

Quantum Communications with Continuous Variables

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Content of this talk

1. Survival Kit on Optical Quantum Continuous Variables

- * Pulsed homodyne detection and quantum tomography Positive and negative Wigner functions
- * Schrödinger's kittens and cats...
	- A. Ourjoumtsev et al, Science 312, 83 (2006), Nature 448, 784 (2007)
- * Quantum Key Distribution with continuous variables Gaussian and non-Gaussian (unconditional) security proofs A. Leverrier et al, PRL 102, 180504 (2009)

2. New Results and Perspectives for Quantum Communications with QCV

- * Delocalized (entangled) Schrödinger's kittens
	- A. Ourjoumtsey et al, PRL 98, 030502 (2007), Nat. Phys. 5, 189 (2009)
- * Non-deterministic Noiseless Amplifier (tomorrow!)
	- F. Ferreyrol, M. Barbieri et al, PRL 104, 123603 (2010)

Optical Quantum Continuous Variables

* What are quantum optical continuous variables ?

* Quantization of the Electromagnetic field *Modes are quantum harmonic oscillators

- * Discrete degrees of freedom (photon number)
- * Continuous degrees of freedom (quadratures $= X$ and P)

* All is about quantized harmonic oscillators !

* Convenient representation : phase space

Homodyne detection

Homodyne detection, Wigner Function and Quantum Tomography

 \mathbf{X}

- • **Quasiprobability density : Wigner function W(X,P)** • **Marginals of W(X, P)** \Rightarrow Probability distributions $P(X\theta)$ • Probability distributions $P(X\theta)$
	- \Rightarrow W(X, P) (quantum tomography)

Homodyne detection, **Wigner Function and Quantum Tomography**

 $QIPC$

Squeezed State: Gaussian Wigner Functions

Homodyne detection, Wigner Function and Quantum Tomography

 Q I P C

 \mathbf{X}

State with negative Wigner function ! (for a pure state W is non-positive iff it is non-gaussian : Hudson-Piquet theorem) **Many interesting properties for quantum information processing**

Wigner function of a single photon state ? (Fock state $n = 1$)

$$
W(p,q) = \frac{1}{2\pi 2N_0} \int dx \, \mathrm{e}^{\frac{\mathrm{i}xp}{2N_0}} \left\langle q - \frac{x}{2} \left| \hat{\rho} \right| q + \frac{x}{2} \right\rangle
$$

where $\hat{\rho} = |1\rangle\langle 1|$ and N₀ is the variance of the vacuum noise :

$$
[\hat{Q}, \hat{P}] \equiv 2iN_0 \qquad \Delta P \Delta Q \ge N_0 \qquad N_0 = \Delta P^2 = \Delta Q^2.
$$

One may have $N_0 = \hbar / 2$, $N_0 = 1/2$ (theorists), $N_0 = 1$ (experimentalists)

Using the wave function of the n = 1 state :
$$
\langle q | 1 \rangle = \frac{q}{(2\pi)^{\frac{1}{4}} N_0^{\frac{3}{4}}} e^{-\frac{q^2}{4N_0}}
$$

one gets finally :
$$
W_{|1\rangle}(q, p) = -\frac{1}{2\pi N_0} e^{-\frac{r^2}{2N_0}} \left(1 - \frac{r^2}{N_0}\right)
$$
 $r^2 = q^2 + p^2$

« Discrete » vs « continuous » Light

Make It Quantum and Continuous

Philippe Grangier

PERSPECTIVES

SCIENCE VOL 332 15 APRIL 2011

Unconditional Quantum Teleportation

A. Furusawa, J. L. Sørensen, S. L. Braunstein, C. A. Fuchs, H. I. Kimble.* E. S. Polzik

23 OCTOBER 1998 VOL 282 SCIENCE

Quantum key distribution using gaussian-modulated coherent

states NATURE | VOL 421 | 16 JANUARY 2003 |

Frédéric Grosshans", Gilles Van Asschet, Jérôme Wenger", Rosa Brouri*, Nicolas J. Cerf+ & Philippe Grangier*

