Quantum Metrology in Open Systems

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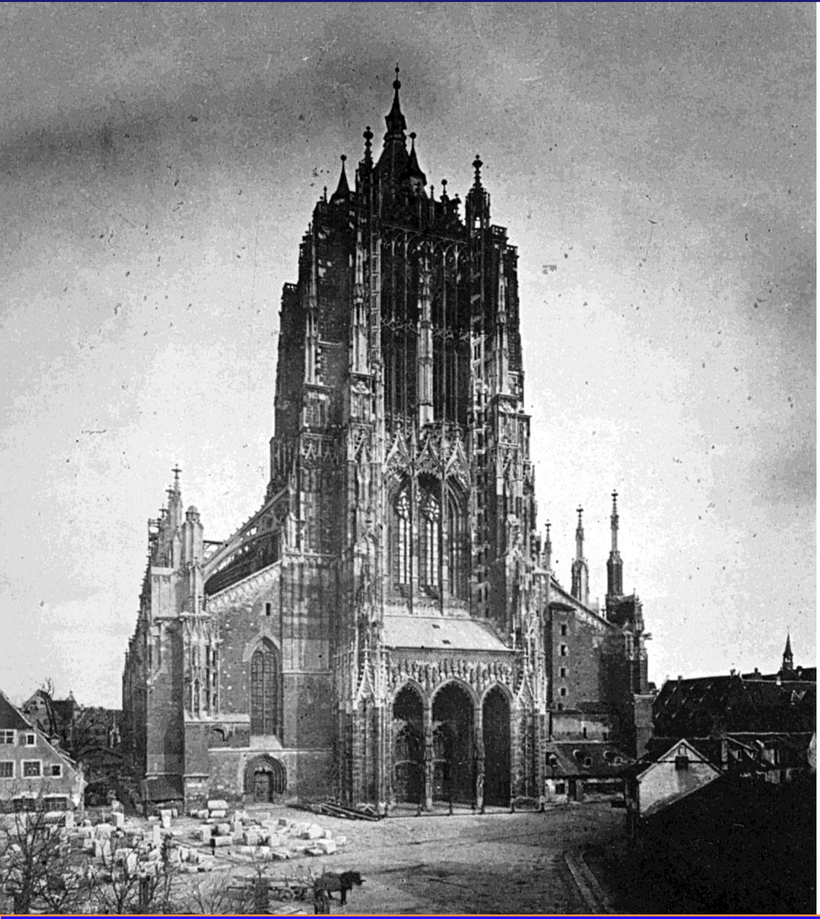

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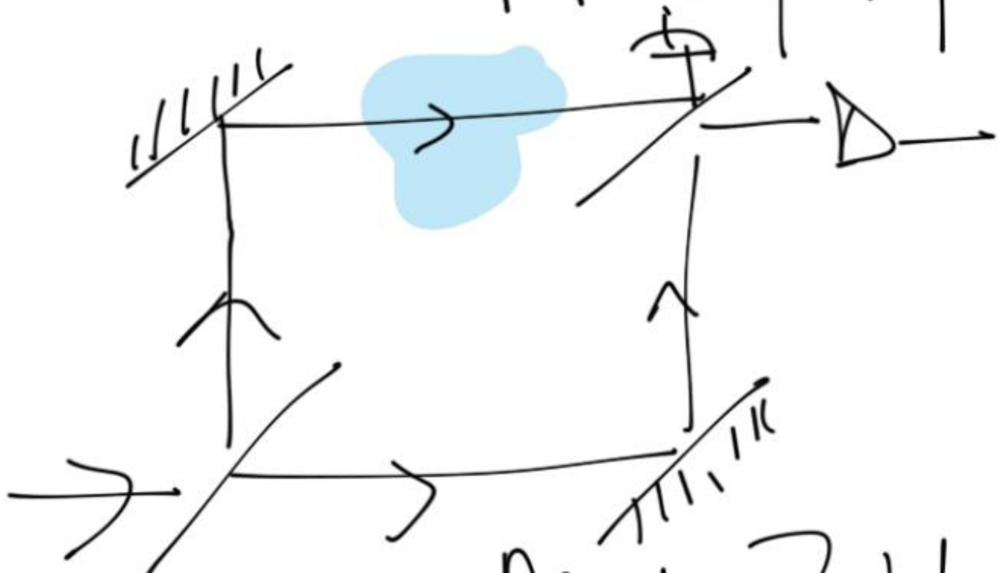
Outline

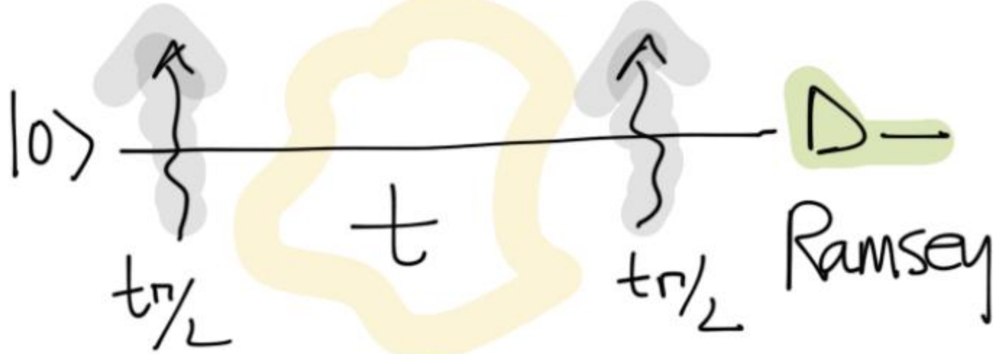
- Fundamental limits
  SQL vs HL
- Open quantum systems
  - Markovian noise
    ↑ metrological equivalence of cat and product state
  - Non-Markov noise
    ↓ equivalence is lifted

