

Graphene: Nobel Prize 2010

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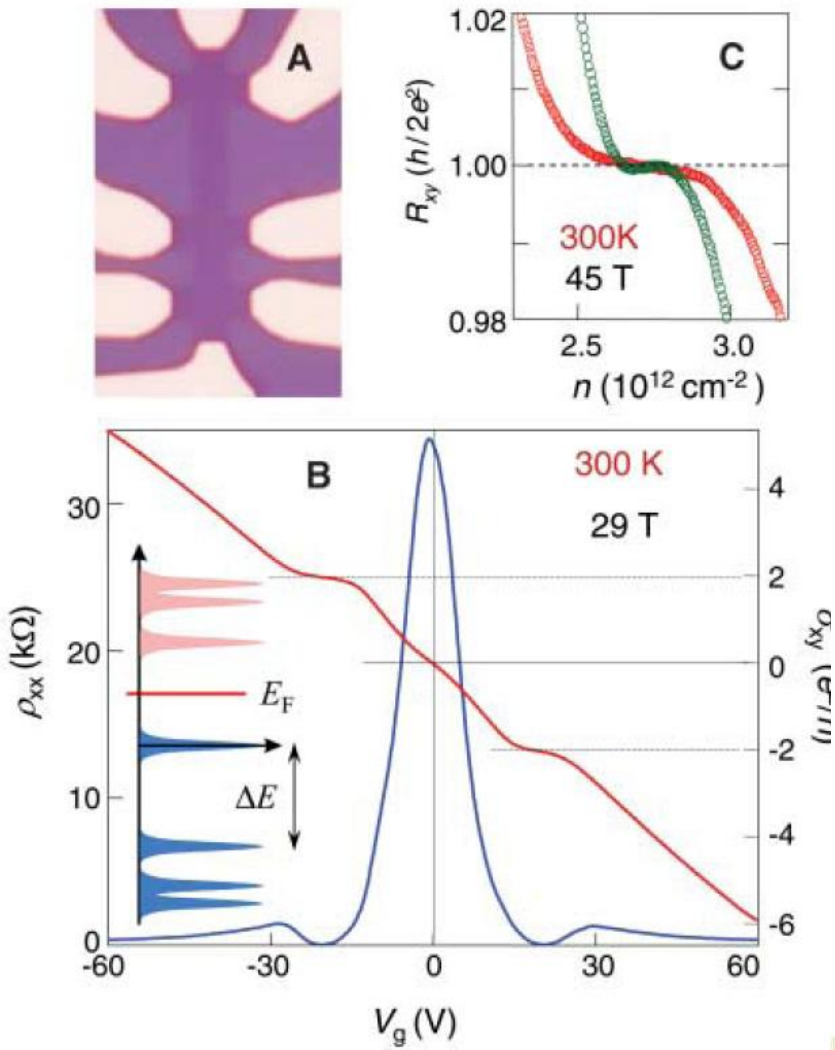
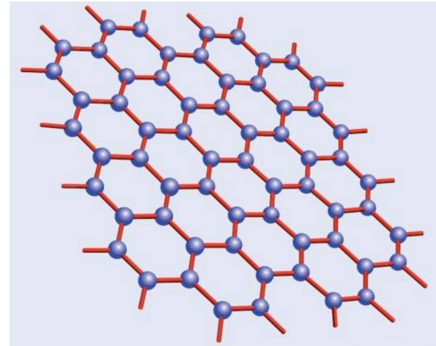
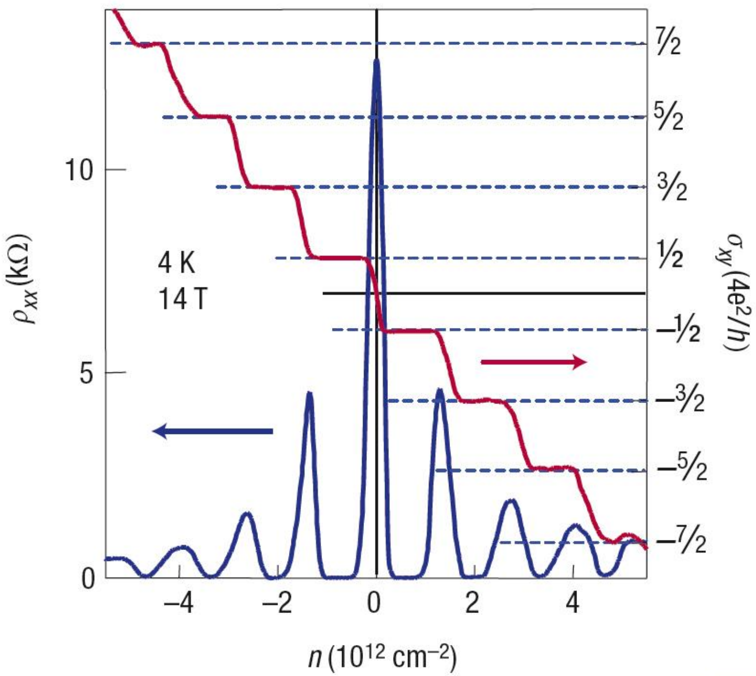


Novoselov et al., Science 315, 2007



$$\sigma_{xy} = \left(4e^2/h\right)\left(N + 1/2\right)$$



Novoselov et al., Nature 438, 2005

→ Cryogen free QHE at 4 K and 2-3 T

Graphene (courtesy JT Janssen NPL)

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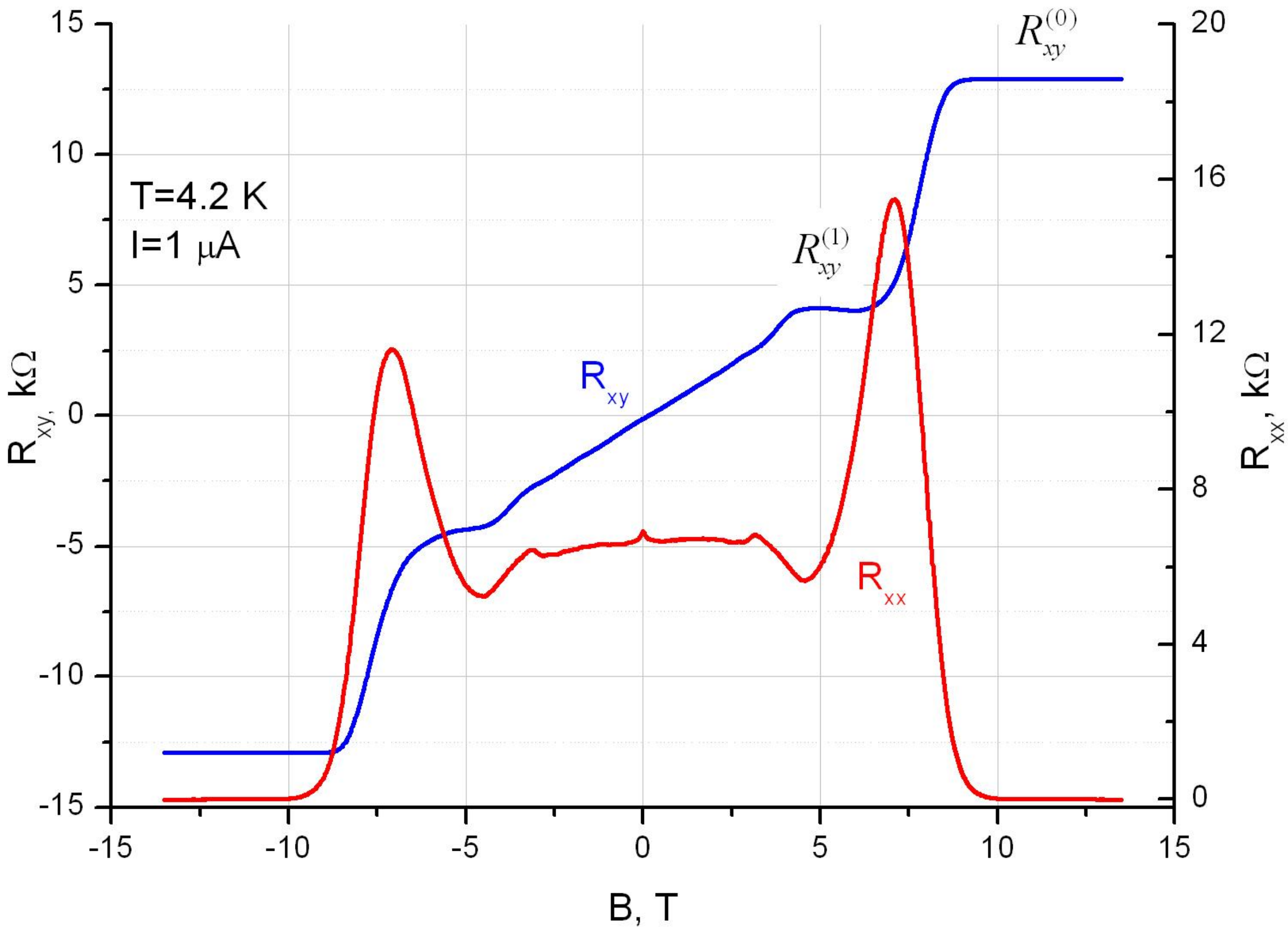

- *Epitaxial single layer*
- R_c = 1.5 Ω

Tzalenchuk, TJBMJ et al. Nature Nanotechnology 5, 186 (2010)

Graphene (courtesy of JT Janssen NPL)

