21-13 July 2014

7th International Summer School of the
SFB/TRR21 "Control of Quantum Correlations in Tailored Matter”

“From trapped ions to macroscopic quantum systems”

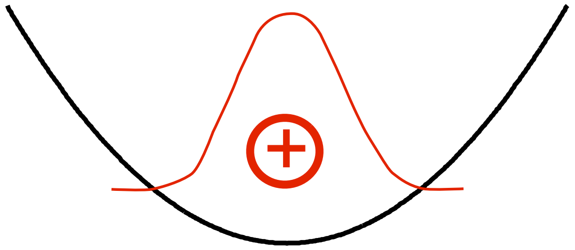
Peter Rabl

Atominstitut
Institute of Atomic and Subatomic Physics

Yesterday ...

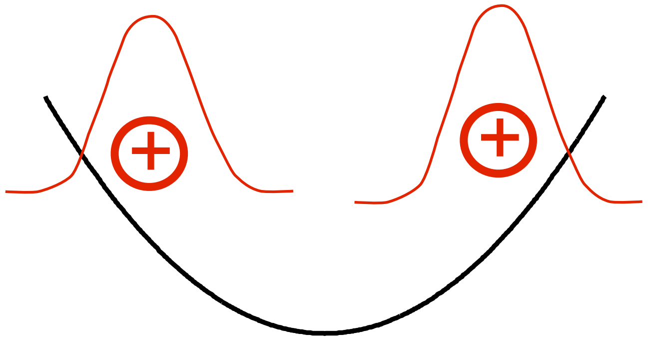
Trapped ions:

quantum
ground state



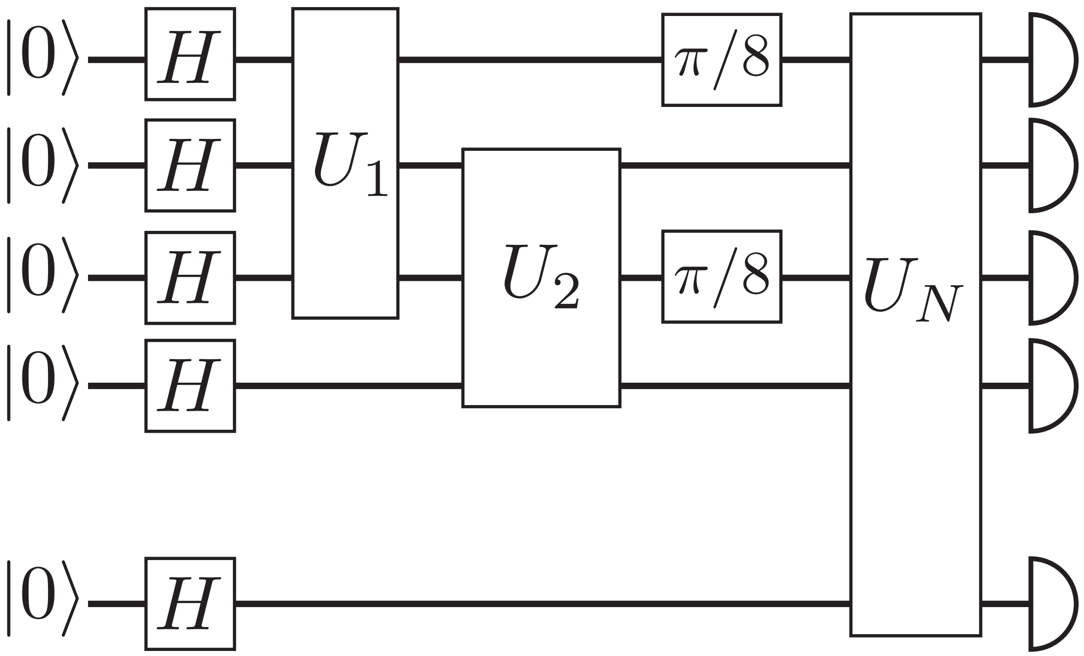
(laser cooling)

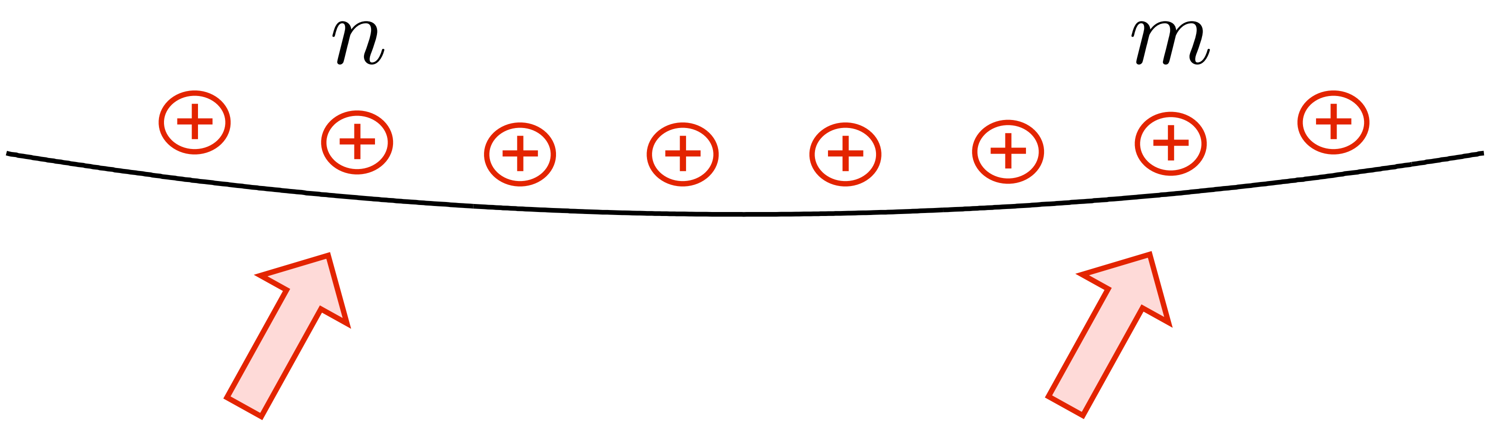
“cat state”