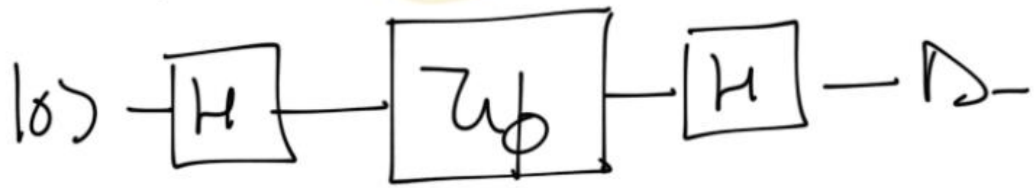
Phase estimation

Ramsey set up



Mach-Zehnder





$$U_\phi = \begin{pmatrix} 1 & 0 \\ 0 & e^{-i\phi(t)} \end{pmatrix}; \quad \phi(t) = \Delta t$$

$$\Delta = \omega - \omega_0$$

$$\rightarrow P_0 = \frac{1}{2}(1 + \cos \Delta t)$$

$$\Delta\phi\,(\equiv \delta\omega_0) = \frac{\sqrt{P_0(1-P_0)/N}}{|dP_0/d\omega|} = \frac{1}{\sqrt{nTt}}$$

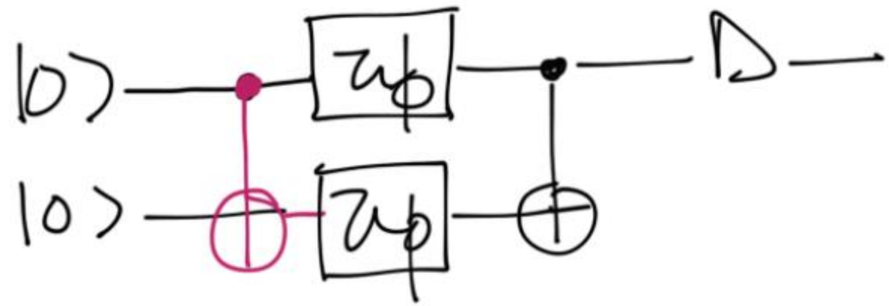
SQL

Use the n particles to create

$$|\Psi\rangle_n = \frac{1}{\sqrt{2}}\left\{|0\rangle^{\otimes n} + |1\rangle^{\otimes n}\right\}$$

$$\equiv |\Psi\rangle^{GHZ}$$

eg: $n=2$



$$P_0 = \frac{1}{2}(1+\cos 2\Delta t)$$

$$\rightarrow P_0^{GHZ} = \frac{1}{2}(1+\cos n\Delta t)$$

and $$\Delta\phi^{GHZ} = \frac{1}{n\sqrt{Tt}} \quad (HL)$$

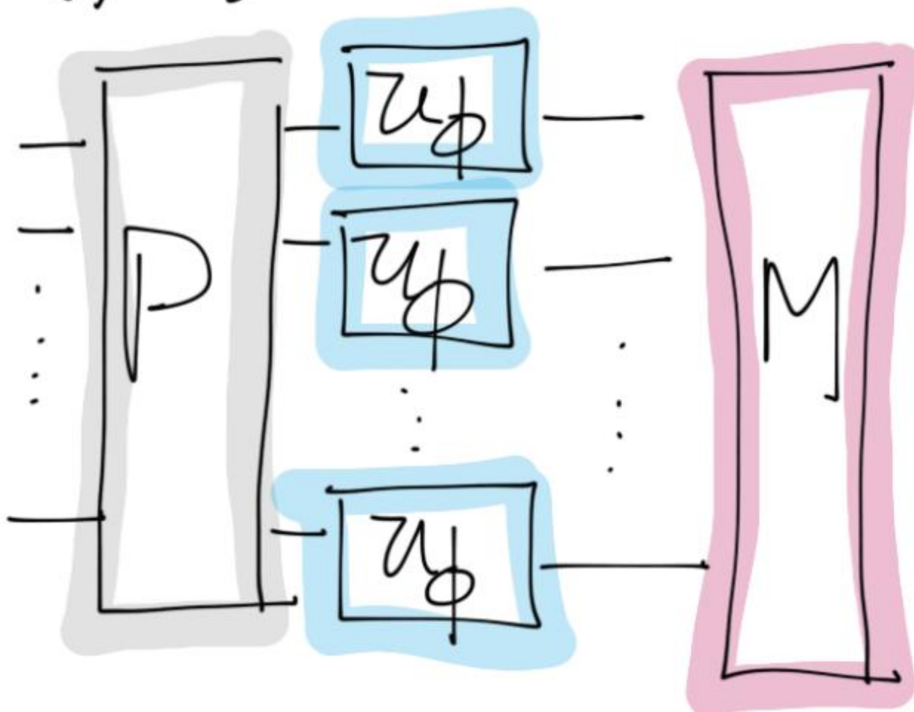
$$\Gamma = \frac{\Delta\phi^{GHZ}}{\Delta\phi_n} = \frac{1}{\sqrt{n}}$$

"Standard Scaling" surpassed using entangled probes

Is this the best that can be done?

How robust is this result to the presence of noise?

(n, T) ← resources



→ $\Delta\phi^{opt}$?