NATURE | VOL 432 | 25 NOVEMBER 2004 | www.nature.com/nature

Experimental demonstration of quantum memory for light

Brian Julsgaard¹, Jacob Sherson^{1,2}, J. Ignacio Cirac³, Jaromír Flurášek⁴ & Eugene S. Polzik¹

Vol 443|5 October 2006|doi:10.1038/nature05136

Quantum teleportation between light and matter

Jacob F. Sherson^{1,3}, Hanna Krauter¹, Rasmus K. Olsson¹, Brian Julsgaard¹, Klemens Hammerer², Ignacio Cirac² & Eugene S. Polzik¹

PHYSICAL REVIEW A 68, 042319 (2003)

Quantum computation with optical coherent states

T. C. Ralph,* A. Gilchrist, and G. J. Milburn W. J. Munro S. Glancy

Generating Optical Schrödinger Kittens for Quantum Information Processing

Alexei Ourioumtsev, Rosa Tualle-Brouri, Julien Laurat, Philippe Grangier* **SCIENCE** VOL 312 7 APRIL 2006

Vol 448 16 August 2007 doi:10.1038/nature06054

Generation of optical 'Schrödinger cats' from photon number states

Alexei Ourjoumtsev¹, Hyunseok Jeong², Rosa Tualle-Brouri¹ & Philippe Grangier¹

Teleportation of Nonclassical Wave Packets of Light

Noriyuki Lee,¹ Hugo Benichi,¹ Yuishi Takeno,¹ Shuntaro Takeda,¹ James Webb,² Elanor Huntington.² Akira Furusawa^{1*}

15 APRIL 2011 VOL 332 SCIENCE

Small sample, many more papers!

Kittens, cats and beyond...

• **The magic of photon subtraction...**

- Classical object in a quantum superposition of distinguishable states
- "Quasi classical" state in quantum optics : coherent state α \rangle

- Resource for quantum information processing
- Model system to study decoherence

Wigner function of a Schrödinger cat state

«""Schrödinger's Kitten »

A. Ourjoumtsev et al, Science 312, 83 (2006)

- Look at small $|\alpha| \sim 1$
- **Very similar to a squeezed single-photon state**
- **Very similar to a photon-subtracted squeezed vacuum state**

Wigner function of a small Schrödinger cat Wigner function of a Photon-subtracted squeezed state

Fidelity between the kitten and the most similar photon-subtracted state

A squeezed state can be « degaussified » by photon subtraction (one single photon in the APD beam)

Also expts at NBI (Polzik), NICT (Sasaki, Furusawa), NIST (Gerrits), LENS (Bellini), Calgary (Lvovsky)…

Experimental Set-up

Experimental Set-up

Femtosecond Ti-S laser and **Constanting the Constanting of Persons pulses 180 fs , 40 nJ, rep rate 800 kHz** $\mathsf{P}(\mathsf{x}_{\scriptscriptstyle \Theta})$ **SHG squeezer** -2 $\chi_{\alpha}^{1} \propto \sqrt[1]{2} - V_1^{1}$ $\overline{2}$ 3 **APD** $P(x_0)$ **Homodyne2-V**
2 **V Time**

Measured Probability Distributions after photon subtraction

Dip in the squeezed quadrature : hint for a negative Wigner function !

Wigner function of the « raw » measured state (no correction)

Radon transform clearly negative ! (no hypothesis, no correction) … but no physical analysis => analytic model

Assuming that $\mu \ll 1$ the Wigner function has the simple generic form :

$$
W(x,p) = \left[2a\frac{x^2}{c} + 2b\frac{p^2}{d} + 1 - a - b\right] \frac{e^{-\frac{x^2}{c} - \frac{p^2}{d}}}{\pi\sqrt{cd}}
$$
 The parameters a, b, c, d,
are simple functions of
s, γ , T, ξ , η , e

The corresponding quadrature probability distribution is :

$$
P(x_{\theta}) = \left[2f\frac{x_{\theta}^2}{g} + 1 - f\right] \frac{e^{-\frac{x_{\theta}^2}{g}}}{\sqrt{\pi g}} \qquad f = \cos^2(\theta)a + \sin^2(\theta)b
$$

$$
g = \cos^2(\theta)c + \sin^2(\theta)d
$$

All parameters can be obtained from the 2d and 4th order moments of $P(x\theta)$

Wigner function of the « raw » measured state (no correction)