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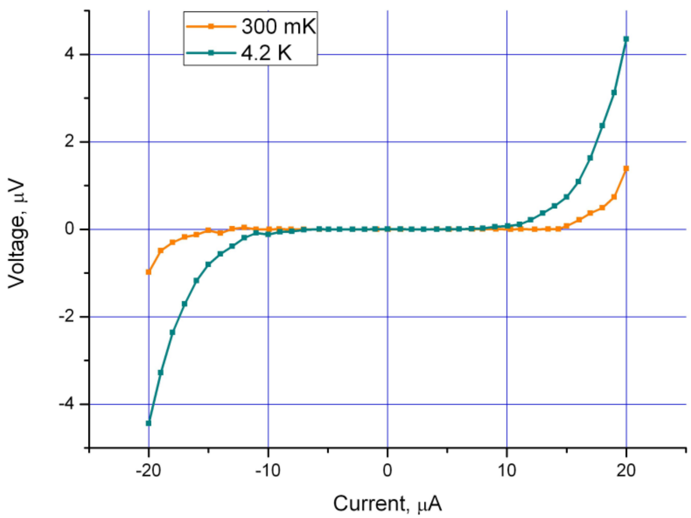
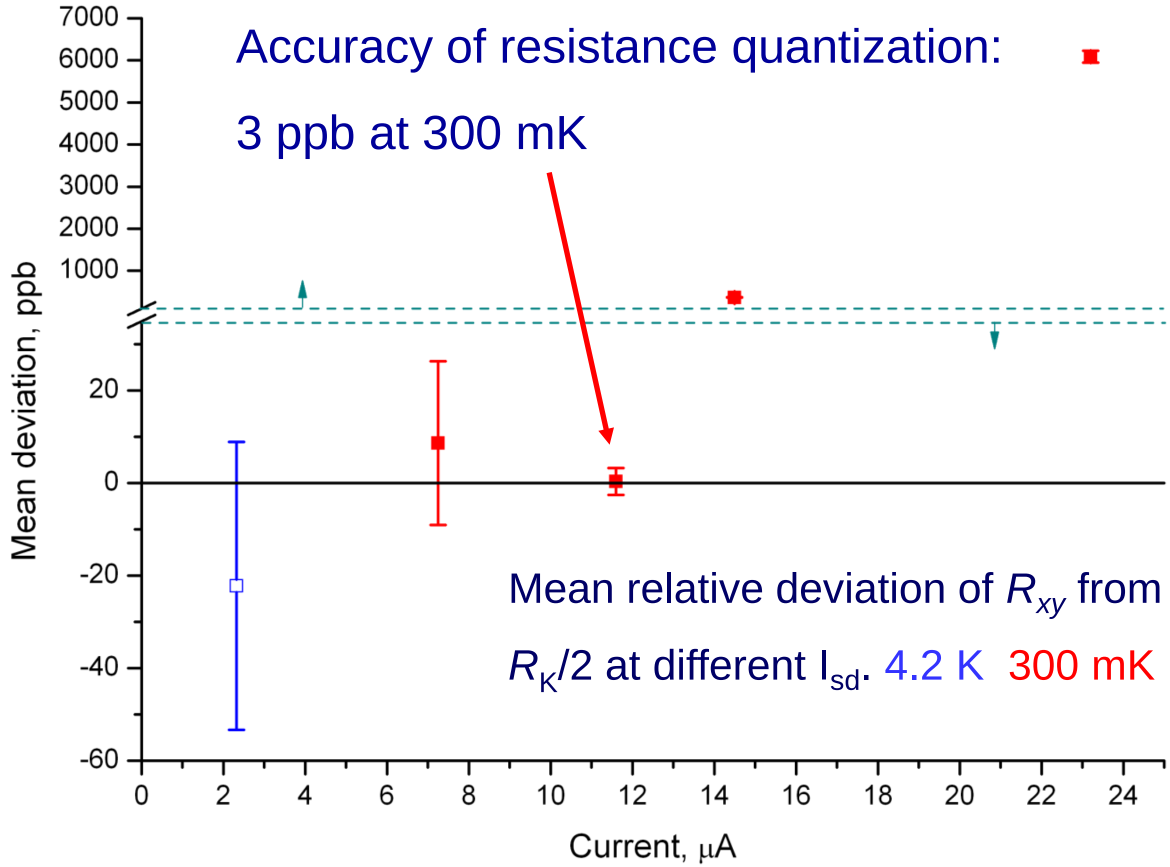
QHE breakdown





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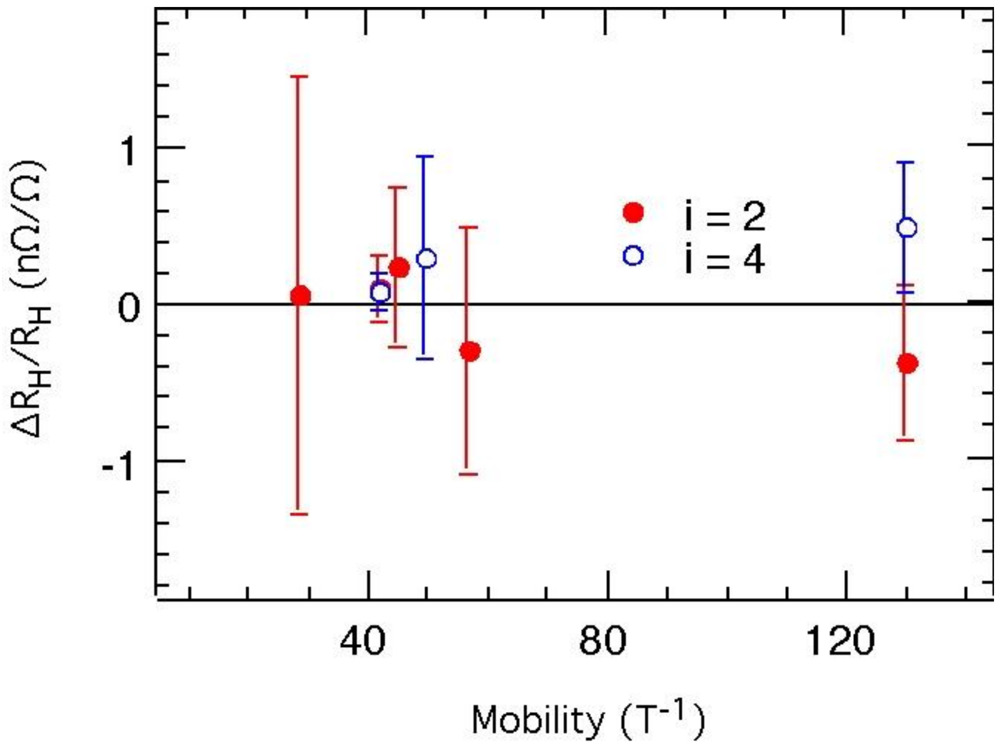
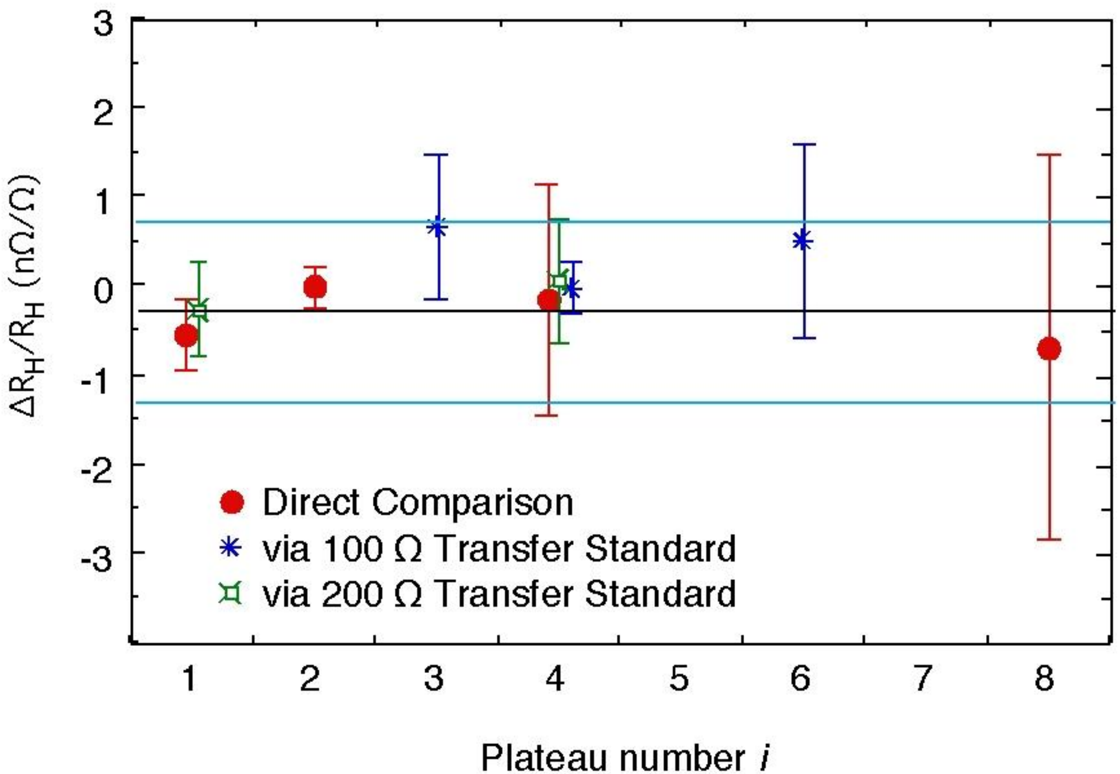
Mobility