Best resolution in the estimation of ϕ given (n, T)

Braunstein & Caves (PRL 97)
State distinguishability
ρ_ϕ vs $\rho_{\phi+\Delta\phi}$

$$\Delta\phi \geq \frac{1}{\sqrt{N F(\phi)}}$$ Cramèr-Rao

$$F(\phi) = \sum_j P_j(\phi) \left\{ \frac{d \ln [P_j(\phi)]}{d\phi} \right\}^2$$

Fisher information

$P_j(\phi) \equiv$ prob. of getting outcome j.

QM $\rightarrow P_j(\phi) = \mathrm{tr}\{\hat{\rho}(\phi) E_j\}$

Optimization over $\{E_j\} \longrightarrow F_Q(\phi)$

$$\Delta\phi \geq \frac{1}{\sqrt{N F_Q(\phi)}}$$

(BC 1994)

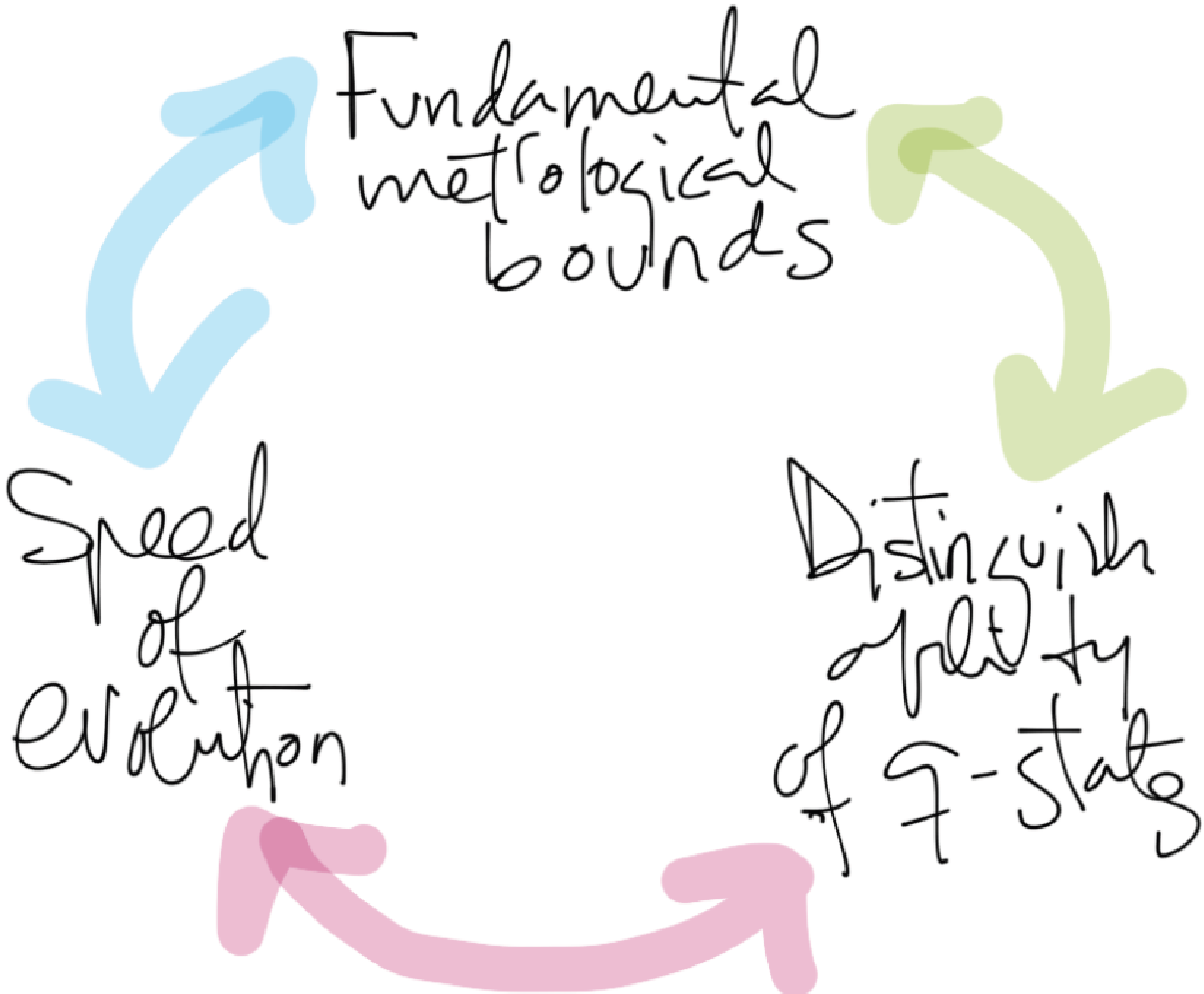
SL Braunstein and CM Caves, PRL **72** 3439 (1994)

$\downarrow$ pure states

$$\Delta\phi \geq \frac{1}{\sqrt{N}\,\Delta H}$$

(uncertainty relation)

Recent review: V Giovannetti, S. Lloyd and L. Maccone, Nat. Photonics **5** 222 (2011)

