Optical Amplification in Quantum (and Classical) Communications Systems

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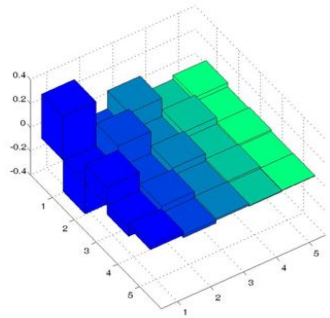
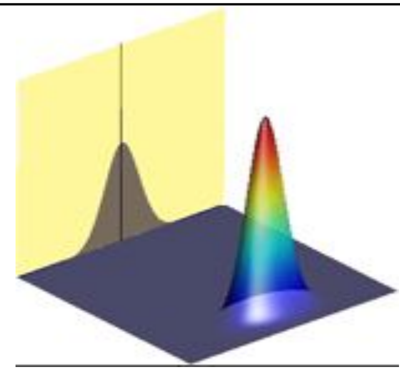
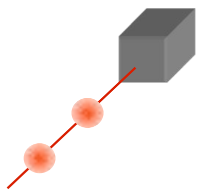
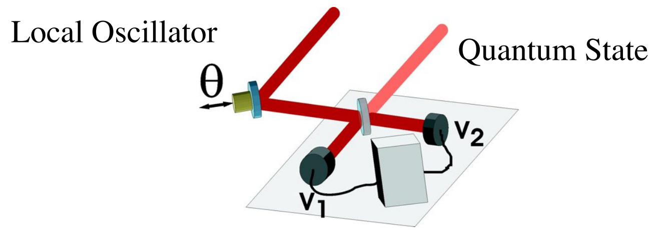
IST / FET / ERANET European Projects :
« COVAQIAL », « COMPAS », « HIPERCOM »

« Discrete » vs « continuous » Light

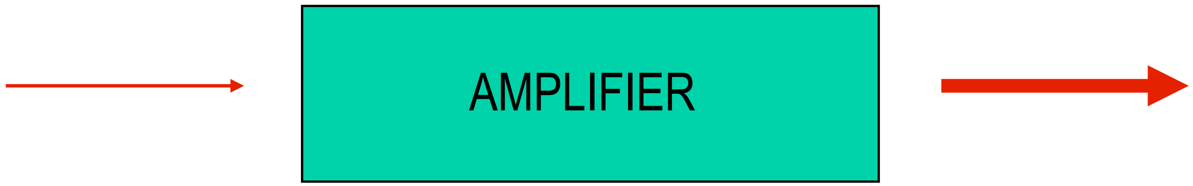
Light is :	Discrete Photons	Continuous Wave
We want to know :	their Number & Coherence	its Amplitude & Phase (polar) its Quadratures X & P (cartesian)
We describe it with :	Density matrix $\varrho_{n,m}$ 	Wigner function W(X,P) 
We measure it by :	Counting: APD, VLPC, TES… 	Demodulating : Homodyne Detection  V_1-$V_2 \propto X_\theta$=Xcosθ+Psinθ
« Simple » States	Fock States	Gaussian States

Basic Properties of Linear Optical Amplifiers

- **Phase sensitive vs phase insensitive amplifier**
- **Some classical and quantum applicatiosn !**

Quantum Limits on Noise in Linear Amplifiers

Carlton M. Caves, Phys. Rev. D 26, 1817 (1982)



Input signal :
Useful amplitude (squared) : $\langle X_{in} \rangle^2$
Fluctuations (noise) : $\langle \delta X_{in}^2 \rangle$
Signal to noise ratio : $\langle X_{in} \rangle^2 / \langle \delta X_{in}^2 \rangle$

Output signal :
Useful amplitude (squared) : $\langle X_{out} \rangle^2$
Fluctuations (noise) : $\langle \delta X_{out}^2 \rangle$
Signal to noise ratio : $\langle X_{out} \rangle^2 / \langle \delta X_{out}^2 \rangle$

Relevant quantity : "Noise Figure" NF of the amplifier :

$$\mathbf{NF} = \frac{\text{Input signal to noise ratio}}{\text{Output signal to noise ratio}} = \frac{\langle X_{in} \rangle^2 / \langle \delta X_{in}^2 \rangle}{\langle X_{out} \rangle^2 / \langle \delta X_{out}^2 \rangle}$$

NF should be small, minimum value = 1 (or 0 dB).

Ultimate limit to the noise of an optical amplifier (1)

Let us consider a "phase-independant" amplifier, which multiplies the input light field amplitude by the factor g (taken as a real number), for any phase of this input field.

What is the associated transformation for the quantum field operators ?

Considering a single mode for simplycity, one may try :

$$a_{out} = g\ a_{in} \qquad a^{\dagger}_{out} = g\ a^{\dagger}_{in}$$

But if $[a_{in}, a^{\dagger}_{in}] = 1$ then $[a_{out}, a^{\dagger}_{out}] = g^2$: impossible !

Other approach :

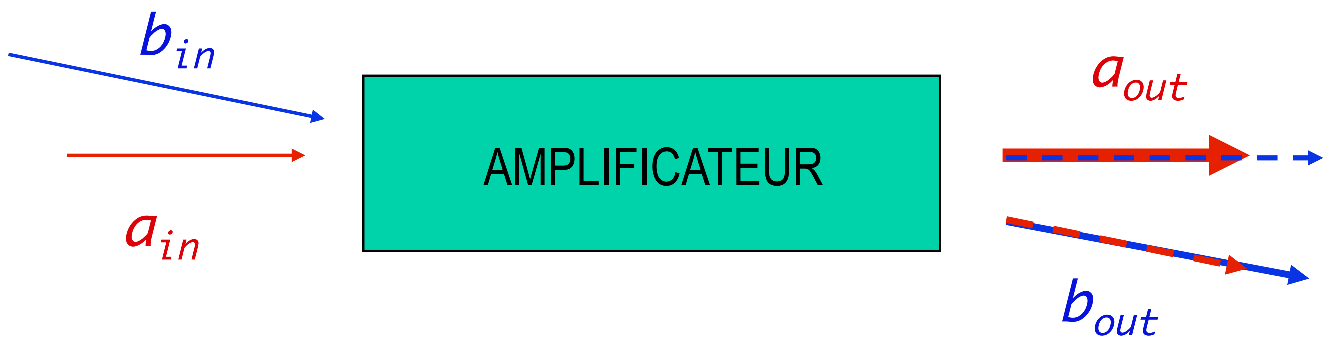
$$a_{out} = g\ a_{in} + \sqrt{g^2 - 1}\ b^{\dagger}_{in} \qquad a^{\dagger}_{out} = g\ a^{\dagger}_{in} + \sqrt{g^2 - 1}\ b_{in}$$

One has then

$$[a_{out}, a^{\dagger}_{out}] = g^2\ [a_{in}, a^{\dagger}_{in}] - (g^2 - 1)\ [b_{in}, b^{\dagger}_{in}] = 1$$

OK !

Spontaneous emission noise



$$a_{out} = g\ a_{in} + \sqrt{g^2 - 1}\ b_{in}^\dagger \qquad a_{out}^\dagger = g\ a_{in}^\dagger + \sqrt{g^2 - 1}\ b_{in}$$

Interpretation : for an amplifier based on a population inversion, it is unavoidable to get "spontaneous emission noise".

Exception : for some amplifiers (parametric amplifiers) one can manage to have the two modes a and b exactly overlapping. The amplifier becomes a "phase sensitive amplifier" or "squeezer")

Ultimate limit to the noise of an optical amplifier (2)

A "phase independant" amplifier with gain g necessarily adds spontaneous emission noise to the input signal :

$$a_{out} = g\, a_{in} + \sqrt{g^2-1}\, b_{in}^{\dagger} \qquad a_{out}^{\dagger} = g\, a_{in}^{\dagger} + \sqrt{g^2-1}\, b_{in}$$

For a signal encoded in the quadrature operator $X = a + a^{\dagger}$ one has :

$$X_{out} = g\, X_{in} + \sqrt{g^2-1}\, X_b \quad \text{with} \quad \langle X_b \rangle = 0 \quad \text{and} \quad \langle \delta X_b^2 \rangle = 1$$

$$\langle X_{out} \rangle^2 = g^2 \langle X_{in} \rangle^2$$

$$\langle \delta X_{out}^2 \rangle = g^2 \langle \delta X_{in}^2 \rangle + (g^2-1) \langle \delta X_b^2 \rangle = (2g^2-1) \langle \delta X_{in}^2 \rangle$$