- R_H independent of the device mobility or the fabrication process to 2 parts in 10^{-10}

metas

Step ratio measurements



- R_H independent of the plateau index to 3 parts in 10^{-10}

$$\frac{i \cdot R_H(i)}{2 \cdot R_H(2)} = 1 - (1.2 \pm 2.9) \times 10^{-10}$$

$$i = 1, 3, 4, 6, 8$$

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Summary

The quantum Hall resitance is a universal quantity independent of:

- Device width
- Device material: MOSFET-GaAs, **Graphene**
- Device mobility
- Plateau index

$$R_H(i, R_{xx} \longrightarrow 0) = h/ie^2$$

…..to a level of 3 parts in 10^{10}

B. Jeckelmann and B. Jeanneret
Rep. Prog. Phys. 64, 1603, 2001

Negligible dissipation:

- Small measuring current $I << I_c = 0.6$ mA/mm
- Low temperature $T < 1.2$ K
- Good quality electrical contacts $R_c < 10\ \Omega$

CCEM Technical Guideline:
F. Delahaye and B. Jeckelmann
Metrologia 40, 217-223, (2003)

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The SI unit: the Ampere

Définition:

L'ampère est l'intensité d'un courant constant qui, maintenu dans deux conducteurs parallèles, rectilignes, de longueur infinie, de section circulaire négligeable et placés à une distance de 1 mètre l'un de l'autre dans le vide, produirait entre ces conducteurs une force égale à 2 x 10^{-7} newton par mètre de longueur.

$$\frac{F}{l} = \frac{\mu_0 \, I_1 \, I_2}{2 \, \pi \, d}$$

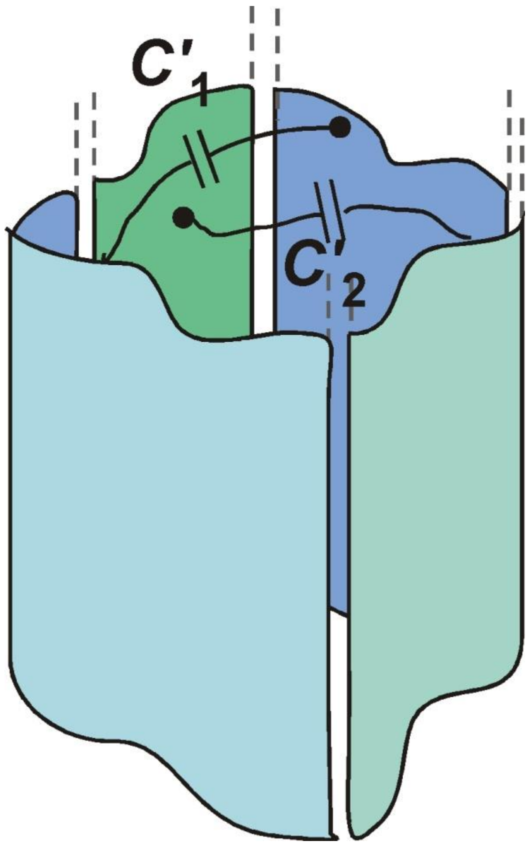
$$\Rightarrow \mu_0 = 4\pi \times 10^{-7} \text{ Vs/Am}$$

$$c^2 = \frac{1}{\varepsilon_0 \mu_0}$$

$c, \ \varepsilon_0, \mu_0$ **are exact!!**

The definition does not lead to a practical realisation of the Ampere!!!

The SI realisation of the Ohm: the calculable capacitor

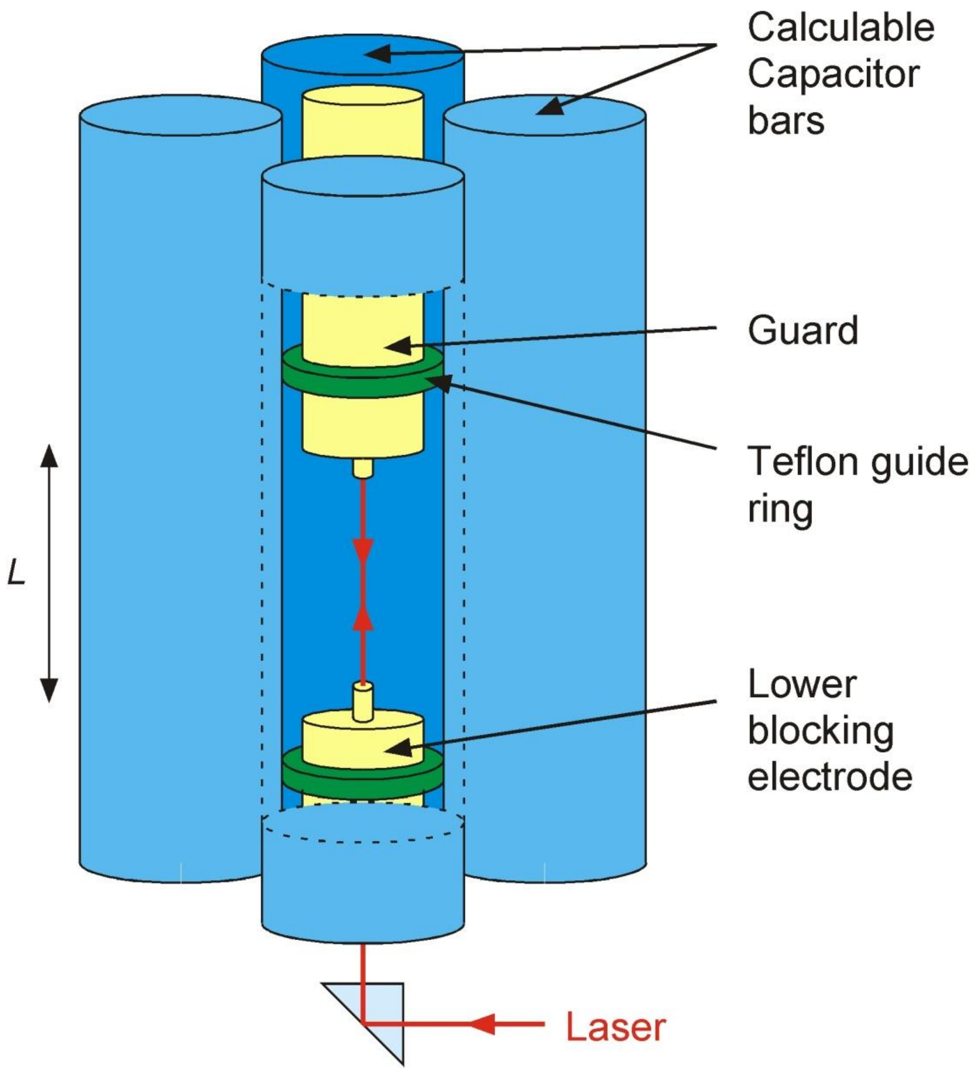


Thompson-Lampard Theorem (1956):

$$\exp\left(-\frac{\pi C_1'}{\varepsilon_0}\right)+\exp\left(-\frac{\pi C_2'}{\varepsilon_0}\right)=1$$

Cross-capacitance identical:

$$C' = \frac{\varepsilon_0 \ln(2)}{\pi} \cong 1.95\,\mathrm{pFm}^{-1}$$

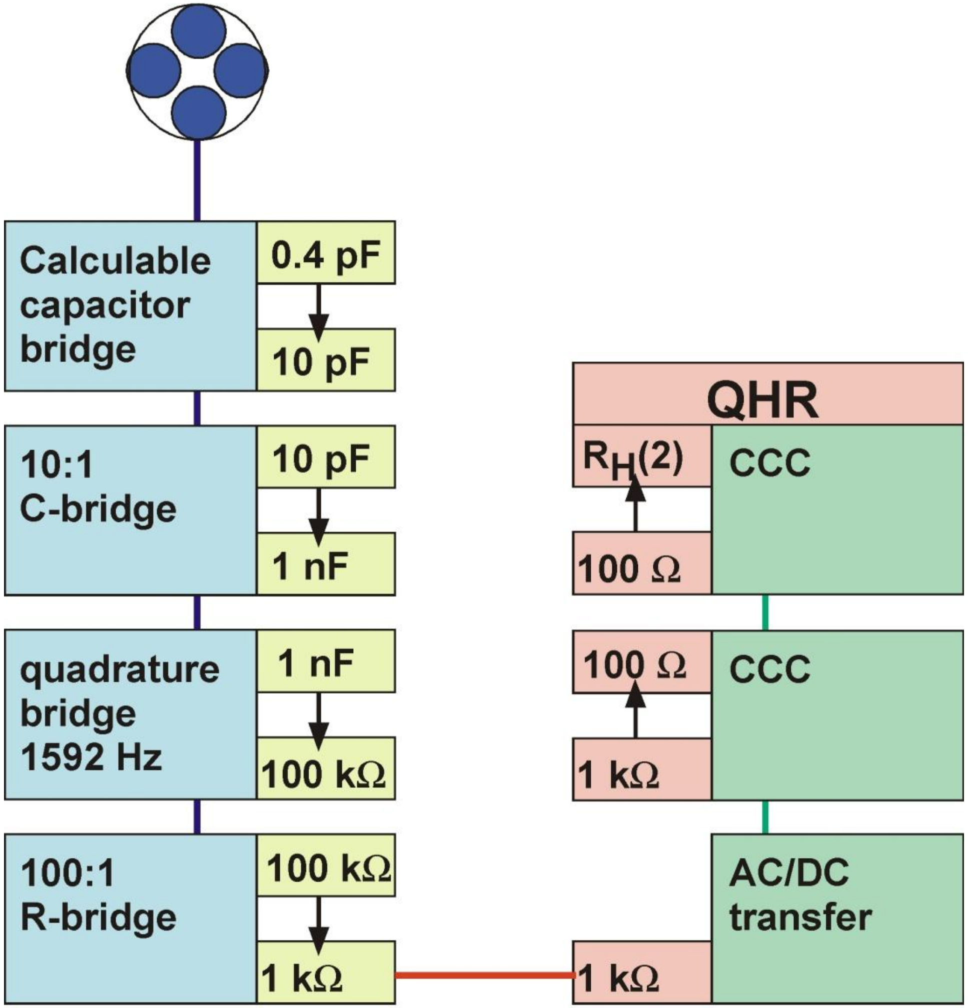


Measurements:

ΔL = 5 - 50 cm

ΔC = 0.1 - 1 pF

$u = 10^{-8}$



CODATA 98:

NPL88: $\Delta R_k / R_k = 5.4 \times 10^{-8}$

NIST97: $\Delta R_k / R_k = 2.4 \times 10^{-8}$

NML97: $\Delta R_k / R_k = 4.4 \times 10^{-8}$

NIM95: $\Delta R_k / R_k = 1.3 \times 10^{-7}$

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A conventional value for R_K

SI Realization of R_K: a few parts in 10^8 uncertainty

Reproducibility of R_H: a few parts in 10^{10} uncertainty

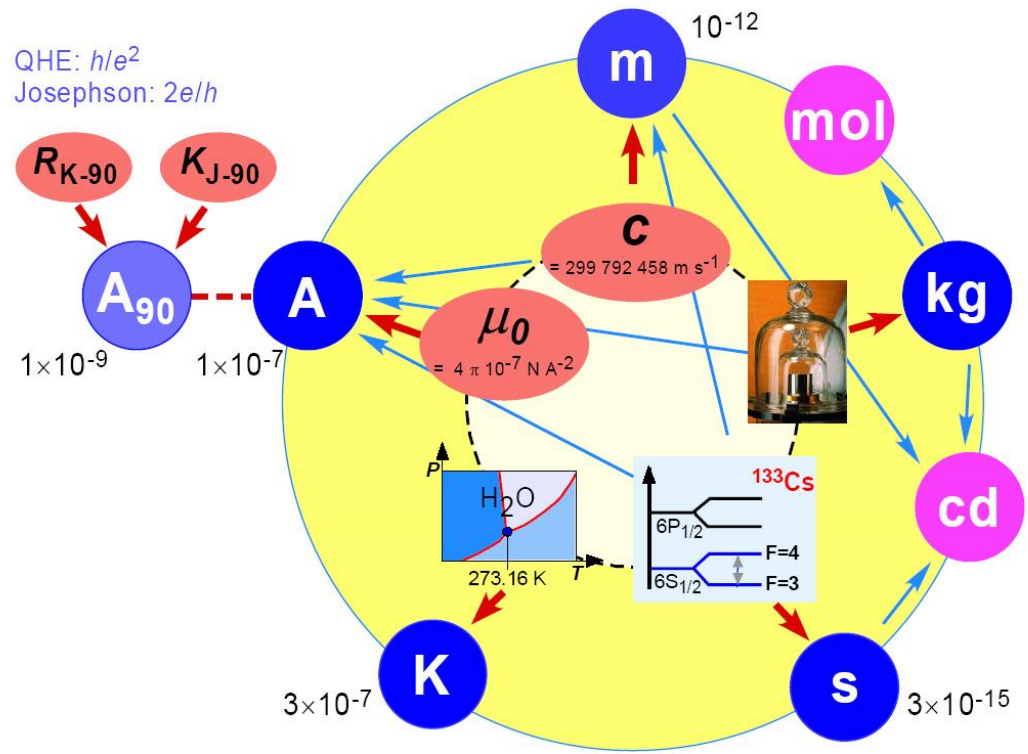
2 orders of magnitude!

Conventional value (exact): CCE 1-1-90

$R_{K\text{-}90}$ = 25 812.807 Ω

$K_{J\text{-}90}$ = 483 597.9 GHz/V

Josephson effect: $K_J = 2e/h$



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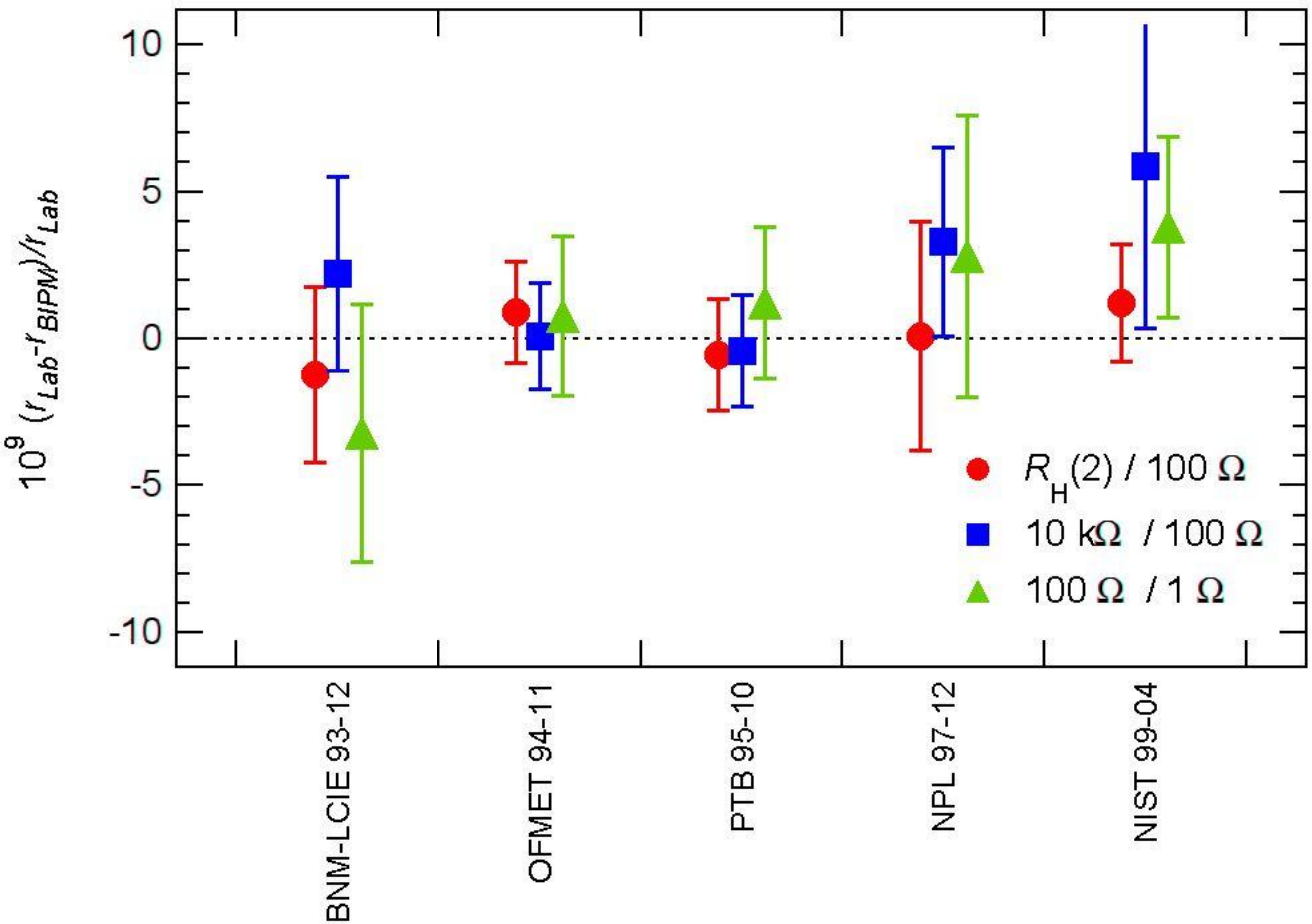
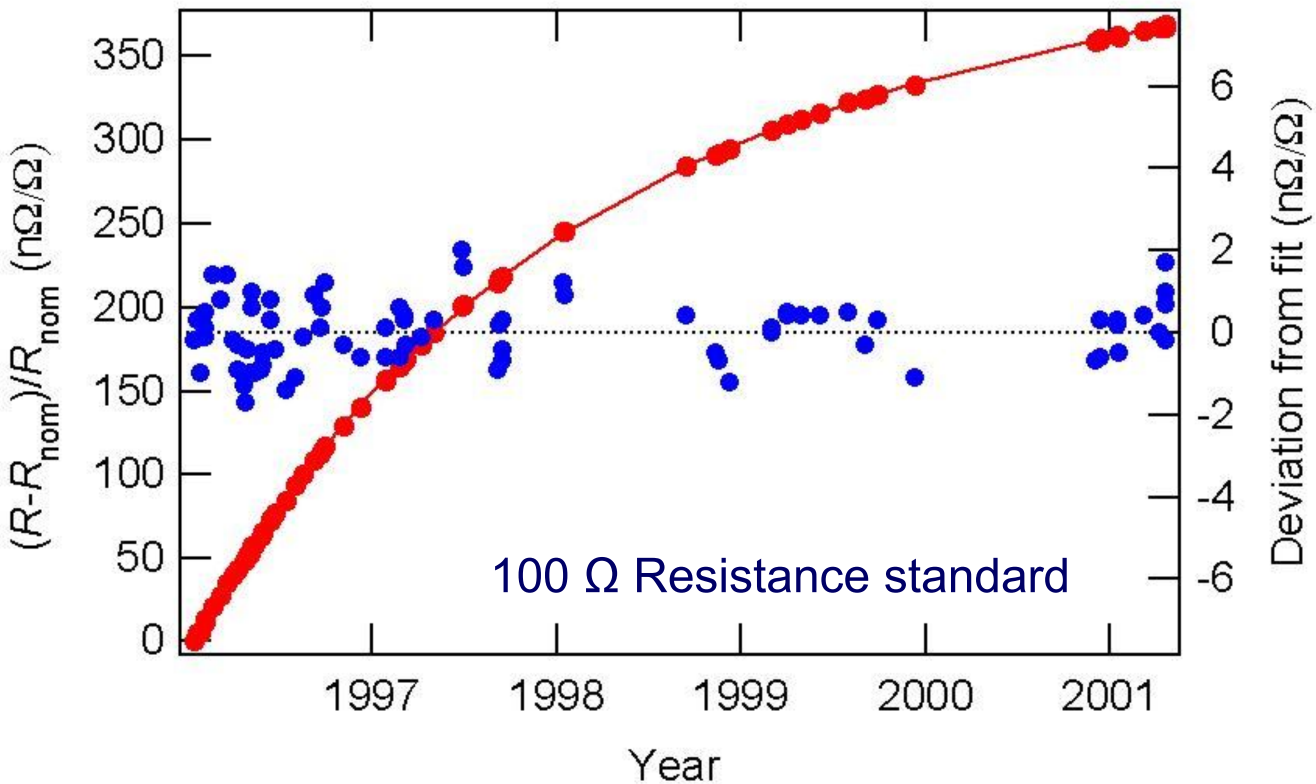
International QHR Key-comparison