« Physical » analytic model fully consistent with Radon transform -> one can reliably correct for the homodyne efficiency

- We are interested in the *generated (propagating) state* => one should correct for the measurement efficiency
- Two possible methods to correct for homodyne losses :
	- numerical method (Maximum Likelihood)
	- using the previous analytical model :
		- 1. From the moments of $P(x_0)$ determine $(s, \gamma, T, \xi, \eta, e)$
		- 2. Check with measured experimental values : OK
		- ! ! **3. Calculate W (s,**), **T,** , , +**, e), compare with Radon : OK**
		- 4. Ideal detection : $\eta = 1$ and $e = 0$
		- **5. Calculate W** (s, T, γ, ξ, 1, 0), compare with MaxLike : OK!

Wigner function of the Kitten (corrected for homodyne efficiency)

 Q I P C

A. Ourjoumtsev et al, Science 312: 83, 2006

Resource : Two-Photon Fock States

What next ?

How to create a Schrödinger's cat ?

Suggestion by Hyunseok Jeong, proofs by Alexei Ourjoumtsev :

$$
S(r)(\vert \alpha \rangle + e^{i\theta}\vert - \alpha\rangle)
$$

 $\alpha^2 = n$ $\theta = n^* \pi$ **3 dB Size : Same Parity as n : Squeezed by :**

The rebirth of the cat

R=50%

- Make a n-photon Fock state
- $50/50$ BS : \approx n/2 photons transmitted

 $|P_0|$ <<1 \Rightarrow OK

Homodyne measurement

- Phase dependence
- Parity measurement : $\langle P_0=0|2k+1\rangle = 0$ Reflected : even number of photons Transmitted : same parity as n

Squeezed cat state (from n=2) = $\sqrt{2/3}$ **| 2** $\rightarrow \sqrt{1/3}$ **| 0** \rightarrow

Resource : Two-Photon Fock States

Squeezed Cat State Generation

Experimental Wigner function

A. Ourjoumtsev et al, Nature 448, 784, 16 august 2007

JL Basdevant 12 Leçons de Mécanique Quantique

Wigner function of the prepared state Reconstructed with a Maximal-Likelihood algorithm Corrected for the losses of the final homodyne detection.

Bigger cats : NIST (Gerrits, 3-photon subtraction), ENS (Haroche, microwave cavity QED), UCSB…

Towards quantum communications and quantum networks ?

- • **Towards quantum communications ?**
- • **First, look at continuous variable entanglement !**

 $(X_A + X_B)$ and $(P_A - P_B)$ are squeezed (commuting operators !) then $(P_A + P_B)$ and $(X_A - X_B)$ are anti-squeezed

> If Alice measures X_A , she will know X_B If Alice measures P_A , she will know P_B and for a large enough squeezing we have :

> > $V(X_B|X_A) V(P_B|P_A) < N_0^2$!!!

 α α **W 2 W 2 W P Extermine P EXEC P EX**

If the squeezing goes to infinity : original EPR state (1935) !

How to produce CV entangled beams ?

Quantum teleportation of coherent states

Experiments :

A. Furusawa et al, Science **282**, 706 (1998) W. Bowen et al, Phys. Rev. A **67**, 032302 (2003) T.C. Zhang et al, Phys. Rev. A **67**, 033802 (2003)

A very useful equivalence : "virtual entanglement"

"Prepare and measure" protocol is equivalent to an entangled state protocol ! This equivalence is extensively used in security proofs

CVQKD : from the idea to unconditionnal security proofs

Coherent States Quantum Key Distribution

* Essential feature : quantum channel with non-commuting quantum observables **! ! -> not restricted to single photon polarization or phase !**

-> Design of Continuous-Variable QKD protocols where :

* The non-commuting observables are the quadrature operators X and P * The transmitted light contains weak coherent pulses (about 10 photons) with a gaussian modulation of amplitude and phase * The detection is made using shot-noise limited homodyne detection