$$|\psi\rangle = |-x_0\rangle + |+x_0\rangle$$

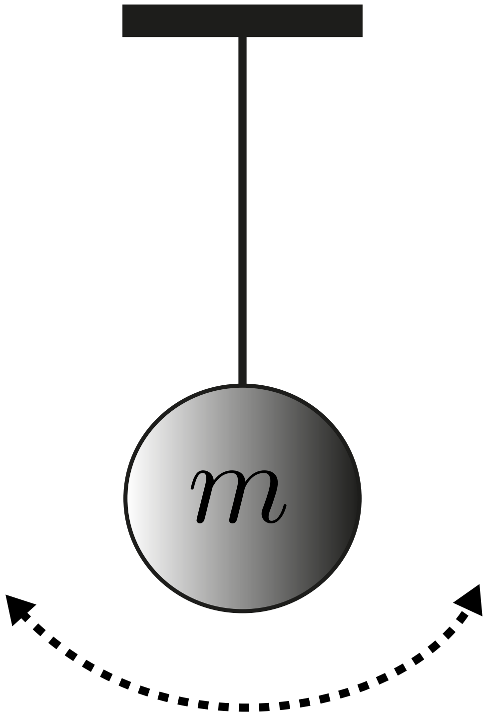
Yesterday ...





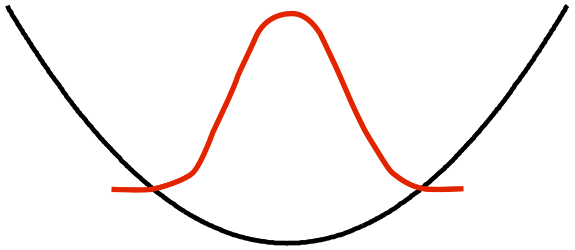
Applications: *Quantum computing, quantum simulators, ...*

Today ...

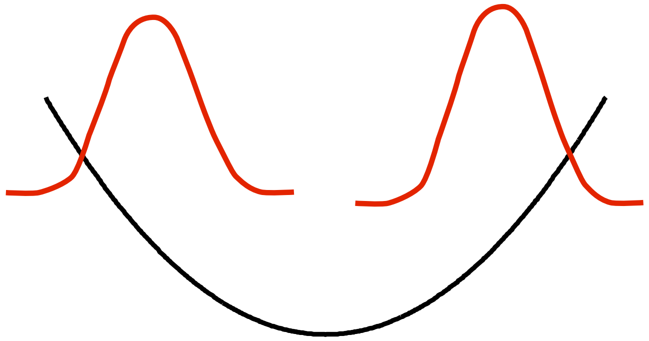


“macroscopic”
objects

quantum
ground state



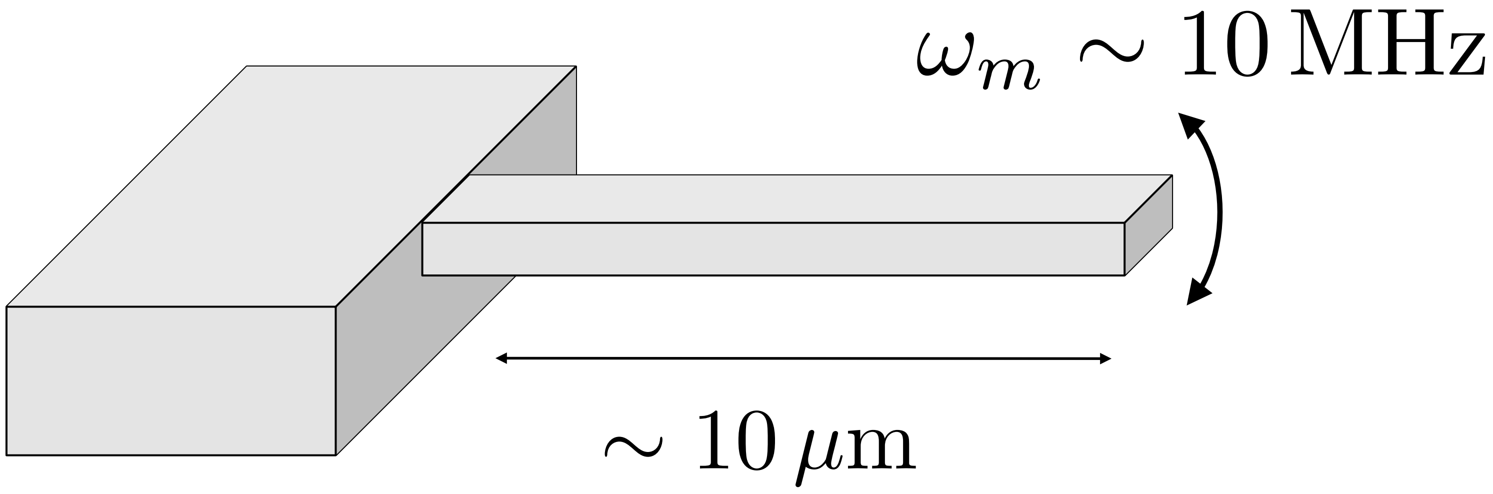
“cat state”