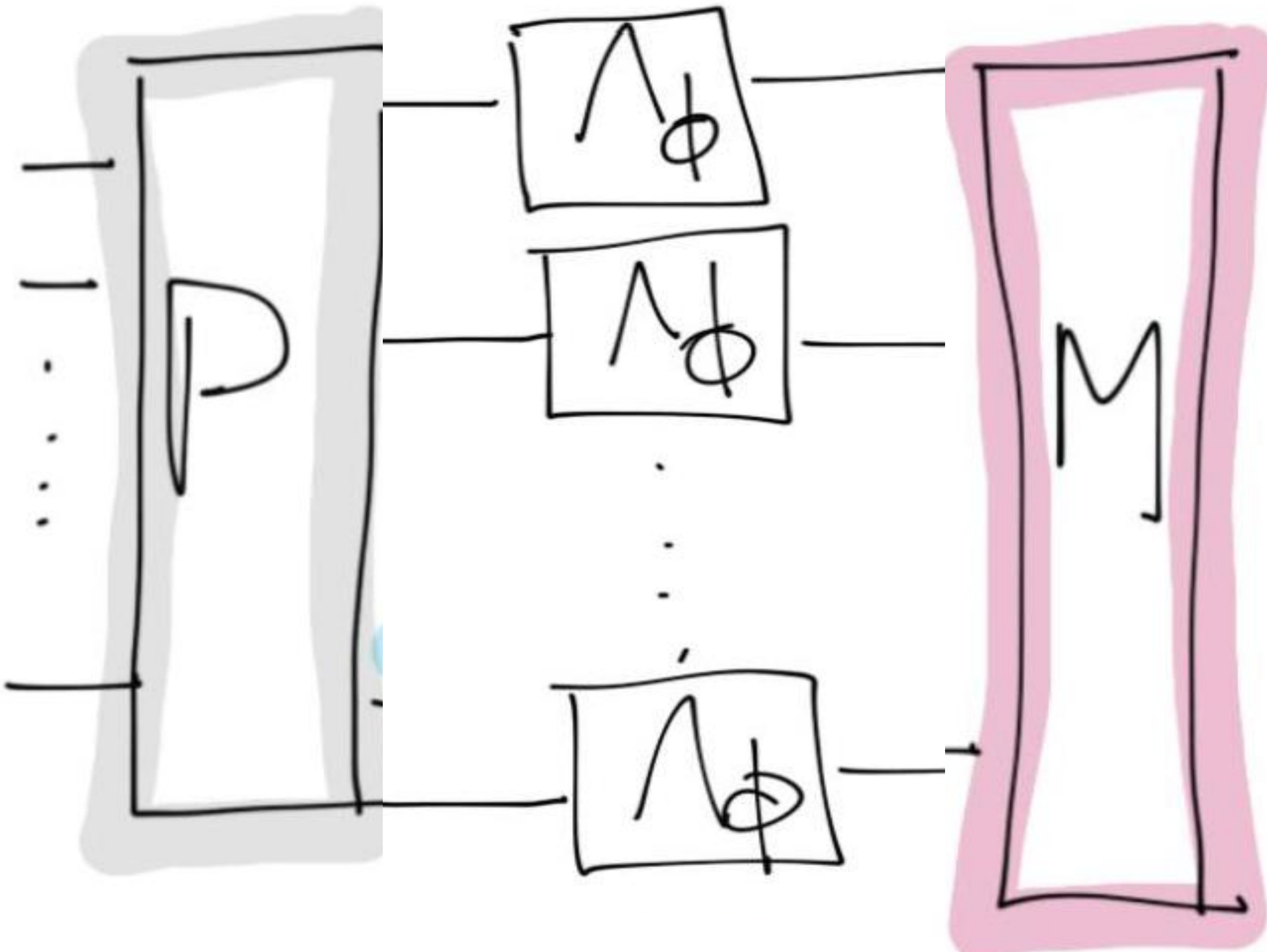

Fundamental metrological bounds
Speed of evolution
Distinguishability of q-states

$$\Delta\phi \geq \frac{1}{\sqrt{N F_Q(\phi)}}$$ saturated by GHZ state

$$\frac{\Delta\phi^{GHZ}}{\Delta\phi^{prod}} = \frac{1}{\sqrt{n}}$$

BUT: What happens in the presence of "noise"?

SH et al, PRL **79** 3865 (1997)


P
Λϕ
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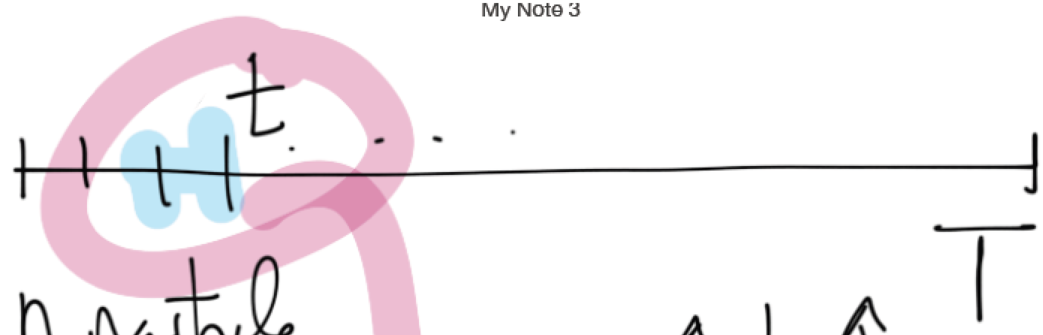


n particles

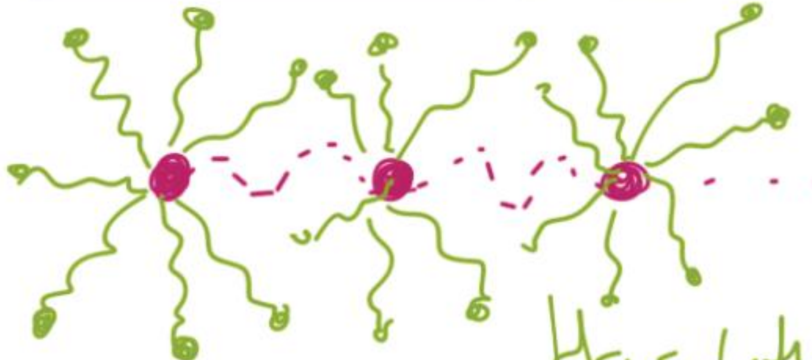
t is optimizable

↓

Depending on input determine $t_{optimal}$ and evaluate $\Delta\phi$

Noise Model

Local Markovian Dephasing



$H_{sys-bath} = \sigma_z \cdot X$

Independent "baths"

$$\frac{d\rho}{dt} = -\frac{i}{\hbar}[H, \rho] + \sum_k \left[\gamma_k A_k \rho A_k^{\dagger} - \frac{\gamma_k}{2} \{ \rho, A_k^{\dagger} A_k \} \right]$$ (Lindblad)