Coherent States Quantum Key Distribution

- Bob reveals measurement choice
- Alice and Bob share a set of Gaussian correlated data
- Further communication to calculate channel parameters and derive secret key based on Bob's data → reverse reconciliation

F. Grosshans et al. Phys. Rev. Lett. 88, 057902 (2002) & Nature 421, 238 (2003)

QKD protocol using coherent states with gaussian amplitude and phase modulation

Efficient transmission of information using continuous variables ? \sim Shannon's formula (1948) : the mutual information I_{AB} (unit : bit / symbol) for a gaussian channel with additive noise is given by

(a) Alice chooses X_A and P_A within two random gaussian distributions. $X^{}_{B}$ $X^{}_{A}$ P₁ X $P₁$ $\rm V_A$ \dot{V}_A I_{AB} = 1/2 $\log_2 [1 + V(\text{signal}) / V(\text{noise})]$ N_0 (b) Alice sends to Bob the coherent state $| X_A + i P_A >$ P. (c) Bob measures either X_B or P_B (d) Bob and Alice agree on the basis choice (X or P), and keep the relevant values. Reminder : $I(X; Y) =$ $H(X) - H(X | Y) =$ $H(Y) - H(Y|X) =$ $H(X) + H(Y) - H(X; Y)$

Data Reconciliation

how to correct errors, revealing as less as possible to Eve ?

Main idea (Csiszar and Körner 1978, Maurer 1993) :

Alice and Bob can in principle distill, from their correlated key elements, a common secret key of size $S > sup(I_{AB} - I_{AE}, I_{AB} - I_{BE})$ bits per key element.

Crucial remark : it is enough that I_{AB} is larger than the **smallest** of I_{AE} and I_{BE} (i.e. one has to take the best possible case).

Data Reconciliation

- If I_{AE} is the smallest, the reconciliation must keep $S = I_{AB} - I_{AE}$ constant : Alice gives correction data to Bob (and also to Eve), and Bob corrects his data : « direct reconciliation protocol »
- If I_{BE} is the smallest, the reconciliation must keep $S = I_{AB} - I_{BE}$ constant : Bob gives correction data to Alice (and also to Eve), and Alice corrects his data : « reverse reconciliation protocol »

Crucial question for Alice and Bob : how to bound I_{AE} and I_{BE} , knowing I_{AB} ?

EPR versus coherent protocol

EPR protocol equivalent to coherent state protocol ! Cf BB84 vs entangled pair (Ekert) protocol

Entropic Heisenberg Inequalities F. Grosshans and N.J. Cerf, PRL **92**, 047905 (2004)

* Mutual Informations can be calculated from conditional entropies

* Conditional entropies are bounded by « entropic » uncertainty relations for X and P:

 $H(X_B|E) + H(P_B|P_A) \ge 2 H_0$

* The security of the protocol follows from a calculation similar to the one used for discrete variables (qubits)

* Important parameters :

- transmission of the channel T_{line}
- "added noise in the channel" $N_{\text{eq}} = N_{\text{losses}} + N_{\text{exc}}$ - "added noise in the channel"

where $N_{losses} = (1 - T_{line}) / T_{line} N_0$ (N_0 is the shot noise) N_{exc} is the "excess noise" (e.g. laser amplifier...)

Security of coherent state CV-QKD protocol

Alice-Bob mutual information : I_{AB}

Eve-Bob mutual information : I_{BE} (Shannon : individual attacks) χ_{BE} (Holevo : collective attacks)

Secret Key Rate : $\Delta I = I_{AB} - I_{BE}$ (Shannon) $\Delta I = I_{AB} - \chi_{BE}$ (Holevo)

EXTERS For both individual and collective attacks Gaussian attacks are optimal \rightarrow Alice and Bob consider Eve's attacks Gaussian and estimate her information using the Shannon quantity I_{BE} or the Holevo quantity χ_{BE}

Fig : V_A = 21 (shot noise units) ε = 0.005 (shot noise units), η = 0.5 M. Navasqués et al, Phys. Rev. Lett. 97, 190502 (2006) R. García-Patrón et al, Phys. Rev. Lett. 97, 190503 (2006)