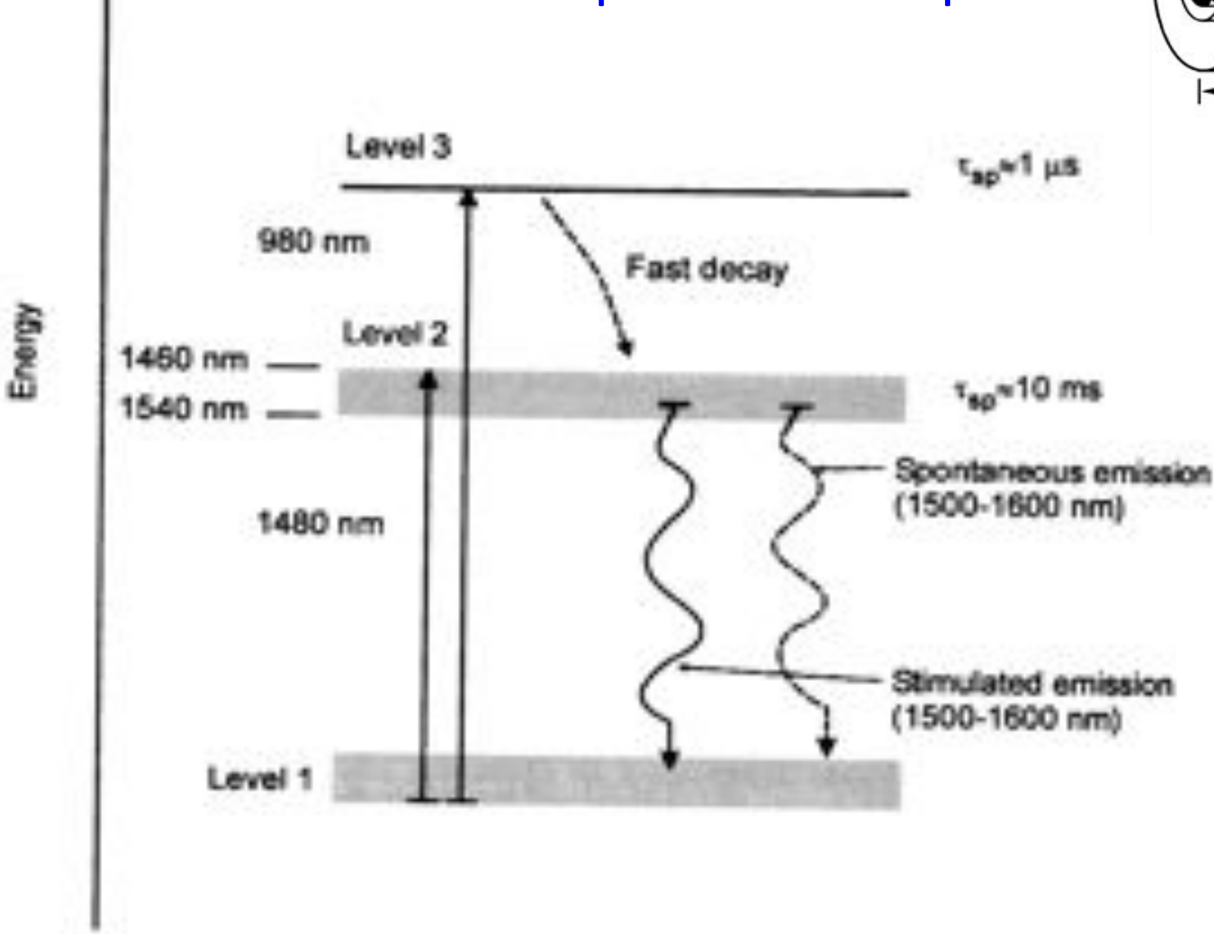
One defines the "noise figure" (NF) of the amplifier :

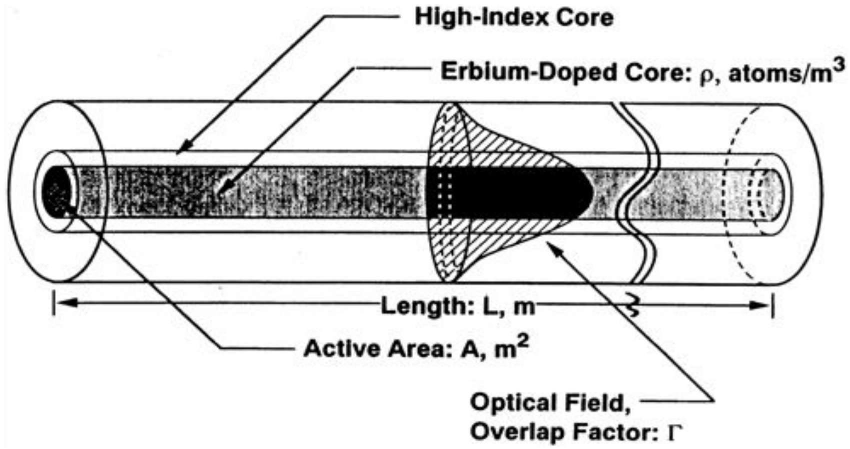
$$NF = \frac{\text{input signal to noise ratio}}{\text{output signal to noise ratio}} = \frac{\langle X_{in} \rangle^2 / \langle \delta X_{in}^2 \rangle}{\langle X_{out} \rangle^2 / \langle \delta X_{out}^2 \rangle} = \frac{2g^2-1}{g^2}$$

$NF \rightarrow 2$ for $g \rightarrow \infty$

Energy levels of Erbium ions in a silica (SiO_2) matrix (optical fiber)

EDFA : Erbium Doped Fiber Amplifier





Calculation of the gain :

$g = \sigma_s (N_2 - N_1)$

$G = \exp(gL)$

with :

$N_1 = 1.8\times10^{17}$ cm^{-3} (lower level)

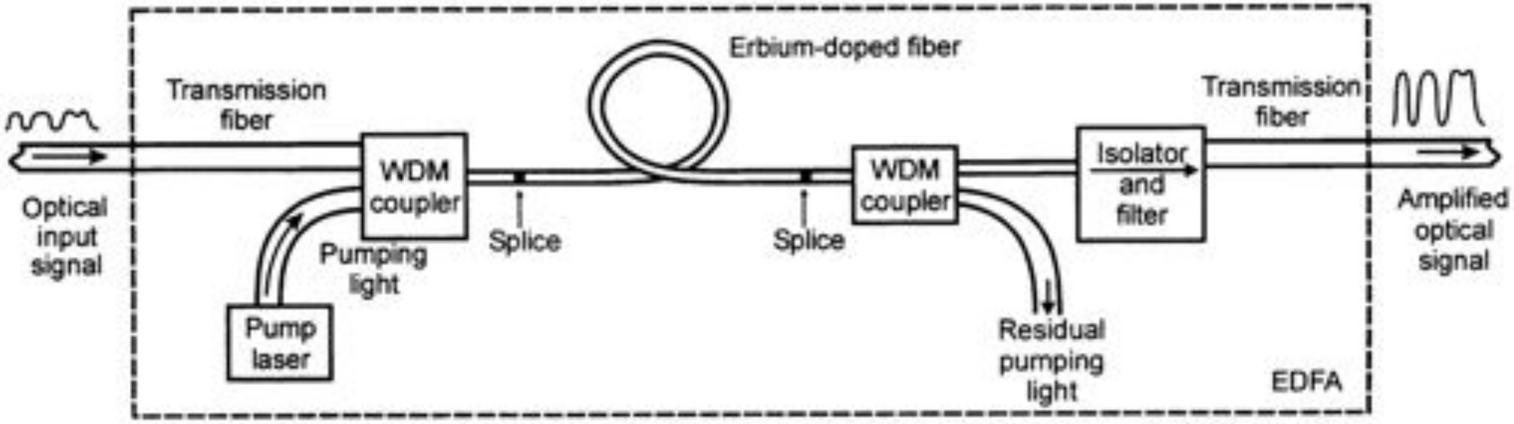
$N_2 = 4.8\times10^{17}$ cm^{-3} (upper level)

$\sigma_s = 7.0\times10^{-25}$ cm^2 (cross section)