Applications: DC Resistance Standard (data METAS)



- Deviation from fit < 2 nΩ/Ω
  over a period of 10 years

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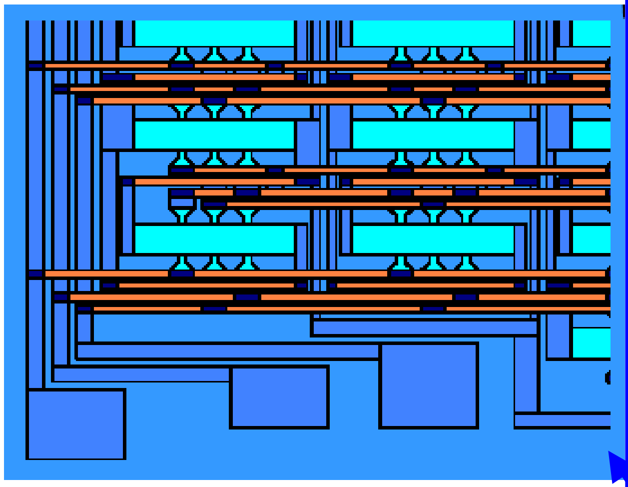
Applications: QHR Arrays

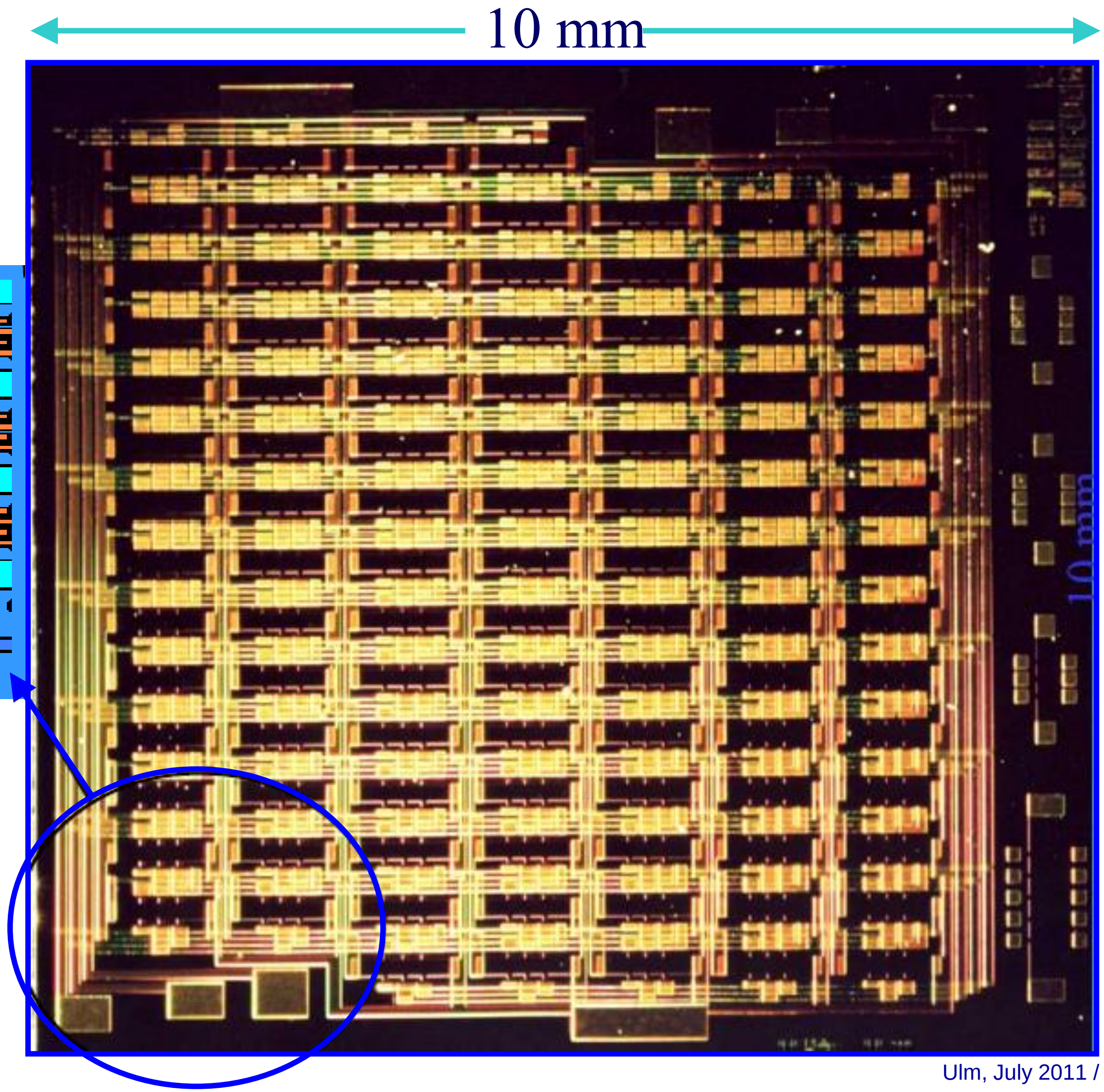
- On chip, large array of Hall bars (up to 100 devices)
- Series-parallel connection scheme: $R = R_K / 200$
- Accuracy of the quantization: 5 parts in 10^9 (T = 1.3 K, i = 2)
- Behave like a single Hall bar
- Transportable resistors for international comparison
- Nominal value of 100 Ω can be fabricated

Applications: QHE array

metas

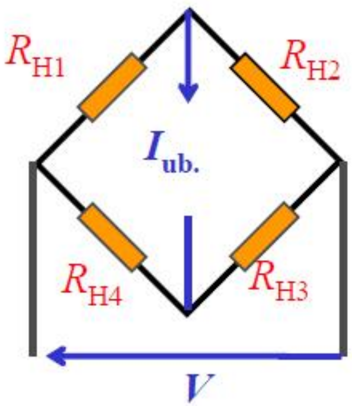
129 Ω @ *i* = 2





W.Poirier et al. LNE

Universality tests: quantum Hall Wheatstone bridge

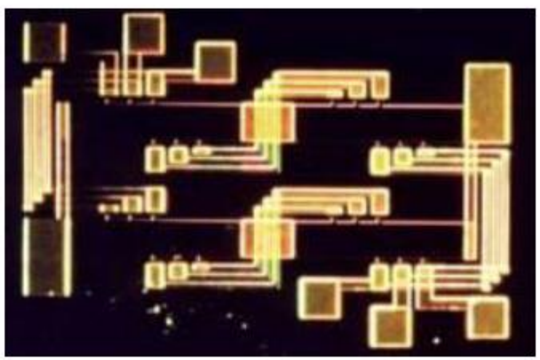


$$I_{ub}/I = V_{ub.}/V \approx \frac{1}{4}\left[(\alpha_1+\alpha_3)-(\alpha_2+\alpha_4)\right]$$

α_j(<<1): relative deviation of the j^{th} resistor to $R_K/2$

Relative deviation of one resistor among the others:
$\Delta R/R = 4\times(I_{ub.}/I) = 4\times(V_{ub.}/V)$

• On-chip fully integrated QHE Wheatstone bridge



LNE-OMMIC sample

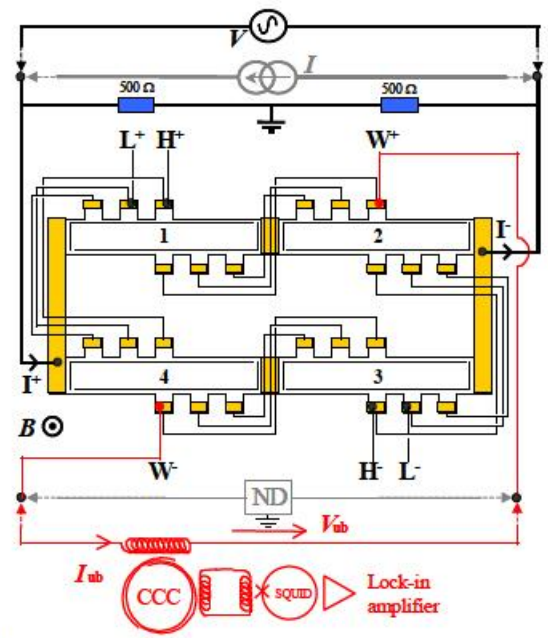
4 Hall bars

Quadruple connection technique

(cancel the resistance of the connections between the four quantum standards)