Reconciliation of correlated Gaussian variables

- Each level has a different error rate
- Non-independent levels
- \rightarrow Error correction performed using multilevel iterative soft decoding with LDPC codes

G. Van Assche et al, IEEE Trans. on Inf. Theory 50, 394 (2004) M. Bloch et al, arXiv:cs.IT/0509041 (2005)

- **Standard privacy** amplification based on universal hash functions
- **Small processing time**

Error correcting codes efficiency

Error correction with LDPC codes, efficiency β

$$
\Delta I^{eff}=\beta I_{AB}-\chi_{BE}
$$

Imperfect correction efficiency induces a limit to the secure distance

Post-processing at SeQureNet

Paul Jouguet, Sébastien Kunz-Jacques, Romain Alléaume

Optimize LDPC codes, use Graphic Processing Units (GPU) rather than CPU

=> Calculation speed is no more limiting the secret bit rate !

 \Rightarrow β is improved from 89% to 95% for any SNR : longer distance (100 km) !

CVQKD : practical implementation and field demonstrations

All-fibered CVQKD @ 1550 nm

Field test of a continuous-variable quantum key distribution prototype S Fossier, E Diamanti, T Debuisschert, A Villing, R Tualle-Brouri and P Grangier *New J. Phys. 11 No 4, 04502 (April 2009)*

Quantum Back-Bone demonstrator SECOQC, Vienna, 8 october 2008

Development of a Global Network for Secure Communication based on Quartism Cryptography winny second, net

Real-size demonstration of a **secure quantum cryptography network** by the European Integrated Project SECOQC, Vienna, 8 october 2008

Node server

Continuous Variables

Id Quantique

The SECOQC Quantum Back Bone

Real-size demonstration of a **secure quantum cryptography network** by the European Integrated Project SECOQC, Vienna, 8 october 2008

Secure Encryption with OUantum key REnewal

- Combining QKD (1 kbit/sec) with fast symmetric encryption (1 Gbit/sec)
- Use 128 bits AES, change key every 10 seconds

Symmetric Encryption with QUantum key REnewal

Thales : Mistral Gbit (fast dedicated AES encryptor)

User window : « sequre drag and drop »

Complete set-up

Field implementation

- ! Fibre link : Thales R&T (Palaiseau) <-> Thales Raytheon Systems (Massy)
- ! Fiber length about 12 km, 5.6 dB loss

Results

On site, 12 km distance, 5.6 dB loss Minimal direct action on hardware (feedback loops, remote control)

2) Key rate (bit/sec)

See http://www.demo-sequre.com

rtment of Electronics

and Telecommunications

CENTER

- Several recent exemples of "quantum hacking" (e.g. Makarov et al.)
- Exploits weaknesses in single photon detectors
- Will NOT work against CVQKD (PIN photodiodes, linear regime)
- Hackers will have to work harder...

$arXiv.org > quant-ph > arXiv:1106.0825$

Quantum Physics

Security of Post-selection based Continuous Variable **Quantum Key Distribution against Arbitrary Attacks**

Nathan Walk, Thomas Symul, Timothy C. Ralph, Ping Koy Lam (Submitted on 4 Jun 2011)

 $arXiv.org > quant-ph > arXiv:1011.0304$

Quantum Physics

Continuous variable quantum key distribution in non-Markovian channels

Search o

Search

Search or

Search or

Ruggero Vasile, Stefano Olivares, Matteo G A Paris, Sabrina Maniscalco

(Submitted on 1 Nov 2010)

 $arXiv.org > quant-ph > arXiv:0904.1694$

Quantum Physics

Feasibility of continuous-variable quantum key distribution with noisy coherent states

Vladyslav C. Usenko, Radim Filip

(Submitted on 10 Apr 2009 (v1), last revised 21 Jan 2010 (this version, v2))

 $arXiv.org > quant-ph > arXiv:0904.1327$

Quantum Physics

Security bound of continuous-variable quantum key distribution with noisy coherent states and channel

Yong Shen, Jian Yang, Hong Guo

(Submitted on 8 Apr 2009 (v1), last revised 29 Jun 2009 (this version, v2))

 $arXiv.org > quant-ph > arXiv:0903.0750$

Quantum Physics

Confidential direct communications: a quantum approach using continuous variables

Stefano Pirandola, Samuel L. Braunstein, Seth Lloyd, Stefano Mancini (Submitted on 4 Mar 2009)

Many other works on CVQKD! \leq **Theory and Experiments:** (incomplete list!)