Pure dephasing: $A_k = \sigma_z^k$

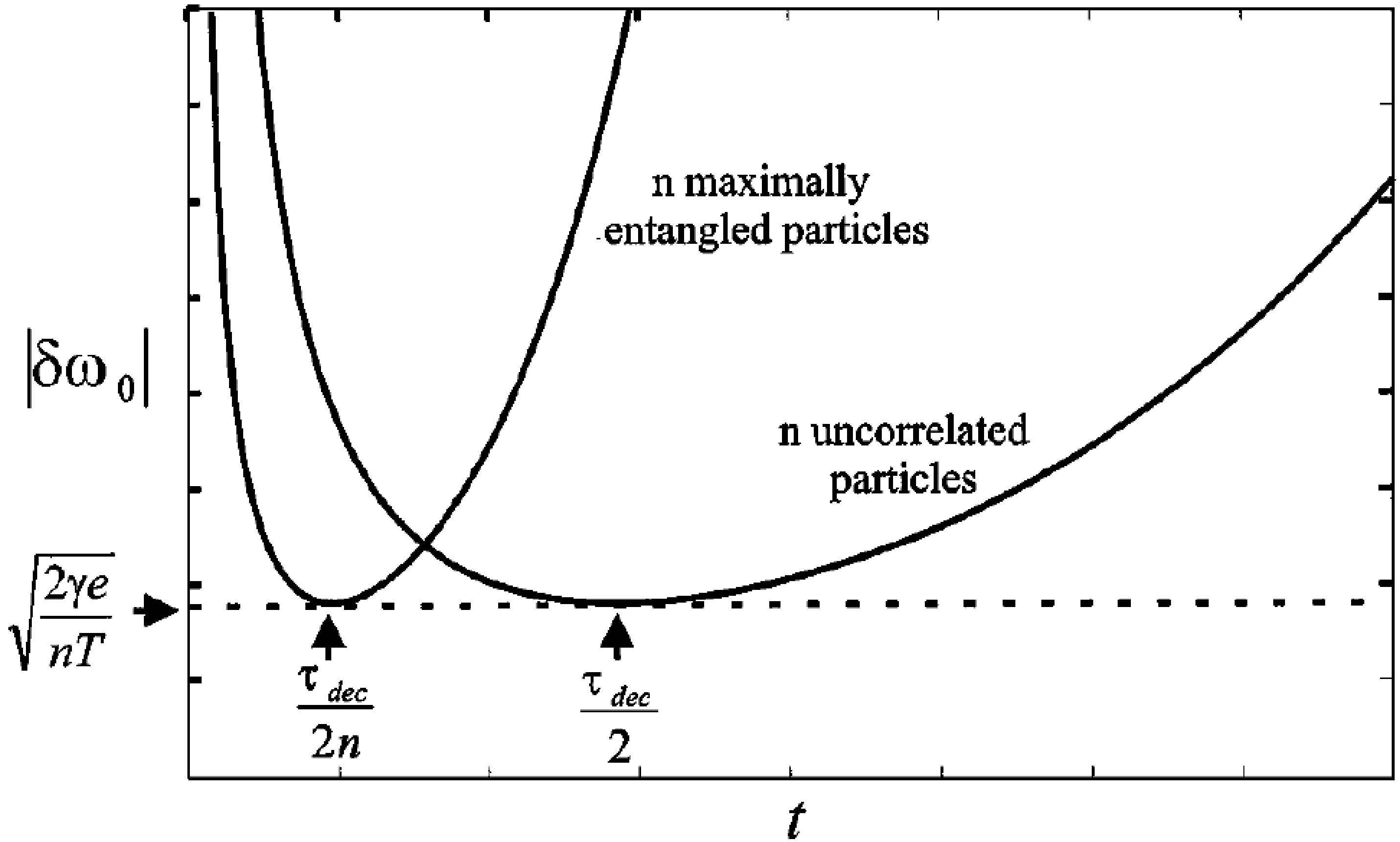


FIG. 3. Frequency uncertainty $|\delta\omega_0|$ as a function of the duration of a single shot t for maximally entangled and uncorrelated particles. Note that the minimum uncertainty is exactly the same for both configurations.