I=78 µA **T=46000 s**

$\Delta R/R$=(2,7 +/- 0,8)x10^{-10}



F. Schopfer et al, J. Appl. Phys. 102, 054903 (2007)

LNE
Le progrès, une passion à partager

Universality tests: quantum Hall Wheatstone bridge

- **QHE Wheatstone bridge with 4 GaAs/AlGaAs LEP 514 Hall bars**

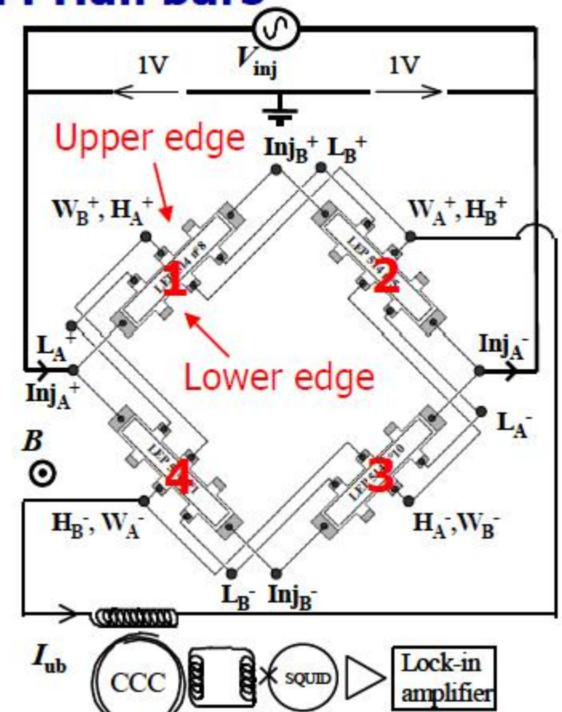
F. Schopfer et al, in: A.H. Cookson, T. Winter (Eds.), Proc. of the CPEM, Boulder, 2008, p. 22.

Current detection

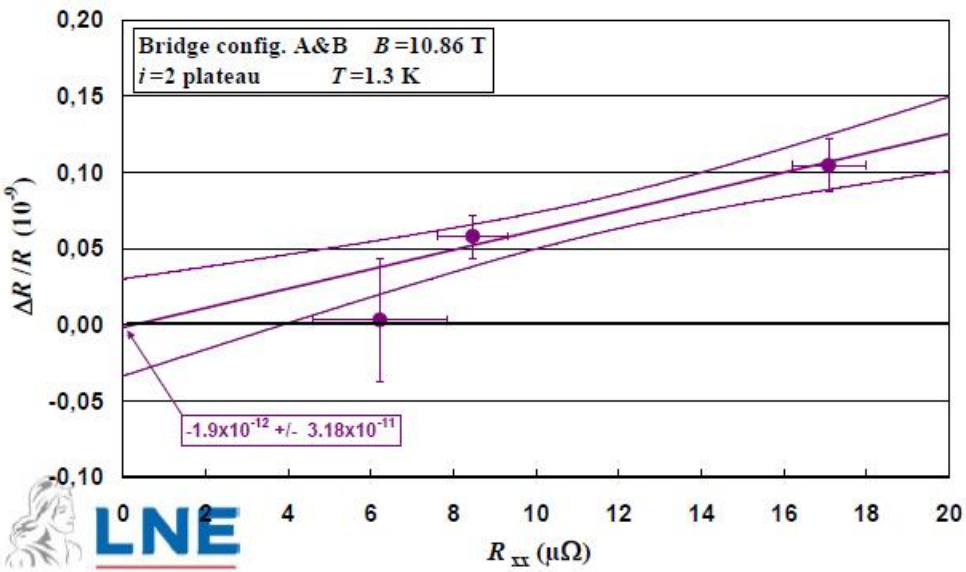
- CCC equipped with a RF SQUID (3436 turns winding)

⇒ **Current resolution: 400 fA/Hz$^{1/2}$**

- Lock-in technique (0.15 Hz to 20 Hz)



- **Extrapolation to zero dissipation state (R_{xx}=0)**



$\Delta R/R$ = -1.9x10^{-12} +/- 3.18 × 10^{-11}

None of the four quantum Hall resistances departs from the others by more than 3 parts in 10^{11}

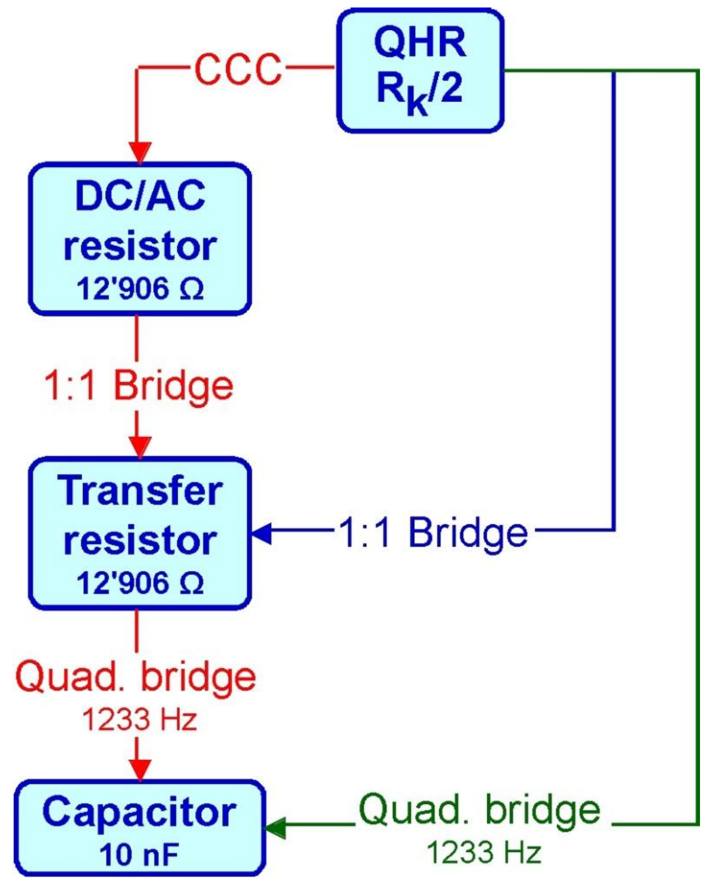
Towards an uncertainty of some parts in 10^{12}

New CCC equipped with a DC SQUID

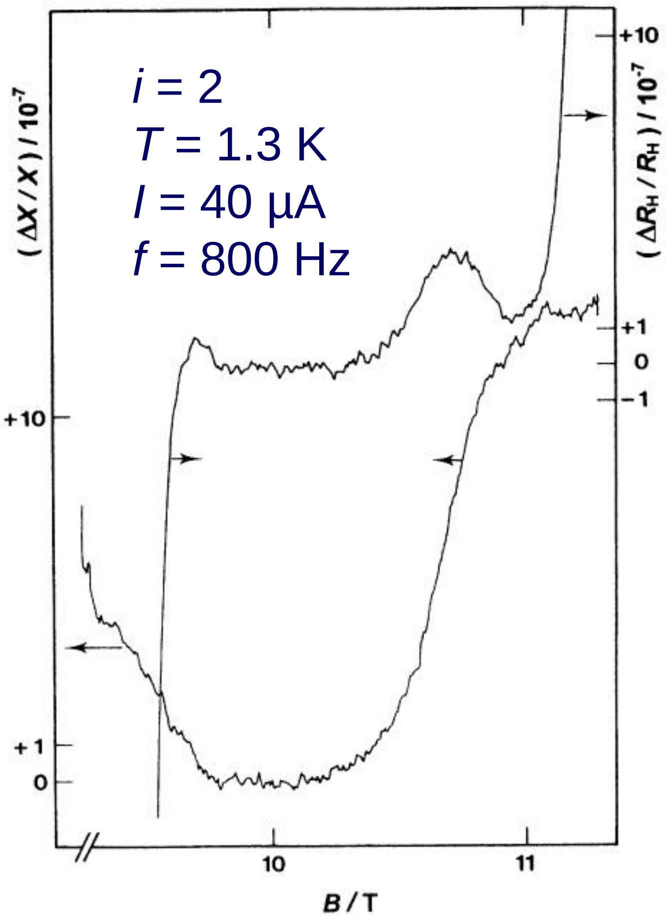
metas

Applications: Capacitance calibration

- SI realisation of the Farad: Calculable capacitor
- Complicated experiment
- Representation of the Farad: DC QHE
- New route: AC measurements of the QHR