$$|\psi\rangle = |-x_0\rangle + |+x_0\rangle$$

How & why ?

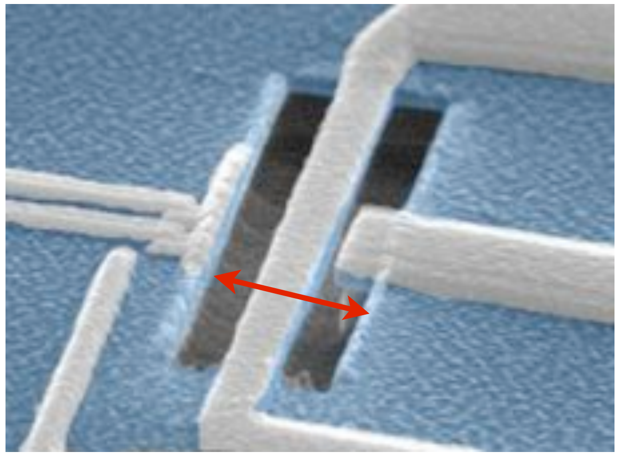
Micro- & nano-mechanical systems



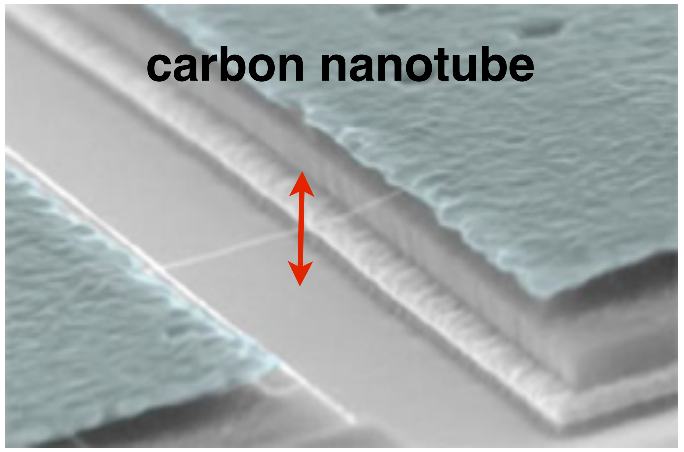
$\omega_m \sim 10\,\mathrm{MHz}$

$\sim 10\,\mu\mathrm{m}$

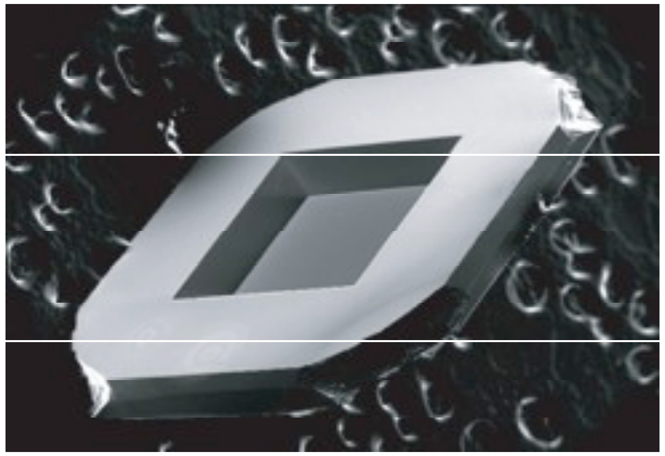
Examples:



$\omega_m \approx 20\,\mathrm{MHz}$

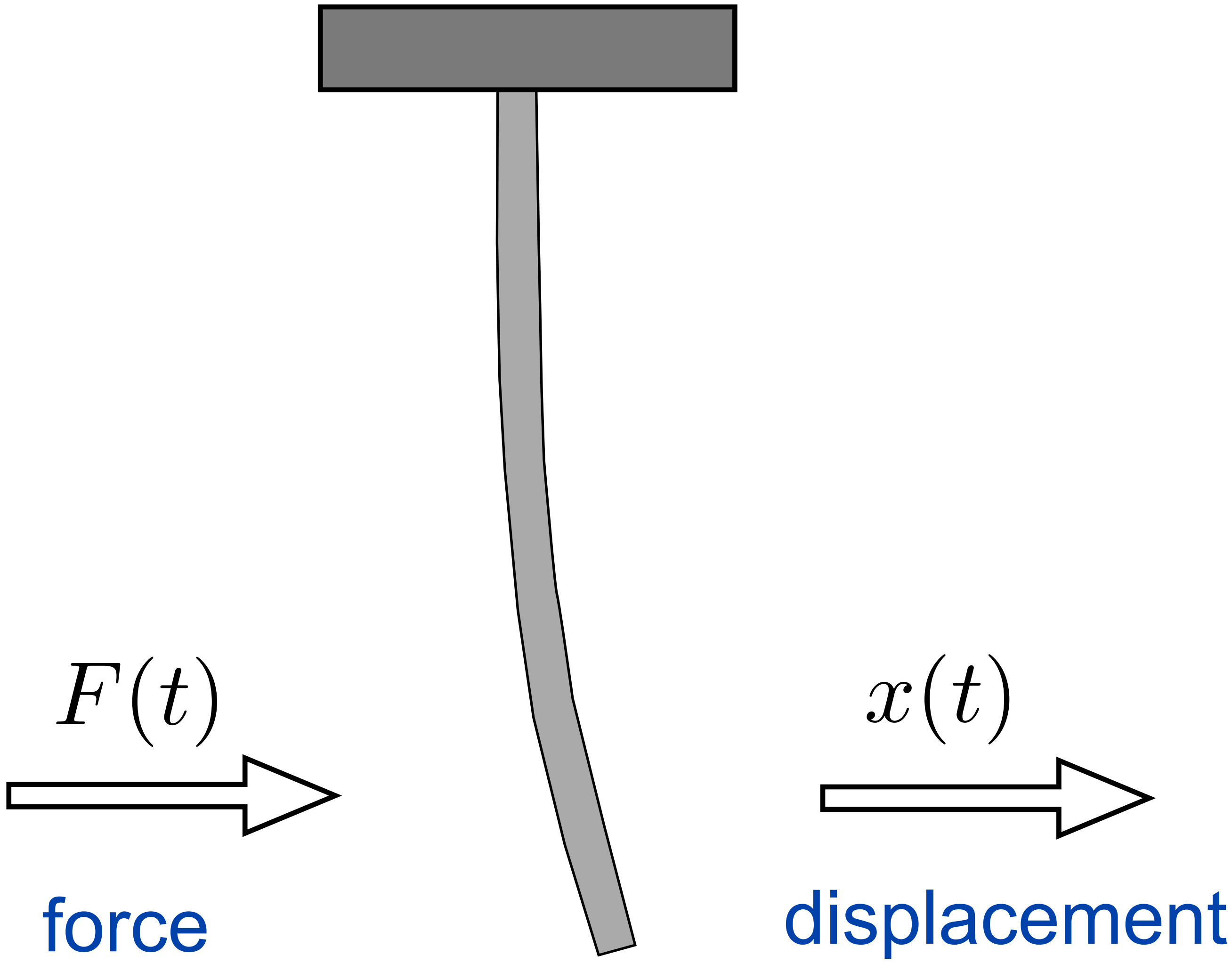


$\omega_m \approx 200\,\mathrm{MHz}$

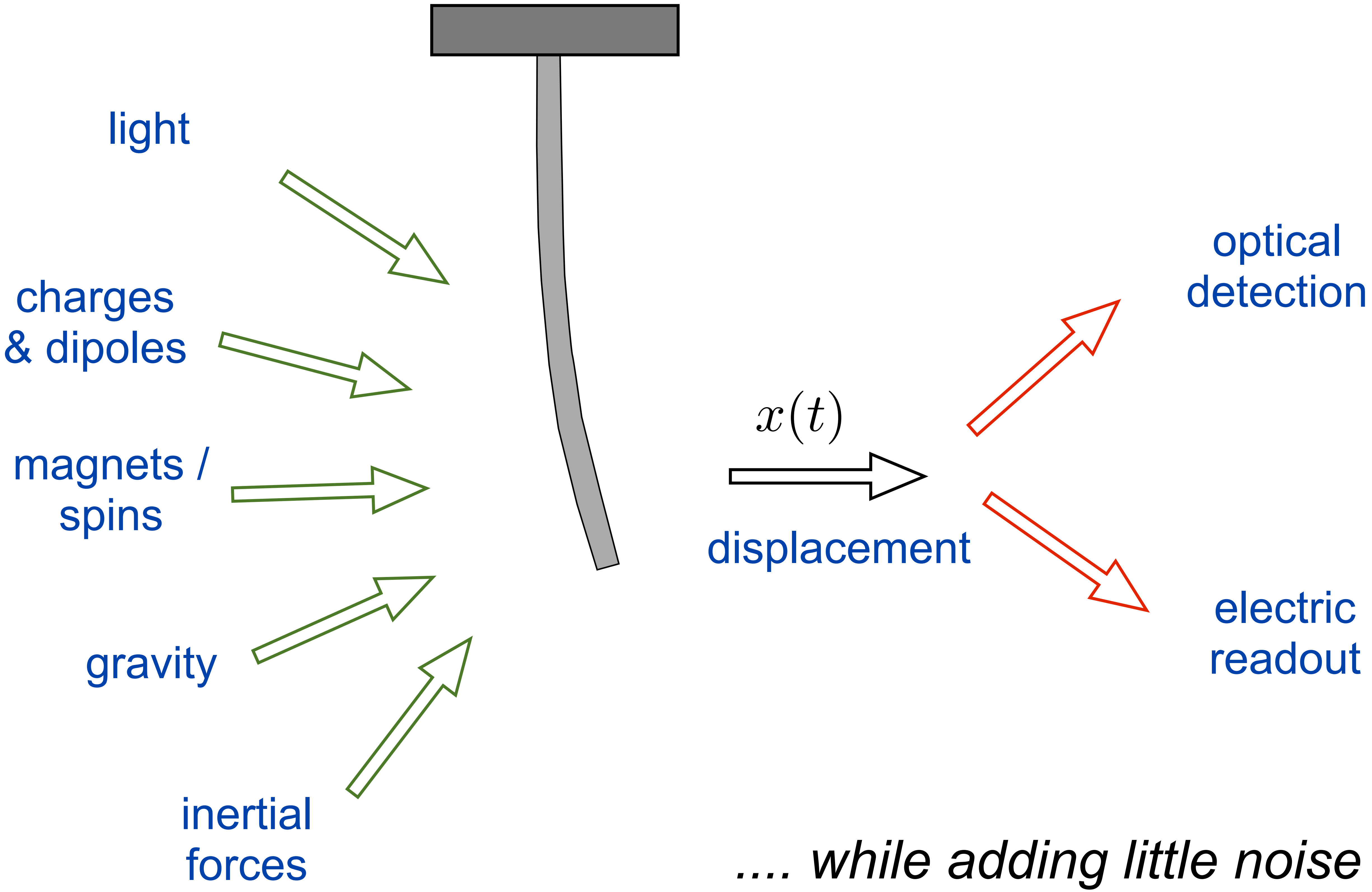


$\omega_m \approx 1\,\mathrm{MHz}$

Mechanical transducers



Mechanical transducers



light

charges
& dipoles

magnets /
spins

gravity

inertial
forces