$g = 2.1\times10^{-3}$ cm^{-1}

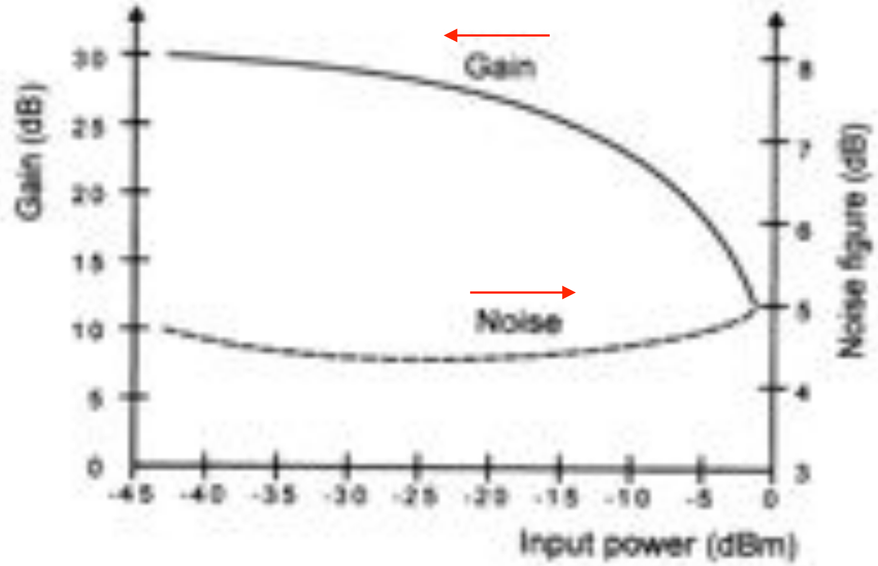
G = 30 dB for L = 33 m

Gain and Noise of an EDFA

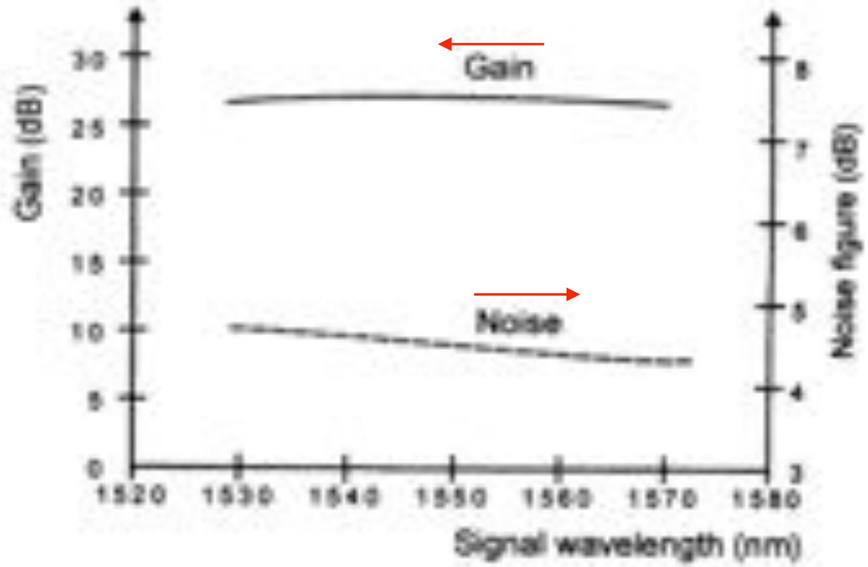


Gain (dB) = 10 $\log_{10}[P_{out}/P_{in}]$

NF (dB) = 10 $\log_{10}[(S/N)_{in}/(S/N)_{out}]$

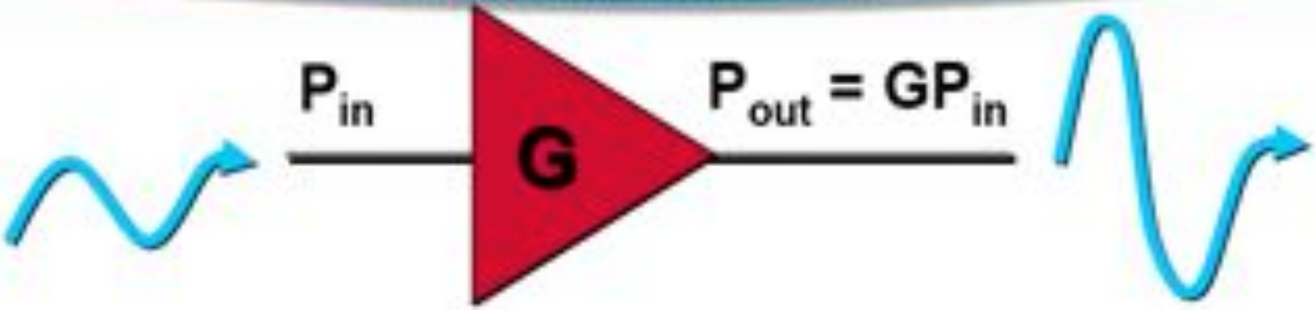


As a function of the signal's input power (gain saturation)



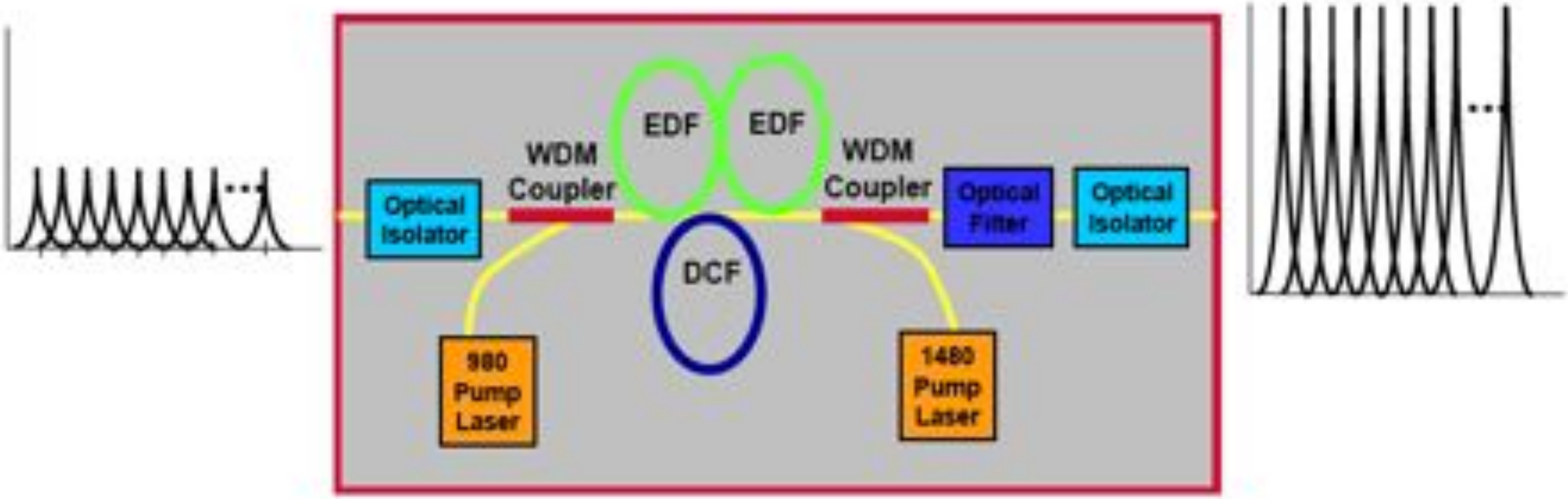
As a function of the signal's wavelength (amplification bandwidth)

Optical Amplifier



- 4 THz of optical bandwidth near 1550 nm
- Nearly ideal noise performance
- Low signal distortion, low cross talk
- High-output saturation power
- Simple and efficient

EDFA
(Erbium Doped Fiber Amplifier)



- Avoids opto-electrical conversion of a repeater
- EDFAs amplify all λs in 1550 nm window simultaneously
- Pump laser is only active part

ALCATEL

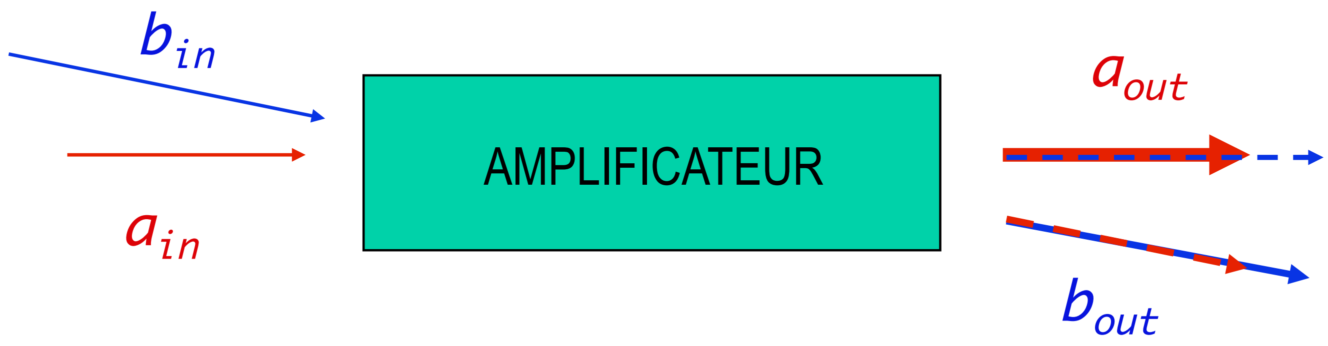
Growth of the world's submarine network (IV)

The project OXYGEN

275,000km cable, 60 cable-boat fleet
262 landing points in 175 countries/locations
100-1,000Gbit/s capacity per fiber, 36 rings

CTR consortium
RFS : end 2003
$14 billion

Spontaneous emission noise



$$a_{out} = g\, a_{in} + \sqrt{g^2 - 1}\, b_{in}^\dagger \qquad a_{out}^\dagger = g\, a_{in}^\dagger + \sqrt{g^2 - 1}\, b_{in}$$

Interpretation : for an amplifier based on a population inversion, it is unavoidable to get "spontaneous emission noise".

Exception : for some amplifiers (parametric amplifiers) one can manage to have the two modes a and b exactly overlapping. The amplifier becomes a "phase sensitive amplifier" or "squeezer")

Ultimate limit to the noise of an optical amplifier (3)

In the "degenerate" case one may fully overlap the modes a and b, then :

$$a_{out} = g\, a_{in} + \sqrt{g^2 - 1}\, a^\dagger_{in} \qquad a^\dagger_{out} = g\, a^\dagger_{in} + \sqrt{g^2 - 1}\, a_{in}$$

The quadrature operators $X = a + a^\dagger$ and $Y = -i(a - a^\dagger)$ become :

$$X_{out} = (g + \sqrt{g^2 - 1})\, X_{in} = g_d\, X_{in}$$

$$Y_{out} = (g - \sqrt{g^2 - 1})\, Y_{in} = (1/g_d)\, Y_{in}$$

X is amplified, whereas Y is deamplified !

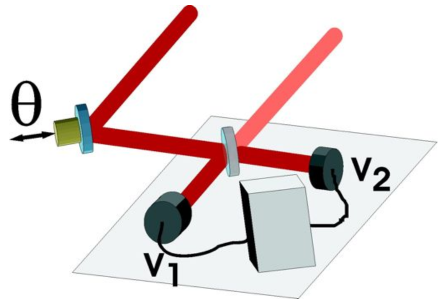
$$\langle X_{out} \rangle^2 = g_d^2\, \langle X_{in} \rangle^2 \qquad \langle Y_{out} \rangle^2 = \langle Y_{in} \rangle^2 / g_d^2$$

$$\langle \delta X^2_{out} \rangle = g_d^2\, \langle \delta X^2_{in} \rangle \qquad \langle \delta Y^2_{out} \rangle = \langle \delta Y^2_{in} \rangle / g_d^2$$

The "noise figure" (NF) of the amplifier is equal to 1 for all gains :

$$NF = \frac{\text{input signal to noise ratio}}{\text{output signal to noise ratio}} = \frac{\langle X_{in} \rangle^2 / \langle \delta X^2_{in} \rangle}{\langle X_{out} \rangle^2 / \langle \delta X^2_{out} \rangle} = 1$$

Application of a phase-sensitive amplifier : improvement of an homodyne detection



Take an imperfect homodyne detection with efficiency $\eta < 1$

This degrades an input signal, for a coherent input the Noise Figure is :

$$NF = 1 / \eta > 1$$

Just before the homodyne detection, add a phase sensitive amplifier on the signal beam with gain $G = g^2$, amplifying the measured quadrature. For a coherent input the Noise Figure is then :

$$NF = 1 + (1 - \eta) / (G \eta) \Rightarrow 1 \quad \text{if } G \gg 1$$

The phase sensitive amplifier makes the homodyne detection perfect !