AC measurements of the QHE

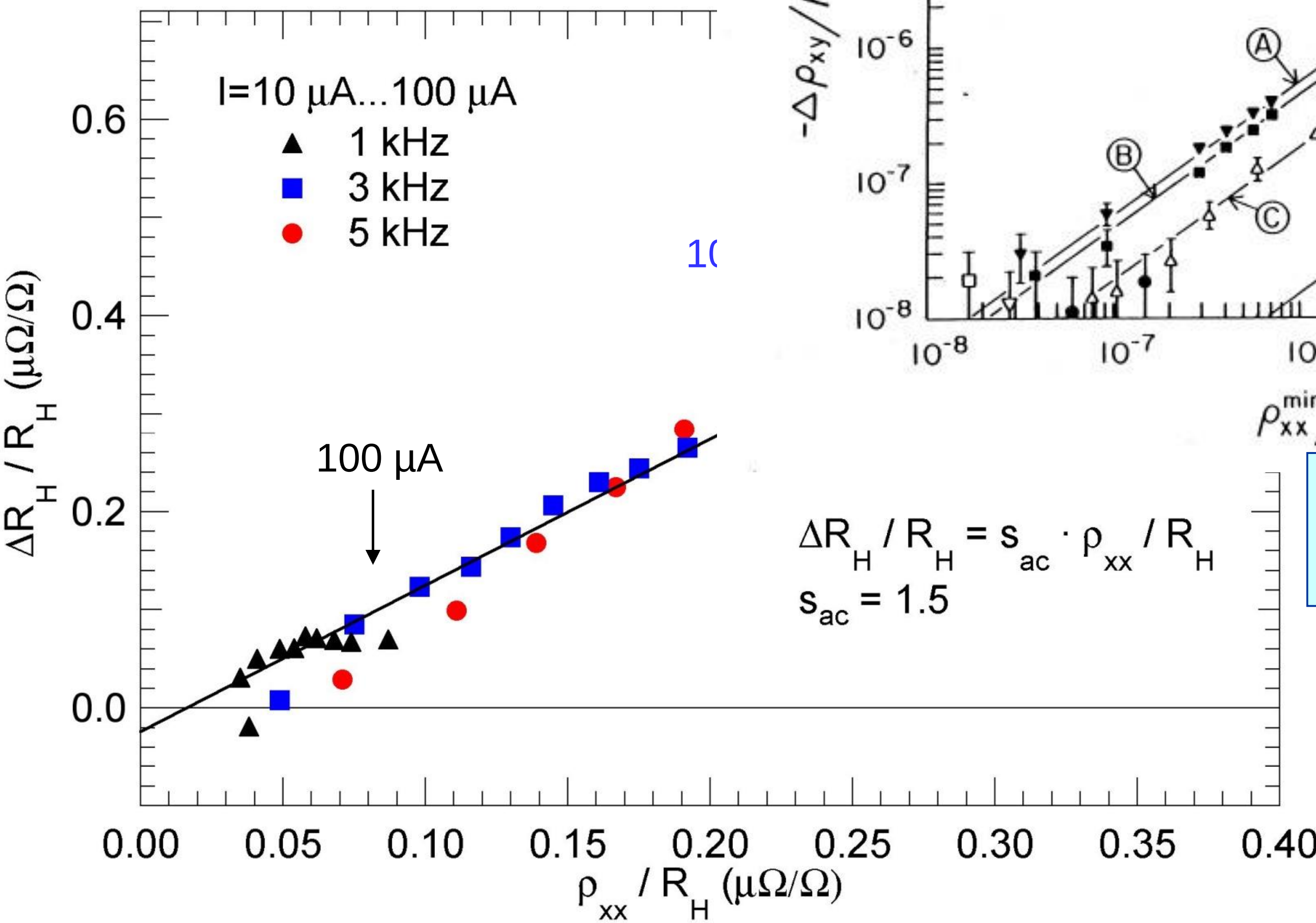


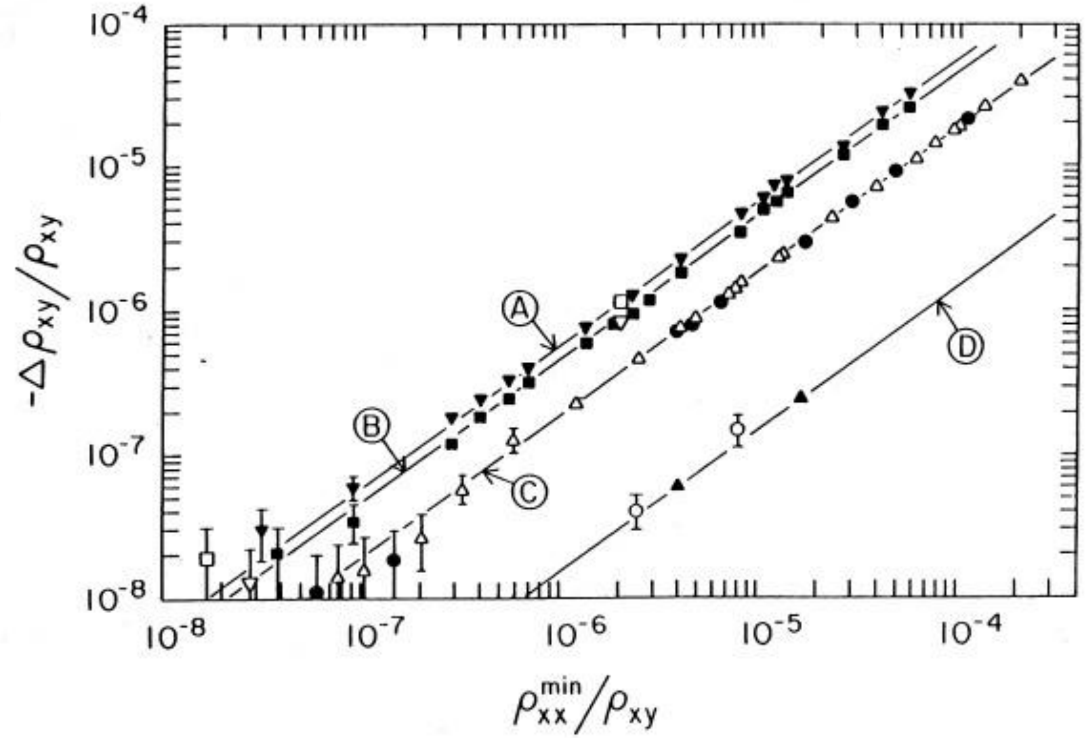
Delahaye 94

- Narrow bumpy “plateau “ (PTB, NPL, NRC, BIPM)
- Frequency dependence: $R_H(i, \omega) = \alpha\ \omega$

  $\alpha = 1 \text{ -}5\ 10^{-7}/\ \text{kHz}$
- Measurements problem: AC Losses

AC measurements of the QI





→ Extrapolate to zero at zero dissipation

AC-QHE: Phenomenological Model

metas

$$\vec{J}(t) = \sigma \vec{E}(t) + d\frac{\partial \vec{D}(t)}{\partial t}$$

d: effective thickness

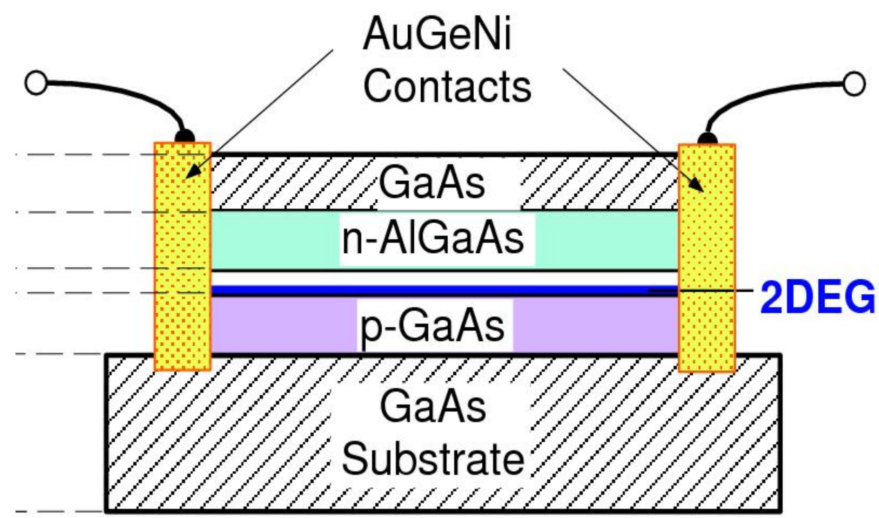
Does not depend on frequency

$\vec{J}_D$ Displacement current sheet density

$$\vec{D}(t) = \epsilon_0 \vec{E}(t) + \vec{P}(t)$$

$$\vec{P}(t) = \epsilon_0 \chi \vec{E}(t)$$

$$\chi = (\chi_r - j\chi_i)\begin{pmatrix} \cos(\varphi) & -\sin(\varphi) \\ \sin(\varphi) & \cos(\varphi) \end{pmatrix}$$