$x(t)$

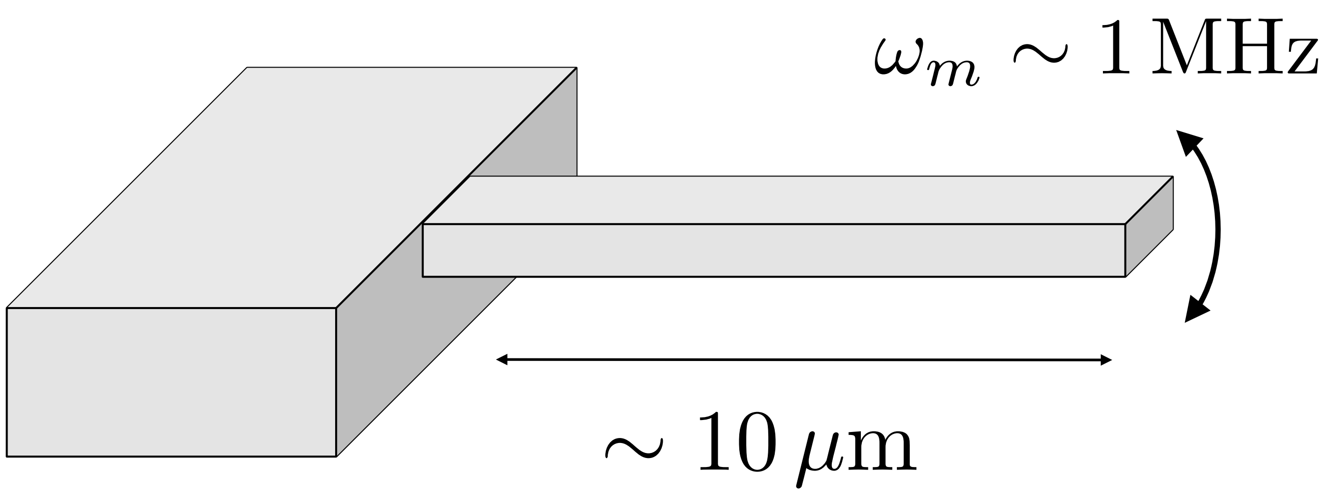
displacement

optical
detection

electric
readout

.... while adding little noise !

Micro- & nano-mechanical systems



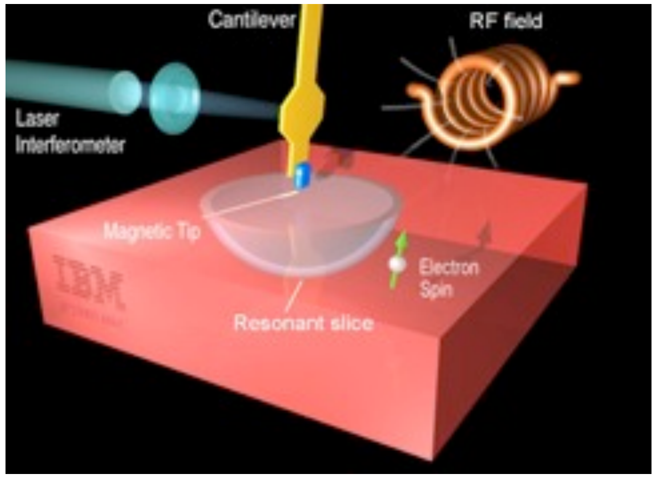
precision measurement:

- detecting single molecules / proteins
- weak force measurements
- bio-sensors, ...