SH et al, PRL **79** 3865 (1997)

$$\delta\omega_0^{u} = \delta\omega_0^{GHZ}$$

Under Markovian noise become metrologically equivalent

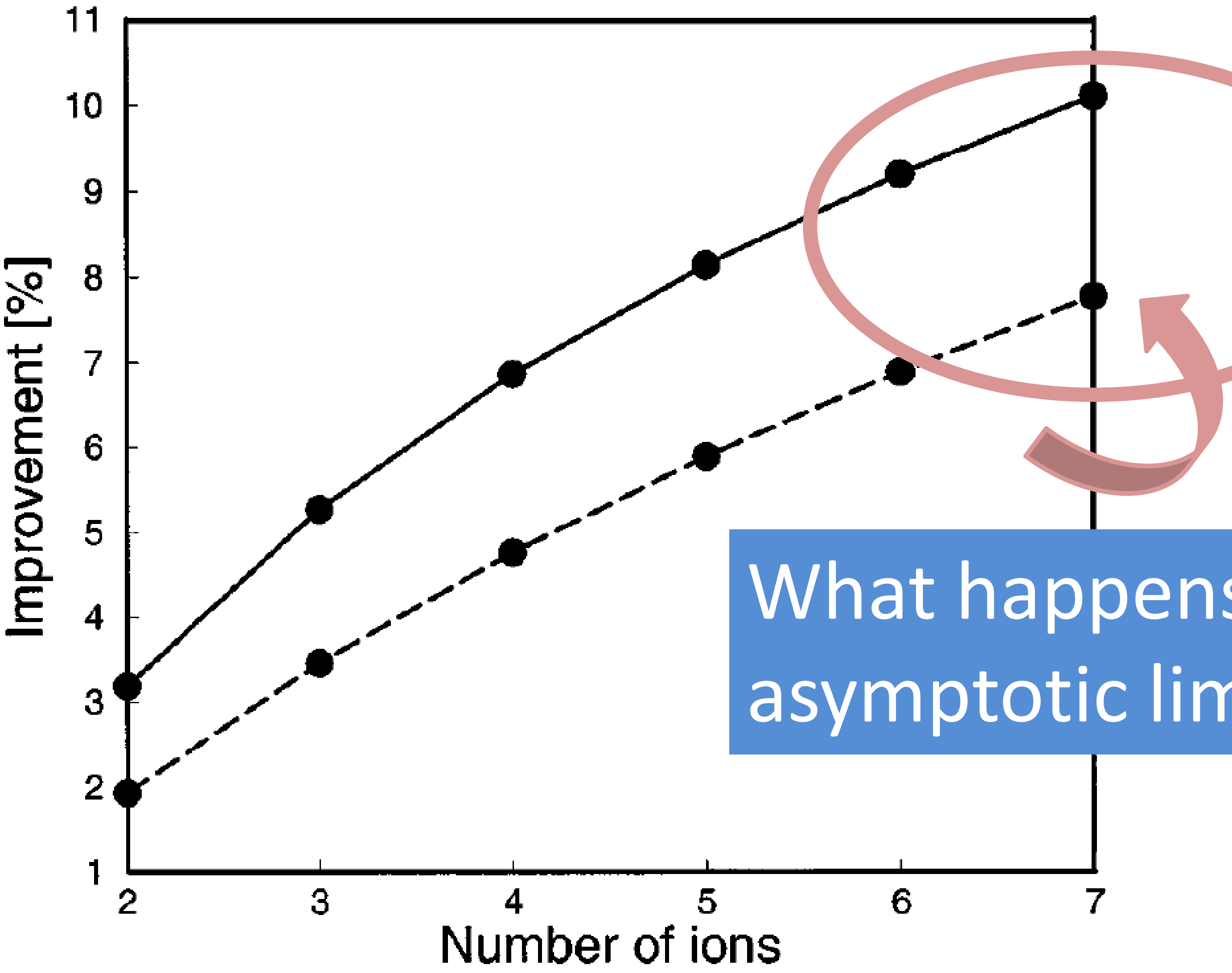
Are entangled states rendered useless in the presence of noise?

NO

Apply Braunstein & Caves algorithm to determine state yielding F_Q^{max}

→ numerical procedure ok for small n

There exist partially entangled states achieving better resolution



What happens in the asymptotic limit?

SFH et al, PRL 79 3865 (1997)

Bounding the resolution $\Delta\phi$ in a Ramsey set up.

$$S_x = \sum_k \sigma_x^k$$

$$\to \Delta\phi^2 = \frac{2n\gamma e^{2\gamma t_{opt}}}{T\langle S_x(t=0)\rangle^2}$$

$$\Rightarrow \Delta\phi \geq \sqrt{\frac{2n\gamma}{T\langle S_x\rangle^2}} \geq \sqrt{\frac{2}{n\tau_{dec}T}}$$

$$\Rightarrow r = \frac{\Delta\phi^{opt}}{\Delta\phi^{u}} = \frac{1}{\sqrt{e}}$$

Open Questions

- Is the bound $\Delta\phi = \sqrt{\frac{2\gamma}{nT}}$ saturable?

- Is the bound tight? i.e can it be recovered from the requirement of maximal $F_Q(\phi)$?

$$\delta\omega_0 \geq \sqrt{\frac{2\gamma}{NT}}$$ (γ dephasing rate)

PRA **64** 052106 (2001)

→ Asymptotic saturation shown by Ulam-Orgikh & Kitagawa

→ Does it coincide with the bound imposed by max F_Q?

Yes: $$NF_Q^{max} \leq \frac{NT}{2\gamma}\left[\frac{2\gamma t N}{1+(e^{2\gamma t}-1)N}\right]$$

$$\leq NT/2\gamma$$ ✓

BM Escher, RL de Matos Filho and L. Davidovich , Nat. Physics **7** 406 (2011)

Markovian Dephasing

* Product and maximally entangled states are rendered METROLOGICALLY EQUIVALENT.