Coherent state continuous variables QKD protocol

- Key information encoded in both quadratures of a coherent state

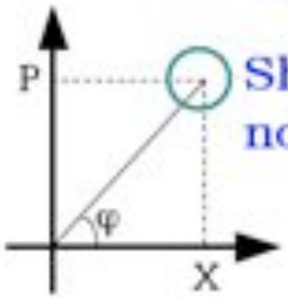
Eve

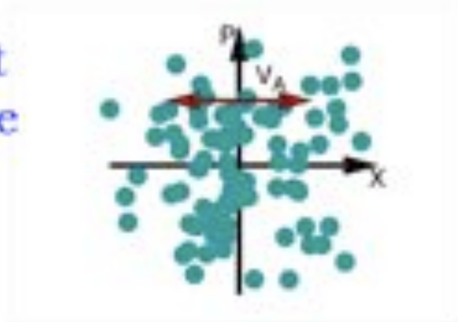
Alice

Quantum channel (T, ε)
+ Classical Channel (auth.)

Bob

Gaussian modulation

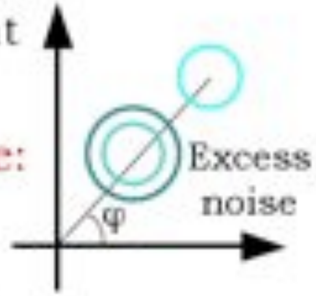




Random measurement of the quadrature of each coherent state (with efficiency η)

Total channel-added noise:

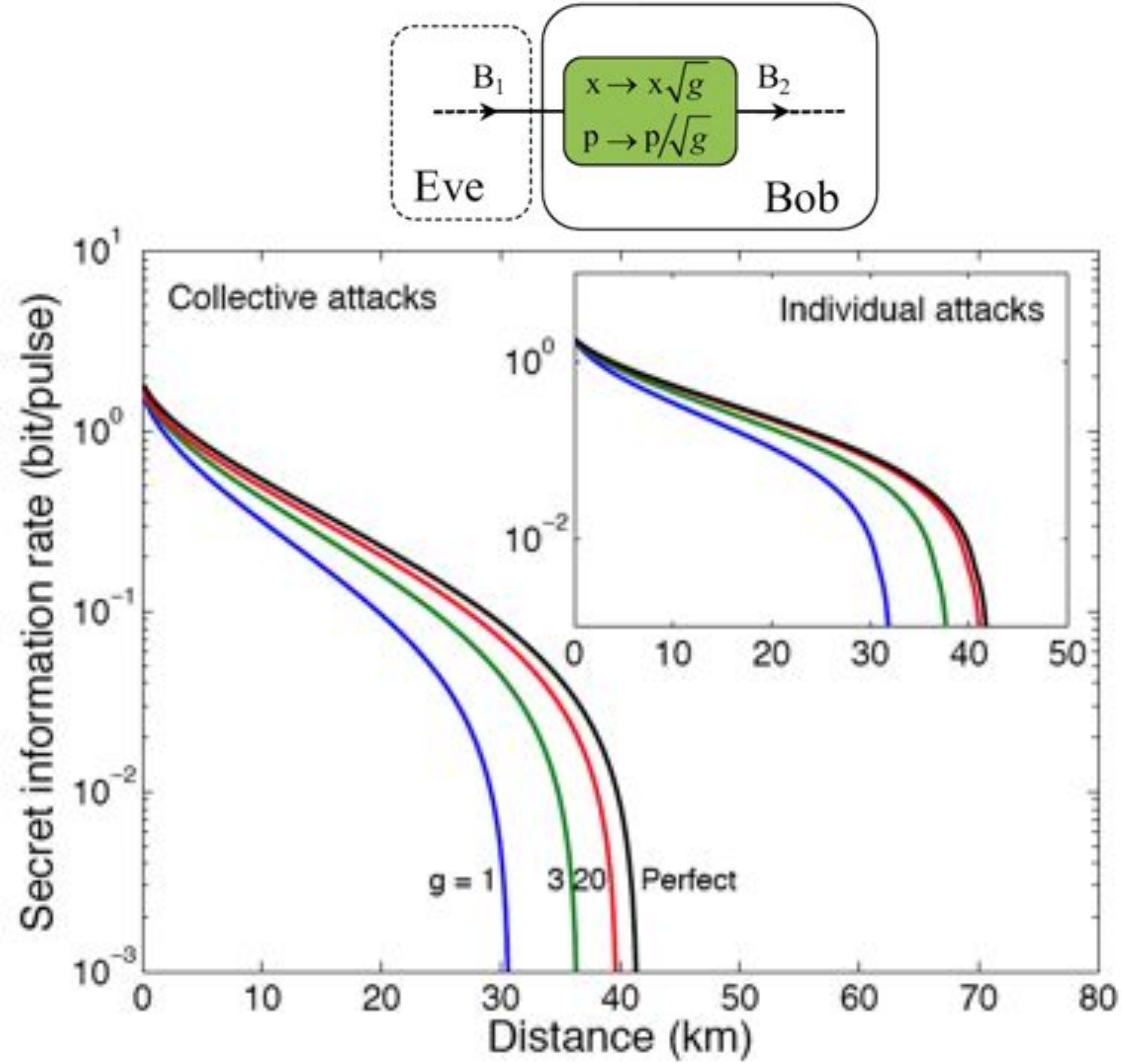
$$\chi = \underbrace{1/T - 1}_{\text{equivalent to photon loss}} + \underbrace{\varepsilon}_{\text{equivalent to errors}}$$



- Bob reveals measurement choice
- Alice and Bob share a set of Gaussian correlated data
- Further communication to calculate channel parameters and derive secret key based on Bob's data → **reverse reconciliation**

F. Grosshans et al, Phys. Rev. Lett. 88, 057902 (2002) & Nature 421, 238 (2003)

Improving CVQKD with a phase-sensitive amplifier ?



In principle, it works !
(theory only : S. Fossier et al, J. Phys. B **42**, 114014 (2009), quant-ph/0812.4314)

"Perfect" : perfect detector

g=1 : imperfect detector with $\eta = 0.6$, no amplifier.

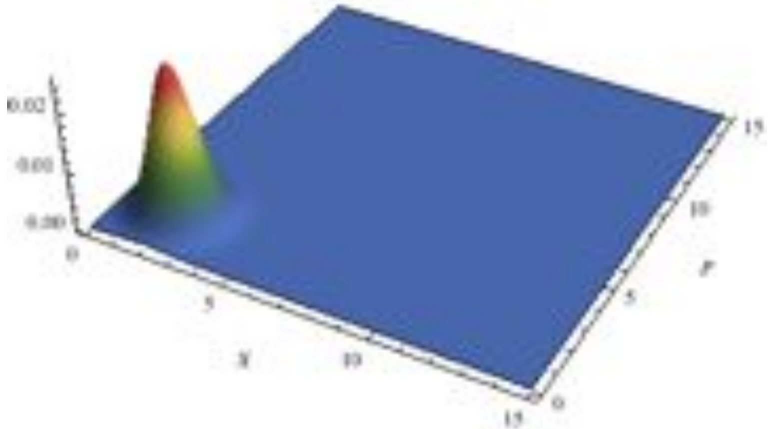
g= 3, 20 : imperfect detector, increasing the amplifier gain : gets closer and closer to perfect !

Towards quantum communications and quantum networks ?

- Longer distances require quantum repeaters and therefore « real » entanglement !
- Can we « amplify » entanglement ?

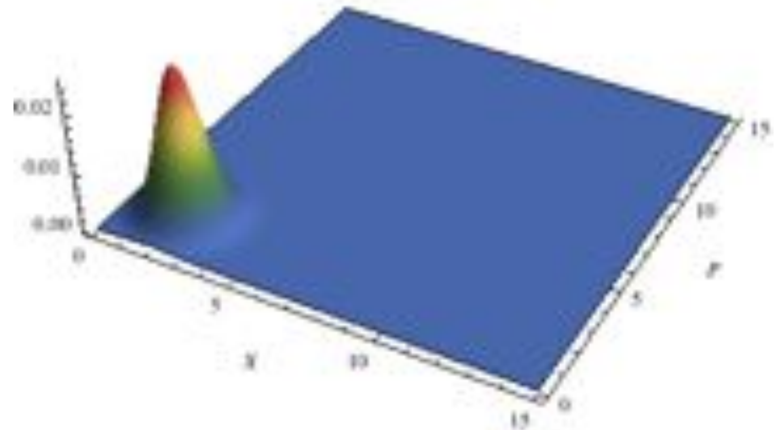
Linear deterministic amplifiers (Caves 1982)

Phase-independent amplifier
(amplifies the amplitude for all values of the phase)



⇒ **Adds excess noise**
(3dB for large gain)
⇒ **Decreases** signal to noise ratio

Phase-dependent amplifier
(amplifies the amplitude a specific value of the phase : “antisqueezing”)



⇒ **No excess noise**
⇒ **Keeps** same signal to noise ratio
⇒ One must know the signal phase

Non-deterministic noiseless optical amplifier

Goal : we want : $|in\rangle = |\alpha\rangle \longrightarrow |out\rangle = |g\,\alpha\rangle$

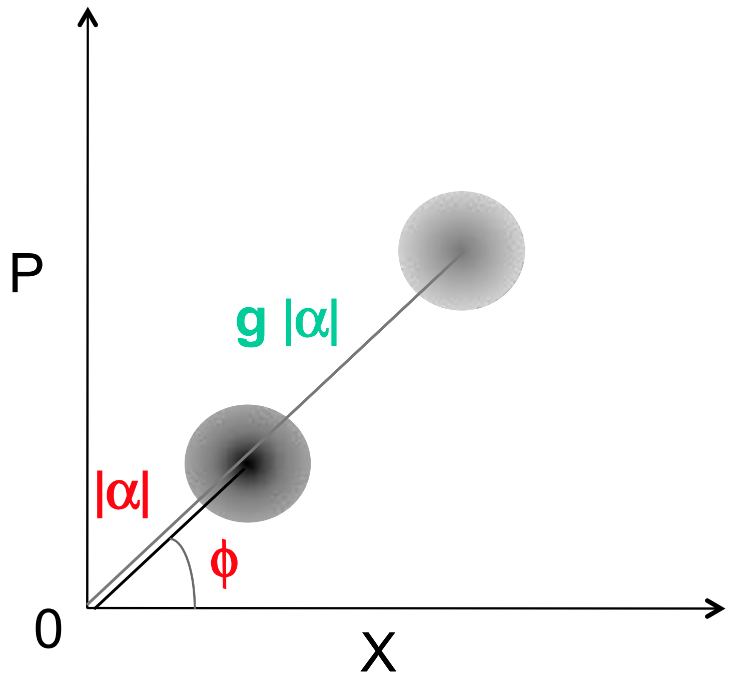
which means

- $\alpha \rightarrow g\,\alpha$
- $\phi \rightarrow \phi$
- $X \rightarrow g\,X$
- $P \rightarrow g\,P$