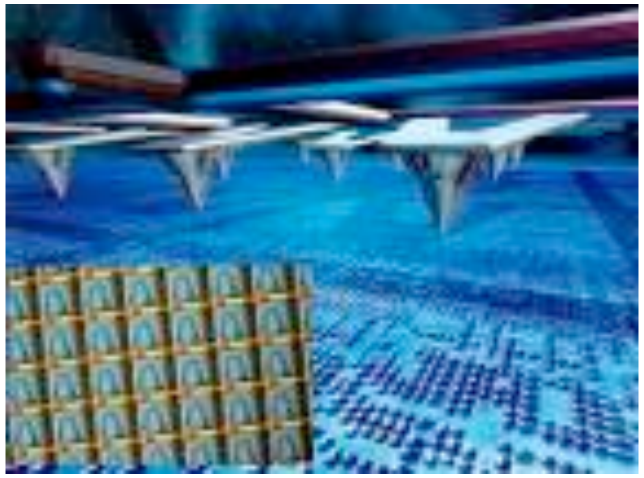
daily life applications:

- delay lines, g-sensors
- signal processing, data storage
- ....

(H. Craighead, Cornell)

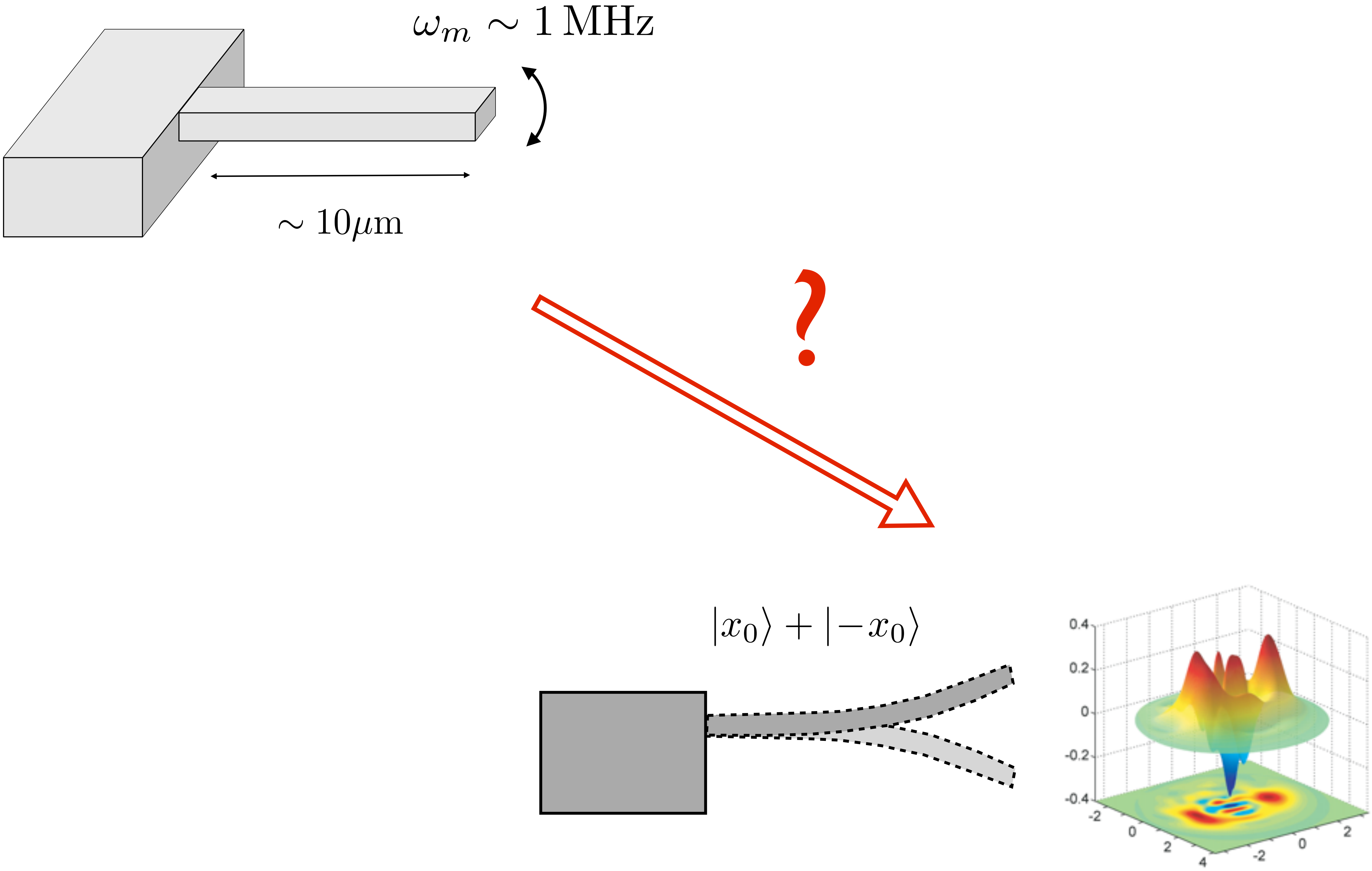


(D. Rugar, IBM)



(“Milipede”, IBM)

“Quantum” mechanical systems ?