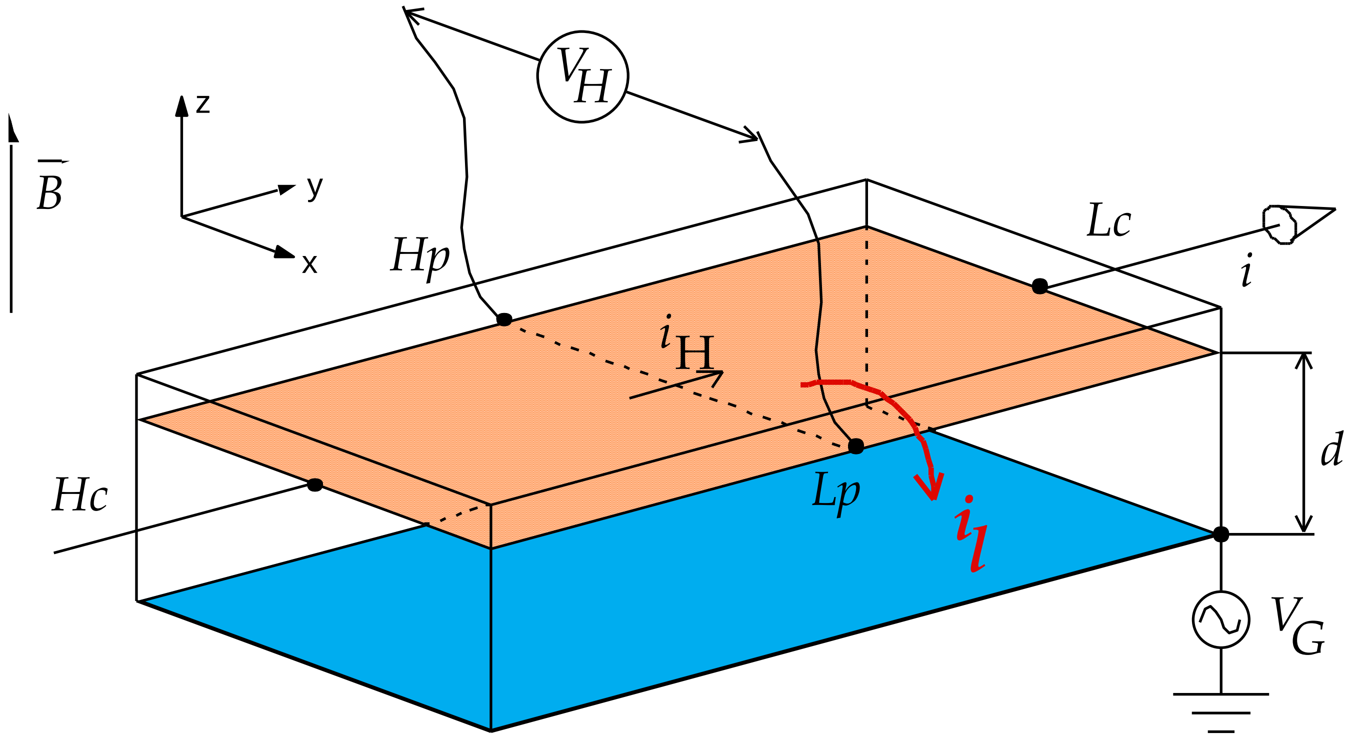


Susceptibility: 2D Model (no gates, no screening electrodes, *d* << *L*, *d* << *w*…..)

→ Model fully explain the measurements (frequency dependence): *B. Jeanneret et al. 2006*

metas

AC Losses

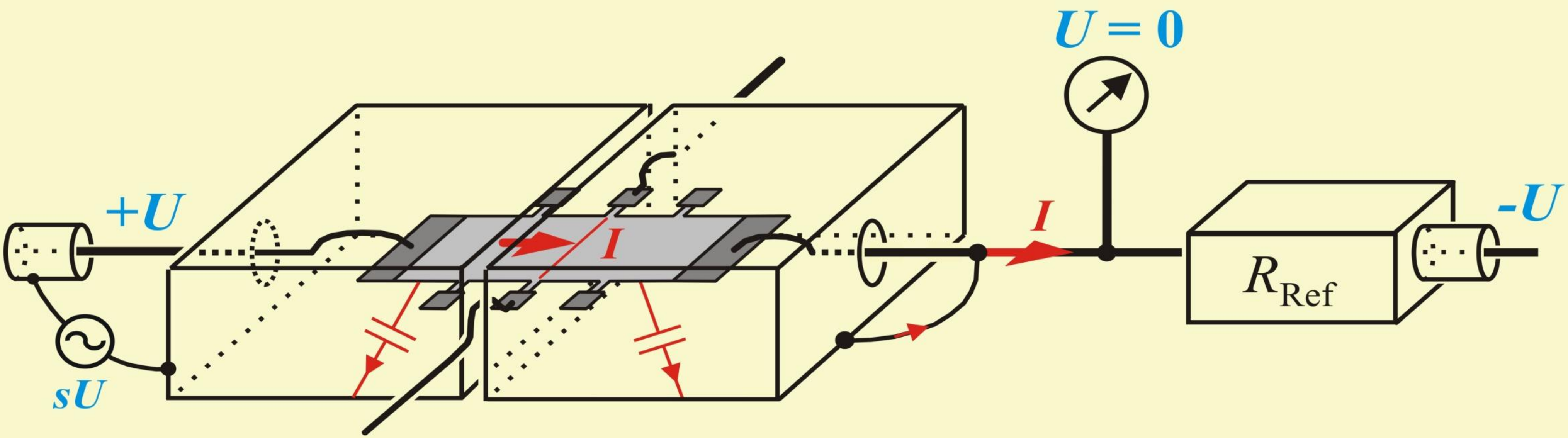


$$Z_{\mathrm{H}} = \frac{V_{\mathrm{H}}}{i} = \frac{V_{\mathrm{H}}}{i_{\mathrm{H}} - i_{l}} \approx R_{\mathrm{H}}\left(1 + \frac{R_{\mathrm{H}}}{V_{\mathrm{H}}} i_{l}\right) = R_{\mathrm{H}}(1 + \Delta)$$

Overney et al., 2003

Double-shielding technique (courtesy J. Schurr, PTB)

metas

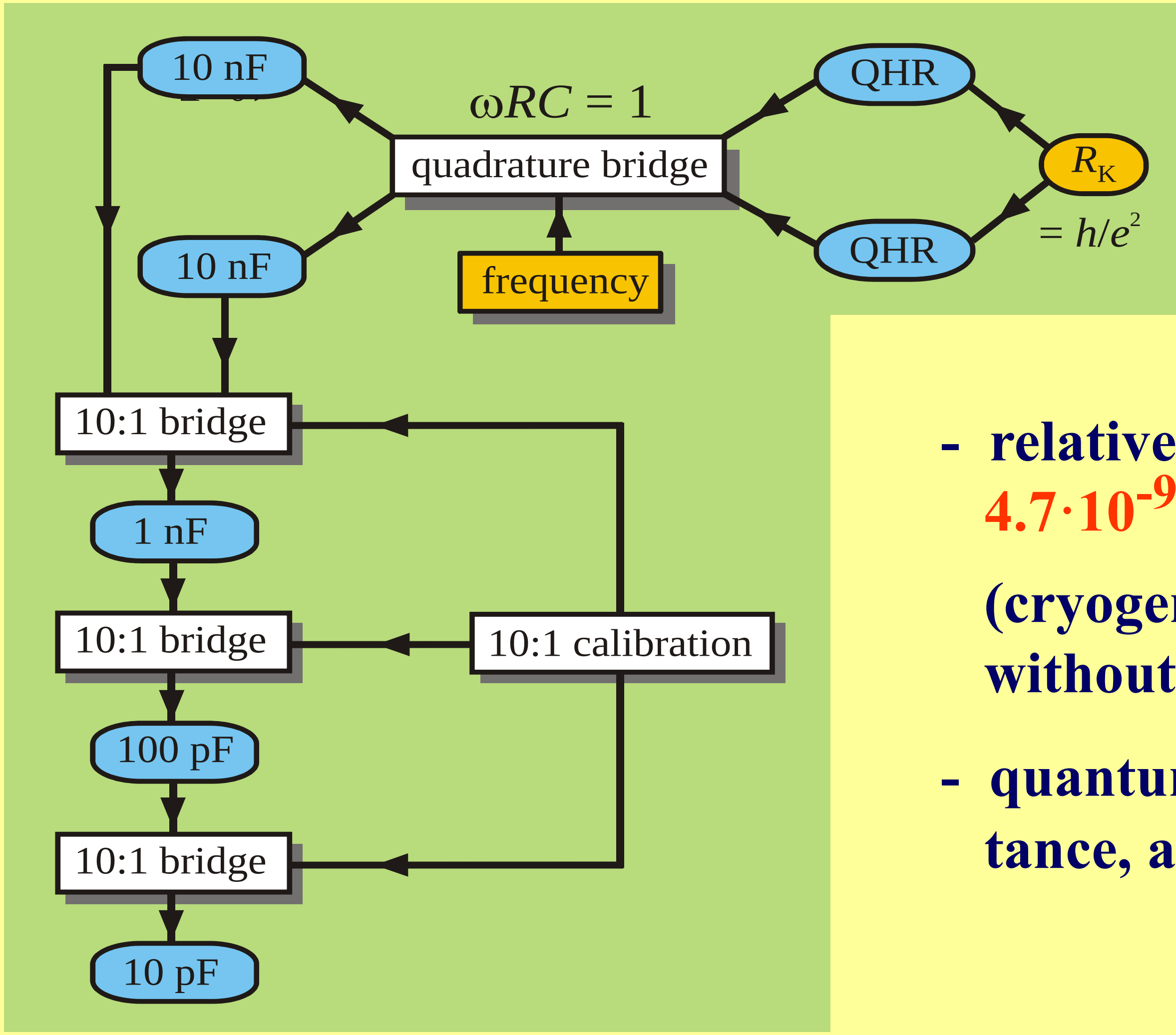


Meet the <u>defining condition</u>: ALL currents which have passed the Hall-potential line are collected and measured.

Adjust the high-shield potential sU so that $\mathrm{d}R_H/\mathrm{d}I = 0$.

B.P. Kibble, J. Schurr, *Metrologia* 45, L25-L27 (2008).

Realization of the capacitance unit (courtesy J. Schurr, PTB)



- **relative uncertainty of 10 pF: 4.7·10^{-9} (k = 1)**

  (cryogenic quantum effect without 'calculable' artefacts)

- **quantum standard of capacitance, analogous to R_{DC}**

J. Schurr, V. Bürkel, B. P. Kibble, *Metrologia* 46, 619-628, 2009

metas

Conclusions

- R_H is a universal quantity
- QHR improved electrical calibration in National Metrology Institutes
- QHR allows a representations of the Farad
- QHR is a primary standard for impedances: AC-QHE
- Future development: **Graphene**