Same noise

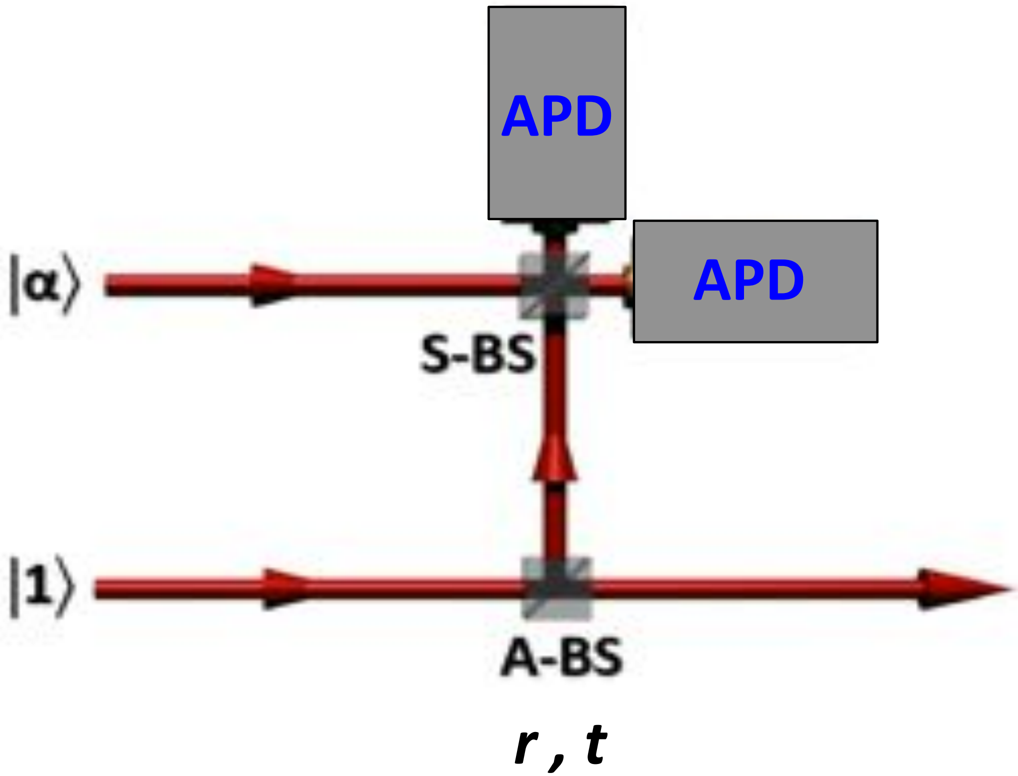
$\Rightarrow$ **Increases** signal to noise ratio ?

$\Rightarrow$ **Breaks all rules ?**



Non-deterministic noiseless optical amplifier

T.C. Ralph and A.P. Lund,
Nondeterministic Noiseless Linear Amplification of Quantum Systems,
arXiv:0809.0326 (2008).



$$|\, g\alpha \rangle \qquad g = \frac{\sqrt{1-r^2}}{r}$$

As such works for small α only : $|\,0\rangle + \alpha|1\rangle \rightarrow |\,0\rangle + g\,\alpha|1\rangle$

Non-deterministic noiseless optical amplifier ?

T. C. Ralph and A.P. Lund, arXiv:0809.0326 (2008) : theory
G. Y. Xiang et al, arXiv:0907.3638 (2009) + Nature Photonics (2010) : theory + expt
J. Fiuracek, Phys. Rev. A 80, 053822 (2009) : theory
F. Ferreyrol et al, arXiv:0912.2065 (2009) + PRL 104, 123603 (2010) : expt
A. Zavatta et al, arxiv:1004:3399 + Nature Photonics (2011) : expt

* If deterministic, the transformation $|\alpha\rangle \Rightarrow |g\alpha\rangle$ is obviously non unitary,

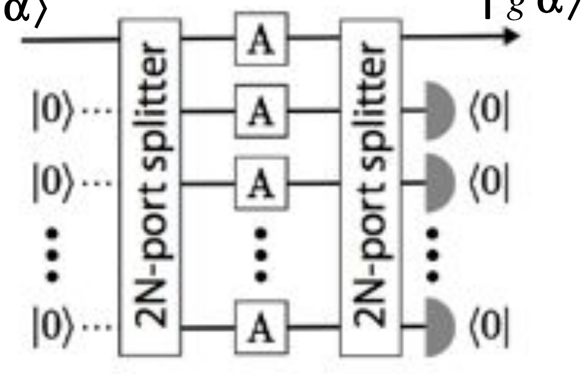
and is not even a positive map : it must be probabilistic !

* Simple approach (Ralph et al) : $|\alpha\rangle\langle\alpha| \Rightarrow P|g\alpha\rangle\langle g\alpha| + (1\text{-}P)|0\rangle\langle 0|$

OK if P is small enough (very small if $|\alpha|$ or g becomes large…)

* Scalable to large $|\alpha|$?

$|\alpha\rangle$ $|g\alpha\rangle$



Yes (Ralph et al)

Amounts to apply the operator g^n (**n** number of photons) which is unbounded

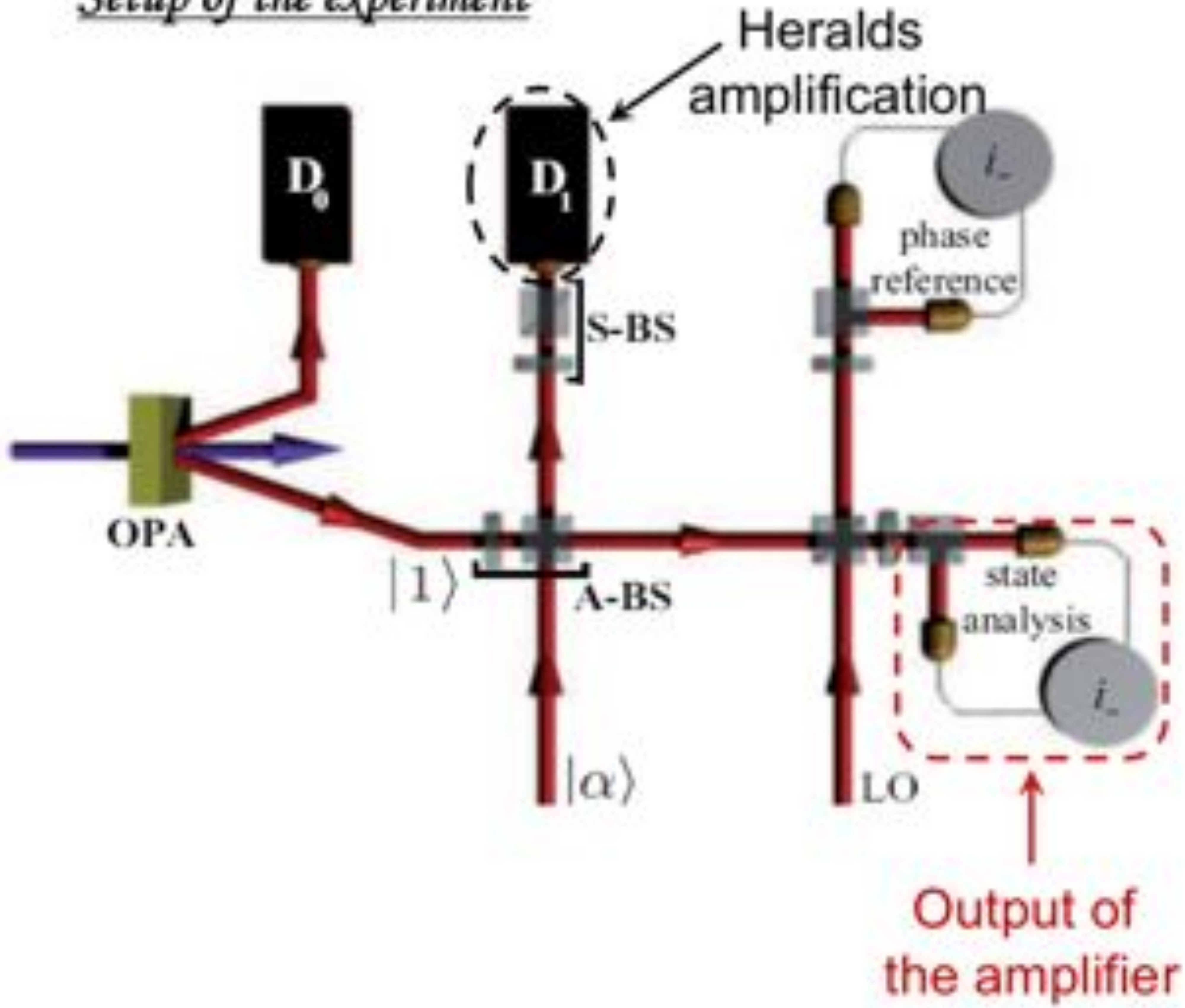
No (Fiuracek)

OK for all practical purposes (e.g. CVQKD : gaussian distribution of coherent states with finite variance)

OK ?