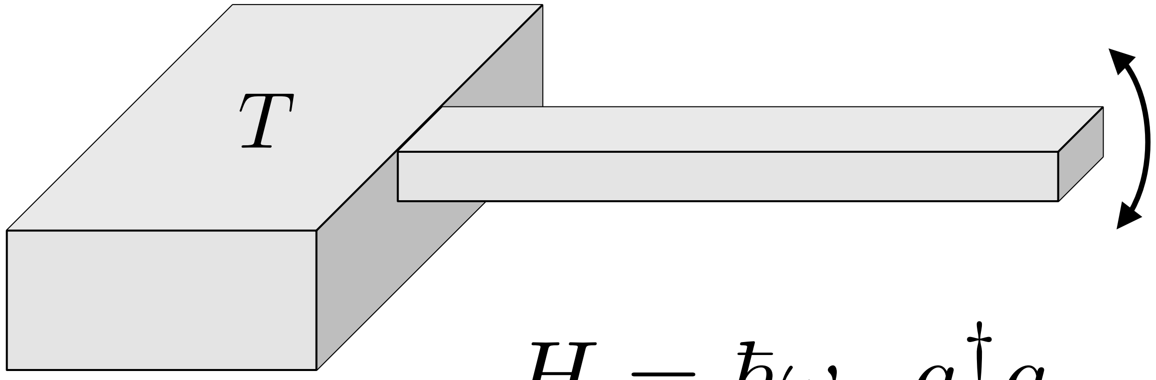
$\omega_m \sim 1\,\mathrm{MHz}$

$\sim 10\mu\mathrm{m}$

?

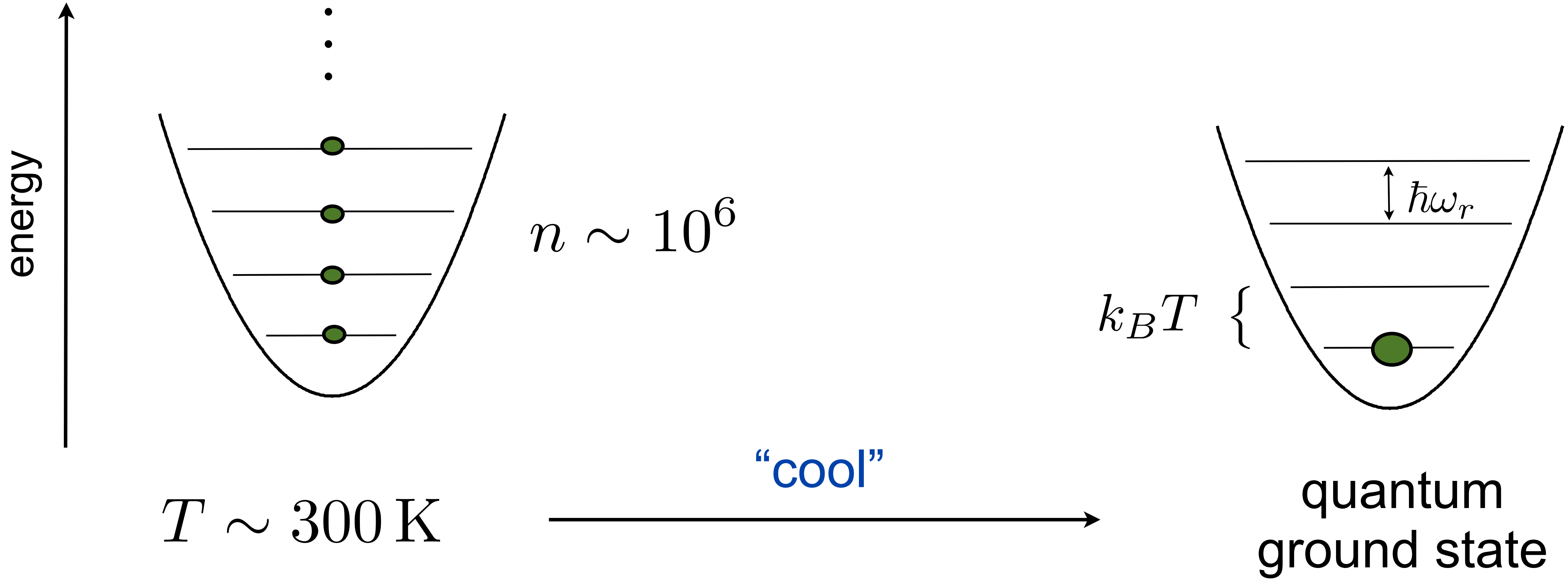
$|x_0\rangle + |-x_0\rangle$

“Quantum” mechanical systems ?



$\omega_m \sim 1\,\mathrm{MHz}$

$$H = \hbar\omega_m a^\dagger a$$

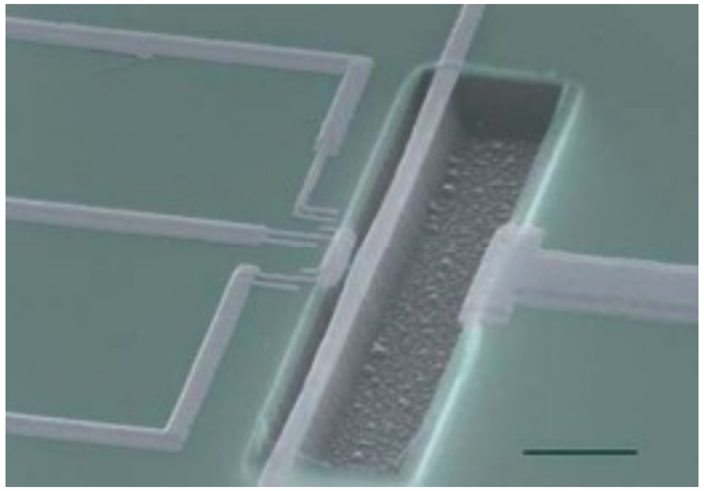


Passive cooling

$$\langle n \rangle = \frac{k_B T}{\hbar \omega_r}$$

$k_B T$ → low temperature (~ 20 mK)