** The presence of noise brings $\Delta\phi$ back to the standard scaling

$$\Delta\phi \sim 1/\sqrt{n}$$

Beyond Markovian noise

$$H_{sys\text{-}env} = \sigma_z \otimes B$$

$$\sigma_z = |1\rangle\langle 1| - |0\rangle\langle 0|$$

$$\rho_{ii}^{sys}(t) = \rho_{ii}^{sys}(0) \quad (i=0,1)$$

$$\rho_{01}(t) = \rho_{01}(0)\, e^{-2\gamma(t)}$$

$$P_0 = \frac{1}{2}\left(1 + \cos(\Delta t)\, e^{-\gamma(t)}\right)$$

$$P_0^{GHZ} = \frac{1}{2}\left(1 + \cos(n\Delta t)\, e^{-n\gamma(t)}\right)$$

$$\delta\omega_0^2 = \frac{1}{NF(\Delta)} \; ; \; N = \frac{T}{t} n$$

$$F = \sum_{0,1} \frac{1}{P_i}\left(\frac{\partial P_i}{\partial \Delta}\right)^2 \text{ (Fisher)}$$

↓ Maximize $F(\Delta)$

$$\Delta t = \frac{k\pi}{2} \text{ (optimal operating point)}$$

$$\delta\omega_0^2\big|_u = \frac{1}{nTt_u} e^{2\gamma(t_u)}$$

$$\delta\omega_0^2\big|_{GHZ} = \frac{1}{n^2 T t_e} e^{2n\gamma(t_e)}$$

A. Chin, SH and MB Plenio, arXiv 1103.1219

optimal interrogation times determined by

$$2t \frac{d\gamma(t)}{dt}\Big|_{t=t_u} = 1$$

$$2nt \frac{d\gamma(t)}{dt}\Big|_{t=t_e} = 1$$

Markov: $\gamma(t) = \gamma_0 t$

→ Recover metrological equiv.

When noise is non-Markov

this equivalence is lifted

Exact model (independent boson model)

$$J(\omega) = \alpha \omega_c^{1-s} \omega^s e^{-\omega/\omega_c}$$

$T = 0$

$s = 1$ (Ohmic)

$$\gamma = \frac{\alpha}{2} \ln(1 + \omega_c^2 t^2)$$

$$\rightarrow \boxed{\gamma = \alpha t^2}$$

consider

$$r = \delta\omega_0^u / \delta\omega_0^{GHZ}$$

$$\Rightarrow r^2 = n \left(\frac{t_e}{t_u}\right) e^{2\gamma(t_u) - 2n\gamma(t_e)}$$

Noiseless: $r = \sqrt{n}$; Markov: $r = 1$

$$\gamma(t_n) = n\gamma(t_e)$$

$$\Rightarrow \exp(\quad) = 1$$

$$r^2 = n\left(\frac{t_e}{t_n}\right) \rightarrow n^{1/\alpha}$$

$$\Rightarrow \boxed{r^2 = n^{\alpha - 1/\alpha}}$$

- $\alpha > 1$ → favours correlated proles
- $\alpha = 1$ → Markov
- $\alpha < 1$ → favours uncorr. proles

Using exact expression for $\gamma(t)$, for short times

$$\gamma(t) \propto t^2 \Rightarrow r = n^{1/4}$$

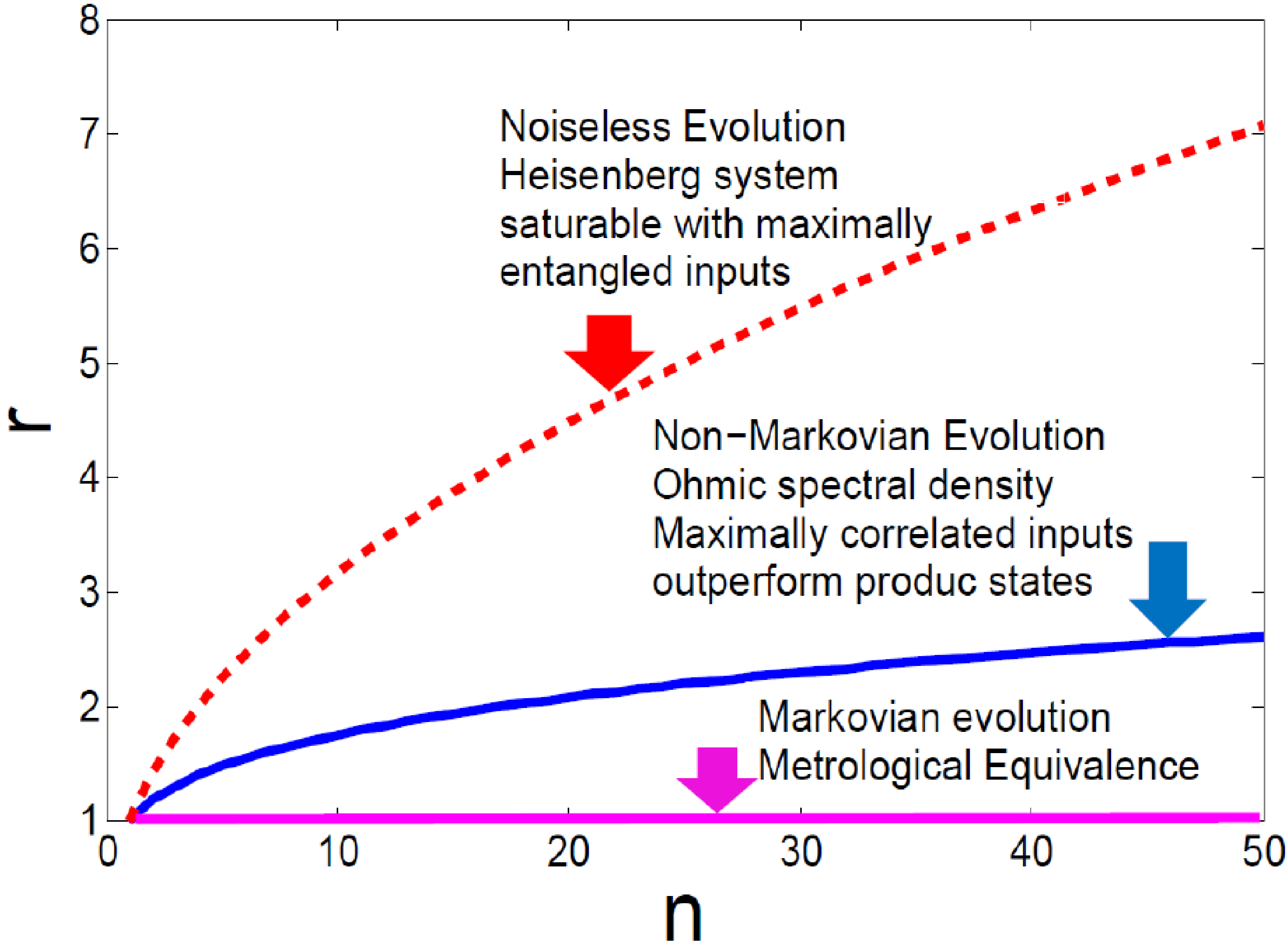
$$t_e \propto (\omega_c \sqrt{n})^{-1}$$