Experimental set-up

Setup of the experiment



Low probability of detection of 2 photons after S-BS
⇒ One APD is sufficient

Polarization encoding
⇒ Preserve phase stability

Success probability:
1% to 6%

F. Ferreyrol et al. Phys. Rev. Lett. 104, 123603 (2010)

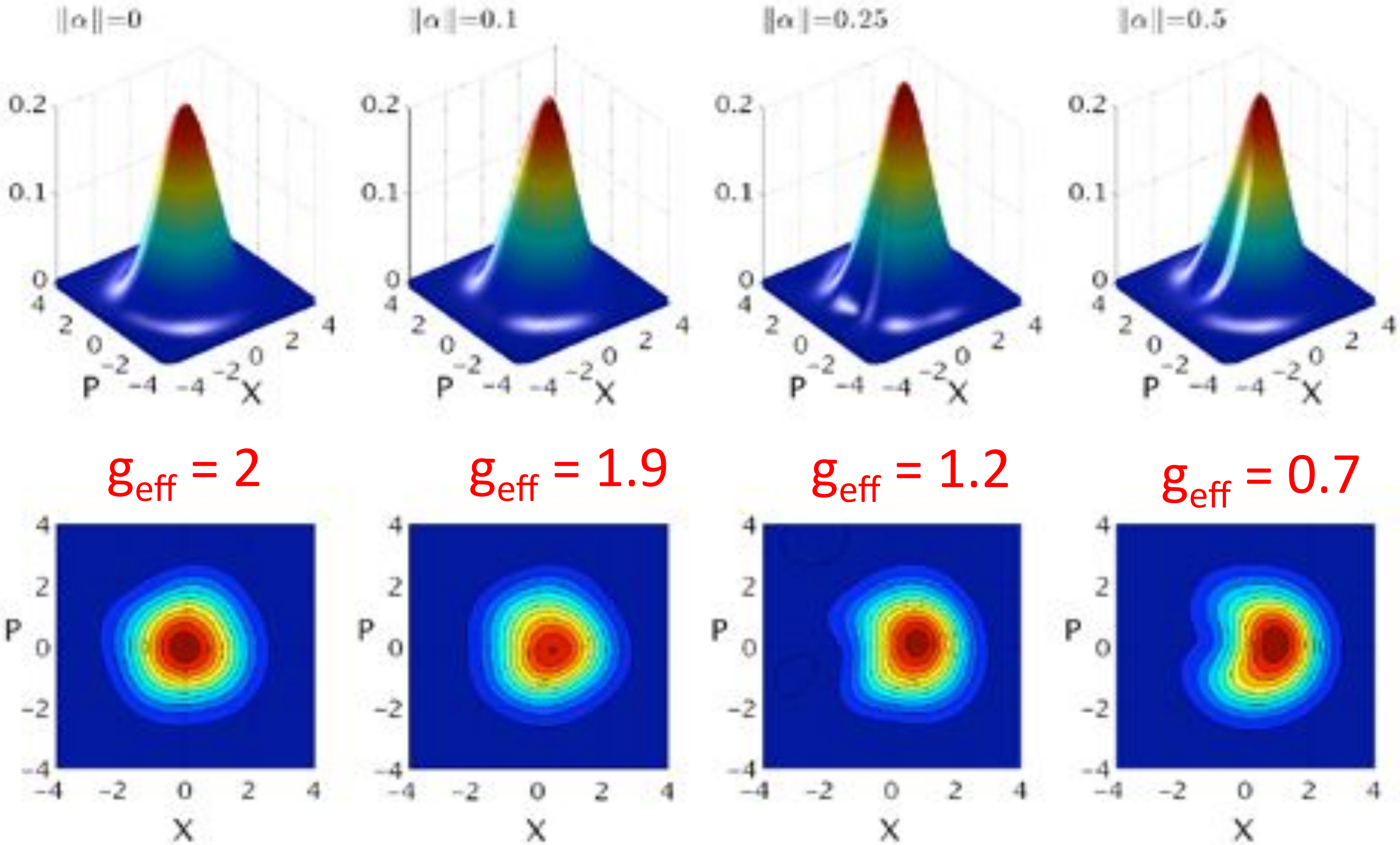
Work with Simon Fossier

INSTITUT d'OPTIQUE GRADUATE SCHOOL

Tomography of the amplified state

CNRS CENTRE NATIONAL DE LA RECHERCHE SCIENTIFIQUE

QIPC

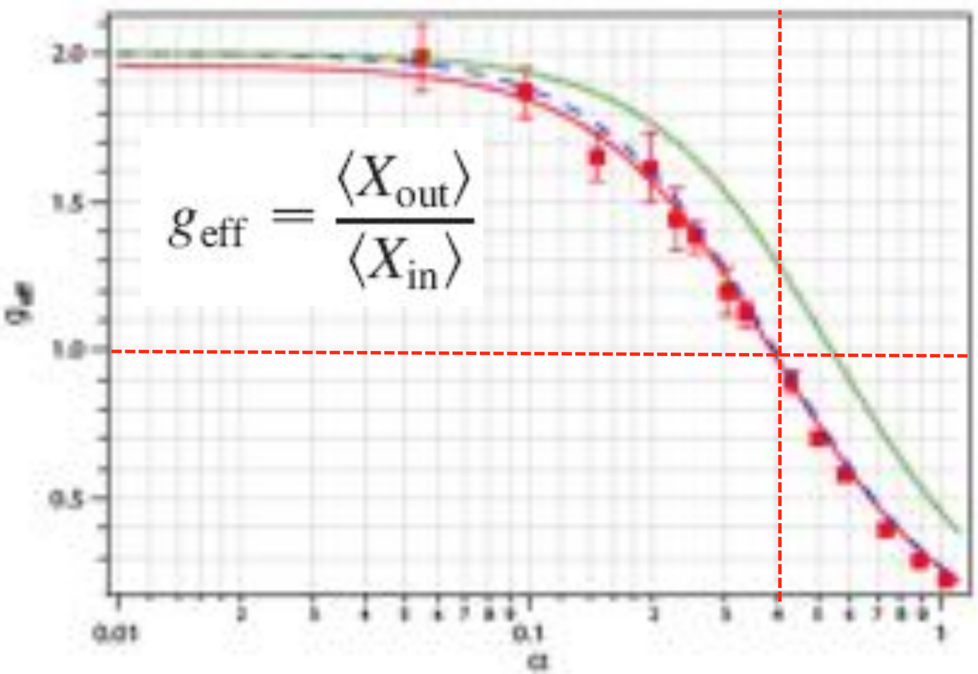


Nominal value of the gain (by adjusting the A-BS) : g = 2

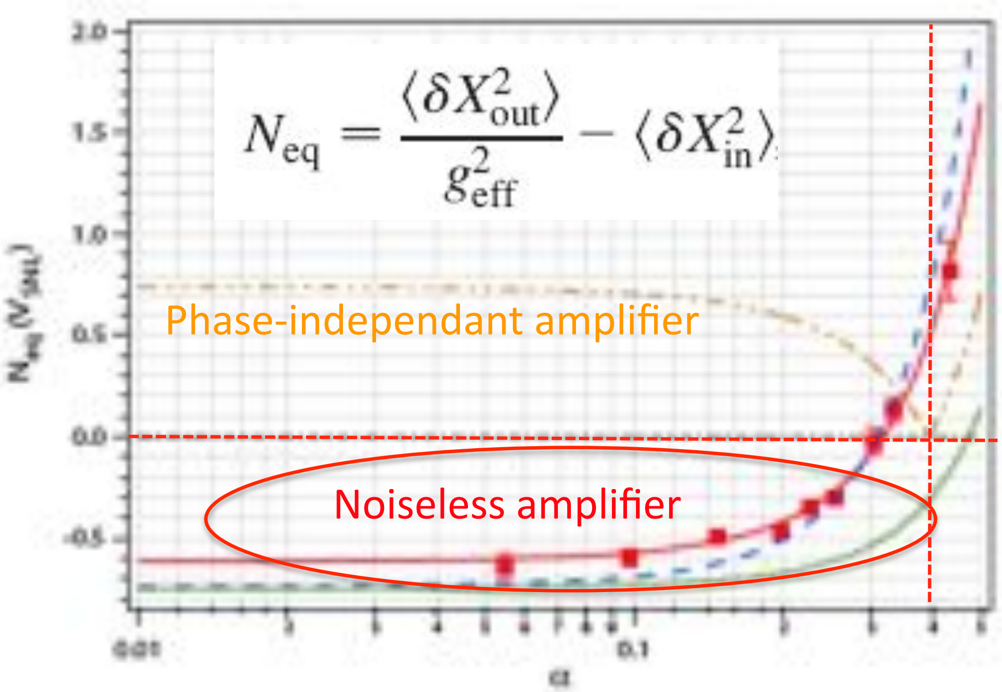
Phase – independant gain !

Gain and Noise

Amplitude gain (g_{eff} = 2 : gain 6 dB)



Equivalent Input Noise N_{eq}



Gain up to 6 dB for small α

Phase independant !

Saturates rather quickly

(ok if α < 0.1)

Negative values of the Equivalent Input Noise !

$$NF = \frac{SNR_{out}}{SNR_{in}} = \frac{V_{in} + N_{eq}}{V_{in}} < 1 \ ???$$

Violation of QM… ? no !

Example : Gaussian modulation with a small amplitude (CVQKD)

Use Shannon's formula for the mutual information :

I_{AB} = ½ Log(1 + SNR)

$\Rightarrow$ $I_{AB,\ ampli}$ = ½ Log(1 + g^2 SNR) > I_{AB} !?!