(Submitted on 28 Nov 2008 (v1), last revised 13 Mar 2009 (this version, v2))

arXiv.org > quant-ph > arXiv:0705.2627

Quantum Physics

Experimental Demonstration of Post-Selection based Continuous Variable Quantum Key Distribution in the **Presence of Gaussian Noise**

Search or

Thomas Symul, Daniel J. Alton, Syed M. Assad, Andrew M. Lance, Christian Weedbrook, Timothy C. Ralph, Ping Koy Lam (Submitted on 18 May 2007)

Towards quantum communications and quantum networks ?

• **Longer distances require « real » entanglement !**

• **"Delocalized" Schrödinger kittens !**

How to create entanglement at a large distance ?

Delocalized photon subtraction

How to avoid the bad effect of losses ? Basic idea : one should not « distribute » the entangled state, but rather create it at a distance

- * Start from two remote ! ! squeezed states $| s \rangle_1$ and $| s \rangle_2$
- * Subtract a photon coherently from the two beams
- * Since subtracting a photon creates a cat, creation of an entangled state :

 $|\Psi\rangle$ = (| s \rangle ₁ | cat \rangle ₂ - | cat \rangle ₁| s \rangle ₂)/ $\sqrt{2}$

"Hamlet state" (to be or not to be... a cat)

Quantum repeaters with entangled coherent states

New method for remote entanglement of cat states Main advantage of this scheme : almost insensitive to transmission losses ! (the non-local cats are never transmitted in the line)

Fidelity for 10 dB losses: $F = 0.4$

Experiment

A. Ourjoumtsev et al, Nature Physics, 5, 189, 2009

Experimental set-up Two-mode probability distributions (two phases ϕ **and** θ **...)**

Results

A. Ourjoumtsev et al, Nature Physics, 5, 189, 2009

$$
|\phi_{\pm}\rangle_{AB} = \frac{1}{\sqrt{M_{\pm}}} (|\alpha\rangle_A |\alpha\rangle_B \pm |-\alpha\rangle_A |-\alpha\rangle_B)
$$

\n
$$
|\psi_{\pm}\rangle_{AB} = \frac{1}{\sqrt{M_{\pm}}} (|\alpha\rangle_A |-\alpha\rangle_B \pm |\alpha\rangle_A |-\alpha\rangle_B)
$$

\n
$$
|\psi_{\pm}\rangle \rightarrow |0\rangle_{\text{out1}} |\text{even}\rangle_{\text{out2}},
$$

\n
$$
|\psi_{\pm}\rangle \rightarrow |0\rangle_{\text{out1}} |\text{odd}\rangle_{\text{out2}},
$$

\n
$$
|\psi_{\pm}\rangle \rightarrow |0\rangle_{\text{out1}} |\text{odd}\rangle_{\text{out2}},
$$

But parity measurements (even / odd) are extremely sensitive to losses…

- -> To avoid errors one has to use kittens rather than cats
- -> Increase of the « failure » probability (getting 0 0)
- -> Overall not significantly better than using entangled photons :-((

Better hardware needed ! (here : deterministic parity measurement)

Conclusion

Many potential uses for Quantum Continuous Variables...

- * Quantum cryptography
- * Coherent states protocols using reverse reconciliation,

secure against any (gaussian or non-gaussian) collective attack

- * Working fine in optical fibers @ 1550 nm (SECOQC project)
- * Conditional preparation of « squeezed » non-gaussian pulses / cats
- * Big family of phase-dependant states with negative Wigner functions !
- * See also many new experimental results by the groups of

A. Lvovsky, M. Bellini, E. Polzik, T. Gerrits, A. Furusawa, M. Sasaki...

- * First step towards : entanglement distillation procedures?
	- new tests of Bell's inequalities?
	- quantum computing ? (QCV version of KLM...)