Ohmic case ($s=1$)

$$r = \sqrt{n}\, f(\alpha, n)$$

GHZ outperform products $\forall n$

$r \to n^{1/4}$ as $n \to \infty$
and/or $\alpha \to \infty$



Noiseless Evolution
Heisenberg system
saturable with maximally
entangled inputs
Non−Markovian Evolution
Ohmic spectral density
Maximally correlated inputs
outperform produc states
Markovian evolution
Metrological Equivalence
r
n
1
2
3
4
5
6
7
8
0
10
20
30
40
50

Main Result:

$\gamma(t) \propto t^2 \implies t_e \sim n^{-1/2}$

beyond specific models

↓

For sufficiently fast interrogation times,

$r = n^{1/4}$

(Zeno limit)

$\implies$ Standard scaling is surpassable

A. Chin, SH and MB Plenio, arXiv 1103.1219

CONCLUSION

Metrological bounds for Open Quantum Systems

- Markovian Noise → Standard scaling: $r = c$ (indep. of n)
- Non-Markovian noise → SS overcome: $r = n^{1/4}$

A. Chin, SH and MB Plenio, arXiv 1103.1219

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Davidovich et al.
Nat. Phys.
Strategy: Minimization over channel purifications

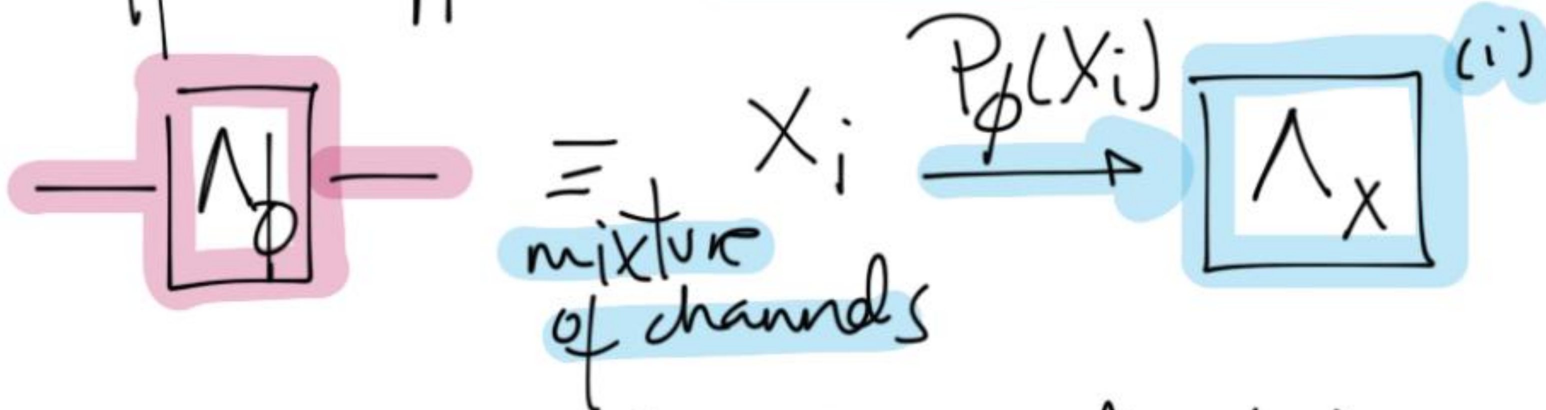
$$F_Q[\Lambda_\phi(\rho)] = \min_{|\Psi_\phi\rangle} F_Q(|\Psi_\phi\rangle)$$

$$\Lambda_\phi(\rho) = \mathrm{tr}_E\{|\Psi_\phi\rangle\langle\Psi_\phi|\}$$

$$= \sum_i K_i(\phi)\,\rho\,K_i^\dagger(\phi)$$

My Note 3

Different approach: arXiv:1201.3940



$$\Lambda_\phi(\rho) = \int dx\, P_\phi(x)\, \Lambda_x(\rho)$$

$$\phi \to P_\phi \to \{X_i\}_{i=1}^{n} \to \tilde{\phi}$$

$$\delta\phi \geq \frac{1}{\sqrt{nF(P_\phi)}}, \quad F = \int dx \frac{\left[\partial_\phi P_\phi(x)\right]^2}{P_\phi(x)}$$