$I_{AB,\ ampli,average}$ = $P_{success}$ $I_{AB,\ ampli}$ = $(1-r^2)$ I_{AB} < I_{AB} **OK !**

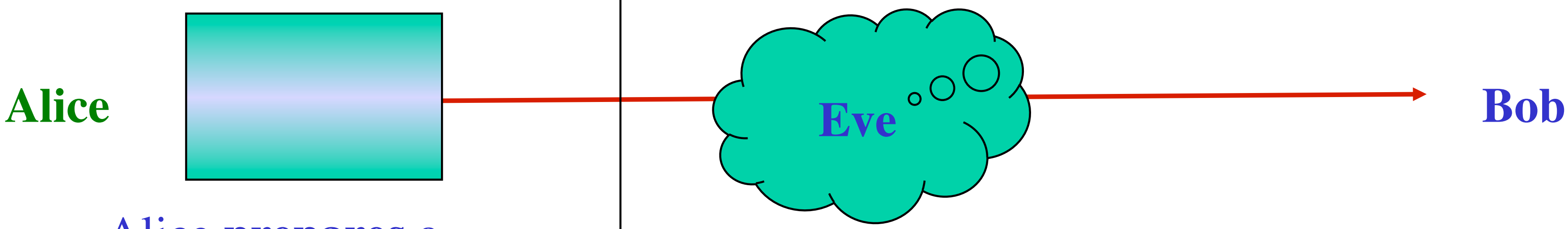
Most interesting use of this amplifier :

increase squeezing or entanglement !

* If the input is a squeezed state, the squeezing is increased, without knowing the direction of the squeezed quadrature !

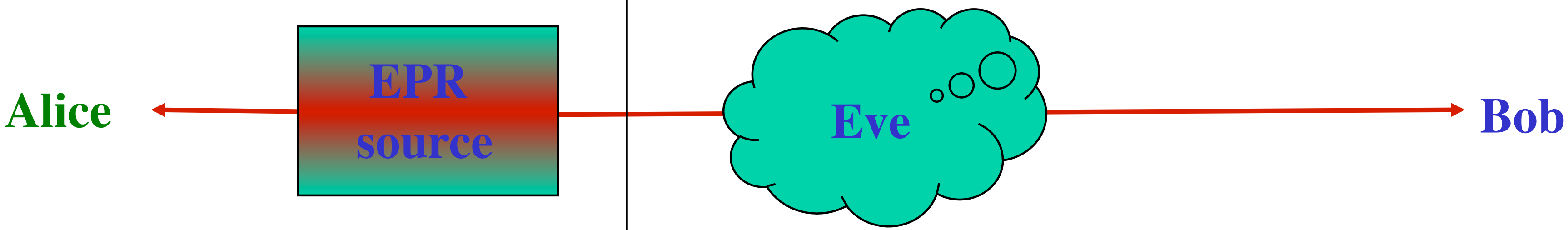
* If the input is an EPR state (two-mode squeezed state), the entanglement is increased (if Bob communicates classically with Alice) !

A very useful equivalence : "virtual entanglement"



Alice **prepares** a random quantum state and sends it to Bob

Eve and Bob receive the states prepared by Alice



Alice **measures** half an EPR state, the other half goes to Bob

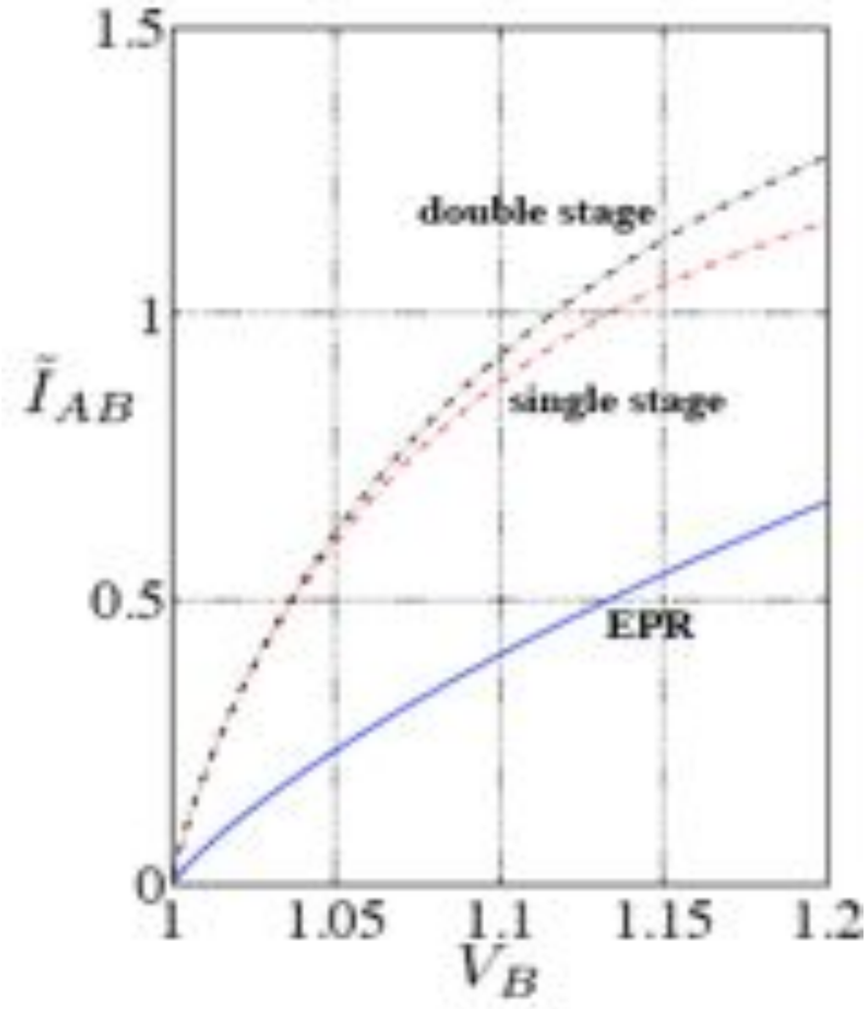
The density matrix received Eve and Bob is the same in both cases : Eve cannot know more !

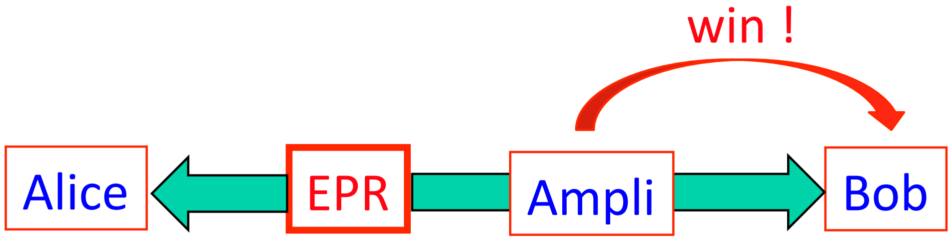
"Prepare and measure" protocol is equivalent to an entangled state protocol !

This equivalence is extensively used in security proofs

« Winning » mutual information

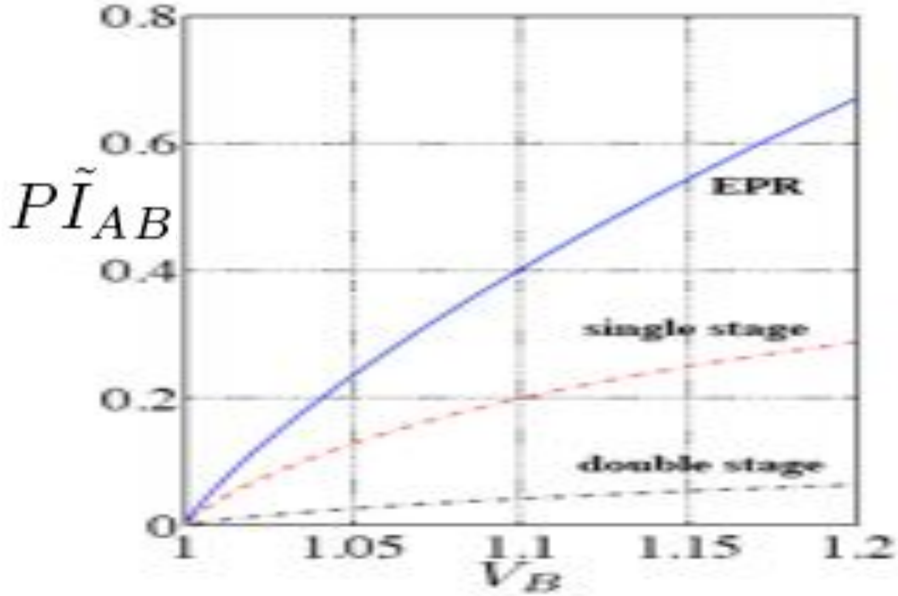
Mutual information
in the « winning » case





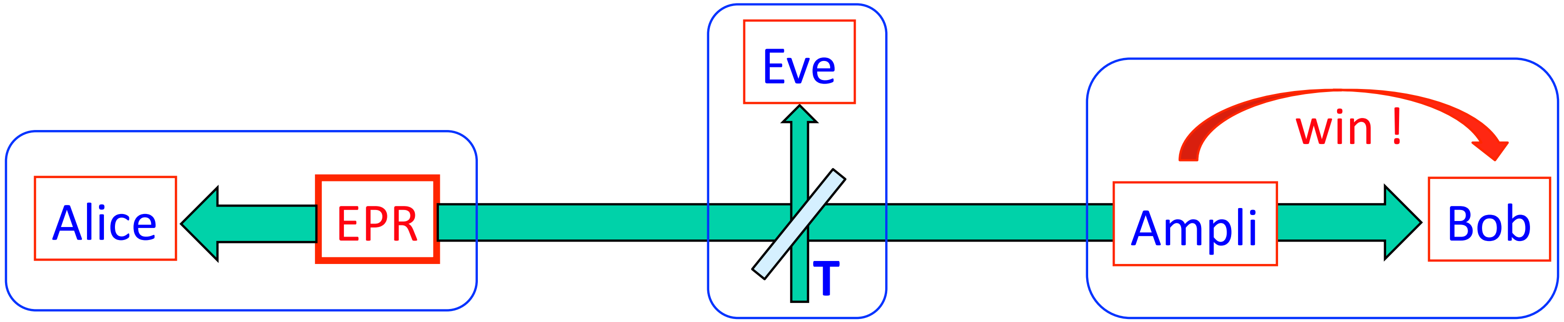
$$\Sigma_n \chi^n \,|\, n, n\rangle \Rightarrow \Sigma_n \,(g\,\chi)^n \,|\, n, n\rangle$$

Averaged mutual information



Can we use this for Quantum Cryptography ?

Can we use the NLA for CVQKD ?



* Gaussian CVQKD, Eve performs a beam-splitting attack + noise
* Consider equivalent ("virtual") entangled scheme with EPR state
* Consider realistic $\beta < 1$
* Can the noiseless amplifier (NLA) be helpful ?

Preliminary question : what is the best possible success rate of the NLA ?

Answer : $1/g^2$

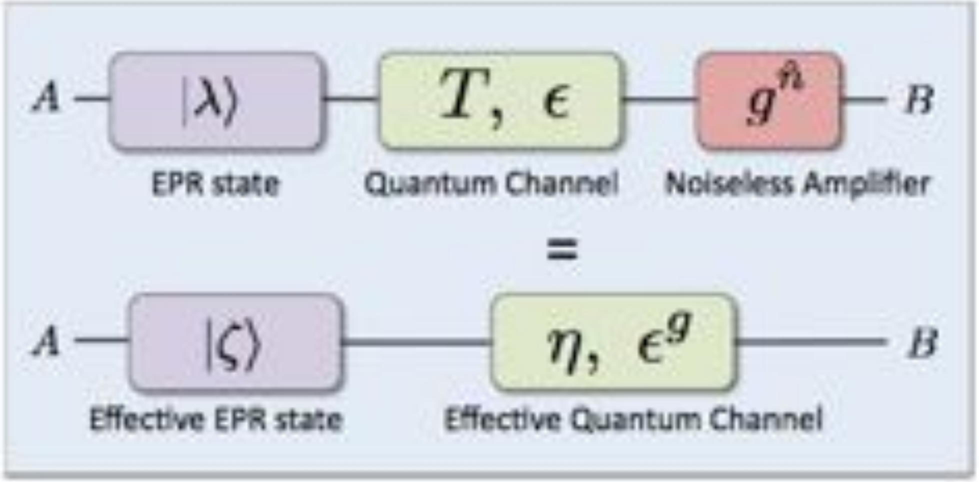
First question : is the NLO useful for $T < 1$ with no excess noise ?

Answer : no ! Better to optimize the variance of the state (i.e. χ)

Second question : is the NLO useful with excess noise in the line ?

Answer : yes ! The NLO is able to "erase" the excess noise.

Can we use the NLA for CVQKD ?

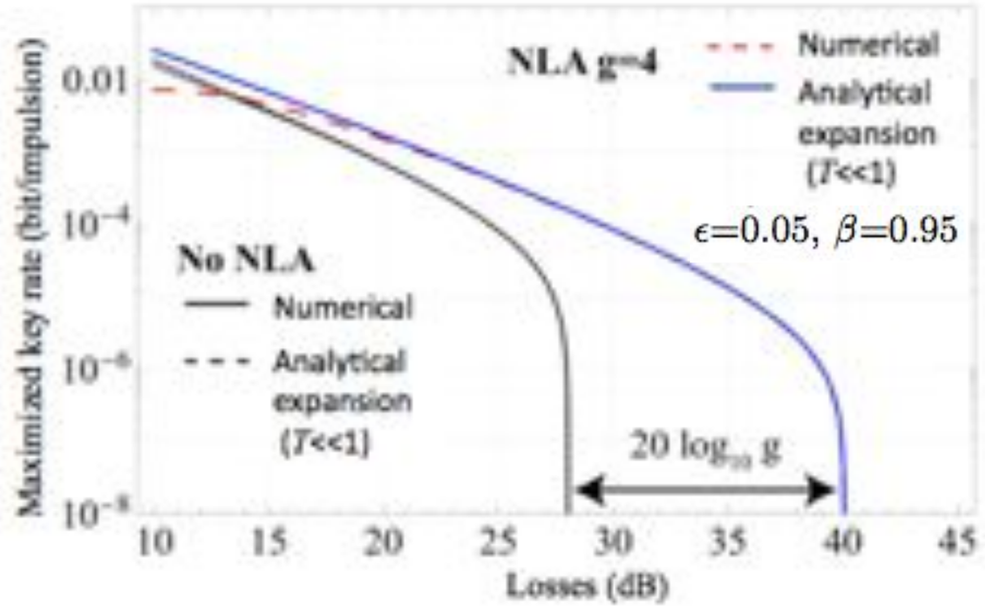


$$\zeta = \lambda\sqrt{\frac{(g^2-1)(\epsilon-2)T-2}{(g^2-1)\epsilon T-2}},$$

$$\eta = \frac{g^2 T}{(g^2-1)T\left(\frac{1}{4}(g^2-1)(\epsilon-2)\epsilon T-\epsilon+1\right)+1},$$

$$\epsilon^g = \epsilon - \frac{1}{2}(g^2-1)(\epsilon-2)\epsilon T.$$

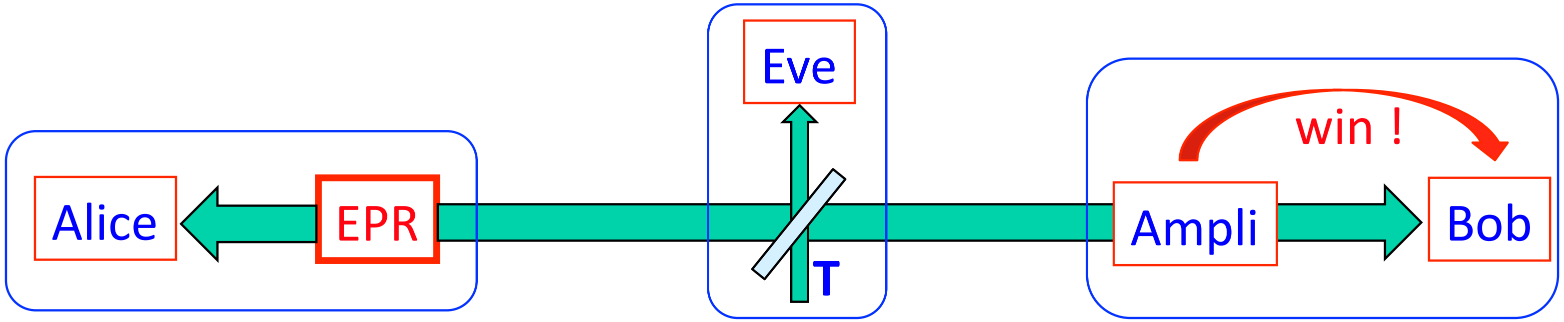
$$\Delta I^g(\lambda, T, \epsilon, \beta) = \Delta I(\zeta, \eta, \epsilon^g, \beta).$$



The maximum tolerable losses are increased by 20 $\log_{10}$ g = 12 for g = 4.

R. Blandino et al, arxiv:1205.0959
Phys. Rev. A 86, 012327 (2012)

Can we use the NLA for CVQKD ?



In this scheme, there is "real" entanglement and "real" amplification

First question : We know that we can use "virtual" entanglement, can we also use "virtual" amplification ?

Answer : yes ! (according to some unpublished preprints)

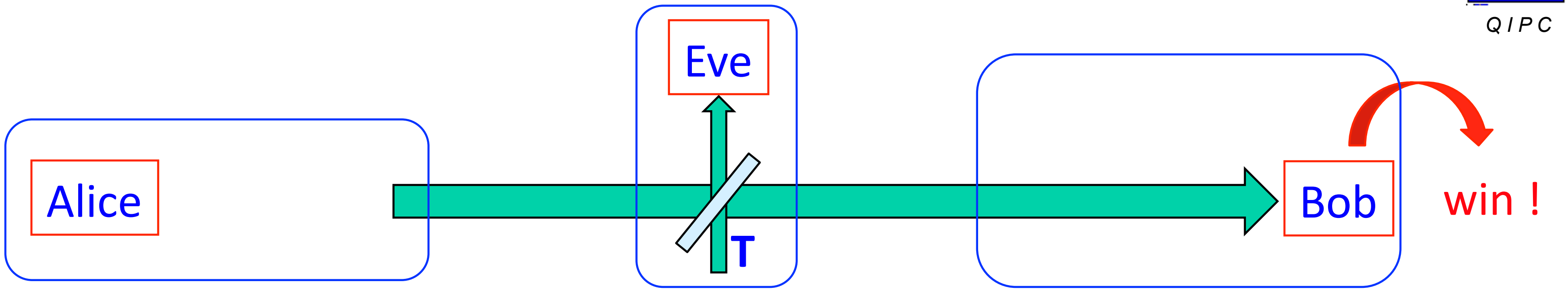
Second question : But how to do it ?

Answer : in a P&M scheme, use (Gaussian) postselection + rescaling

See : J. Fiuracek and N.J. Cerf , arxiv:1205.6933

N. Walk, T.C. Ralph et al, , arxiv:1206.9936

Can we use the NLA for CVQKD ?



In this scheme, there is no "real" entanglement neither "real" amplification

First question : We know that we can use "virtual" entanglement, can we also use "virtual" amplification ?

Answer : yes ! (according to some unpublished preprints)

Second question : But how to do it ?

Answer : in a P&M scheme, use (Gaussian) postselection + rescaling

See : J. Fiuracek and N.J. Cerf , arxiv:1205.6933

N. Walk, T.C. Ralph et al, , arxiv:1206.9936

Conclusion

Many potential uses for Quantum Continuous Variables...

* Quantum cryptography

* Coherent states protocols using reverse reconciliation,

secure against any (gaussian or non-gaussian) collective attack

* Working fine in optical fibers @1550 nm (SECOQC / SEQURE projects)

* Towards long distance / repeaters

* Use entangled / non-gaussian states (with negative Wigner functions)

* Many experimental results by our group, and many others :

A. Lvovsky, M. Bellini, E. Polzik, T. Gerrits, A. Furusawa, M. Sasaki...

* First steps towards : - entanglement distillation procedures ?

- new tests of Bell's inequalities ?

- quantum computing ? (QCV version of KLM...)

Thank you for your attention !

QIPC