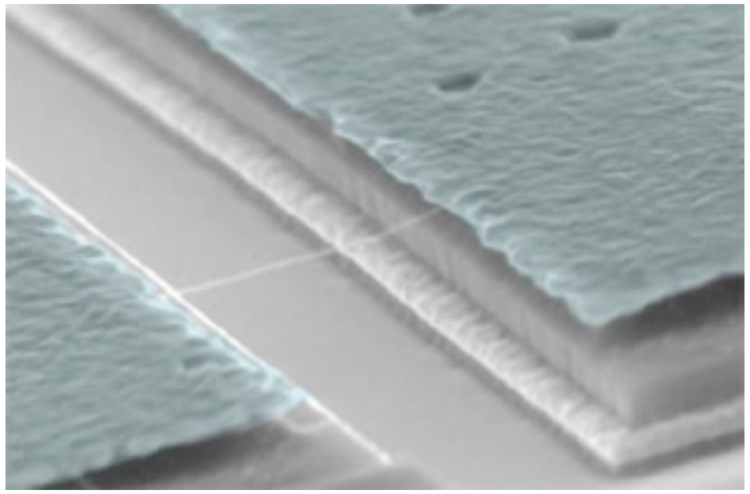
$\hbar \omega_r$ → high vibration frequency !



Si nano-beam
(K. Schwab, Caltech)

$$\omega_r \approx 20\,\mathrm{MHz}$$

$$\langle n \rangle \approx 15$$



Carbon nanotubes
(H. van der Zandt, Delft)

$$\omega_r \approx 200\,\mathrm{MHz}$$

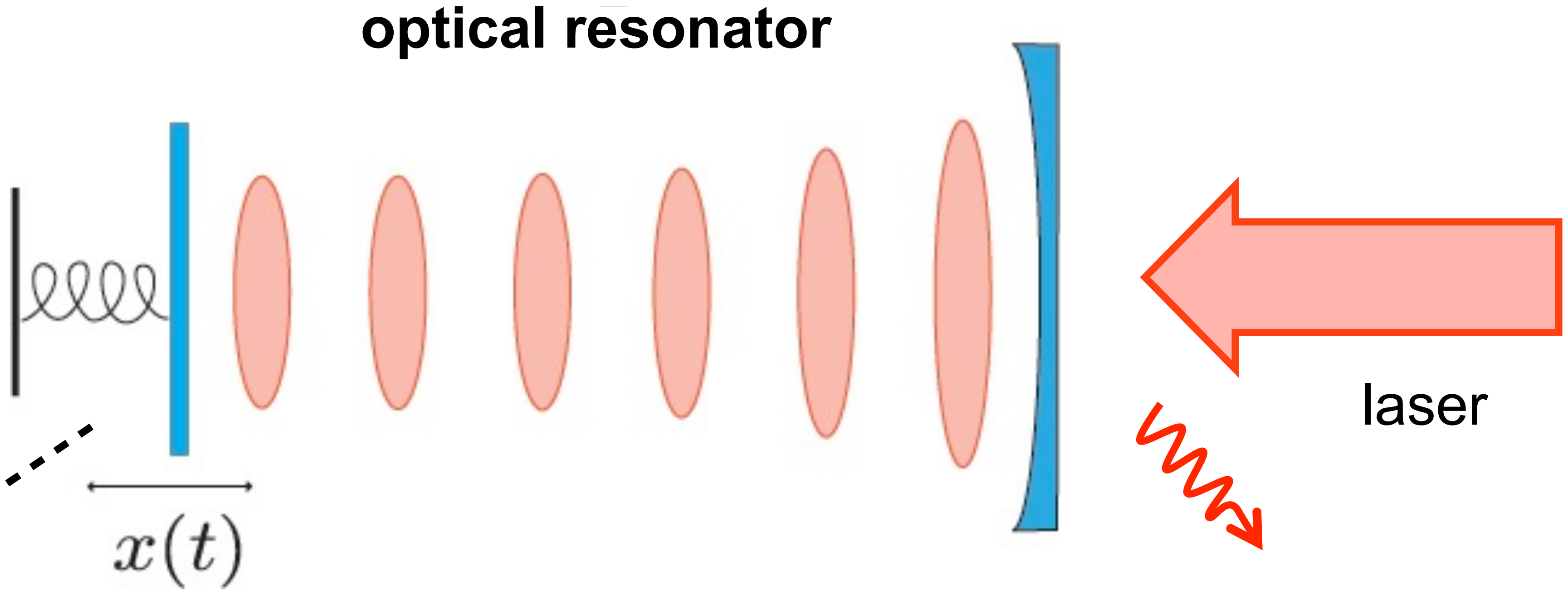
$$\langle n \rangle \approx 1$$

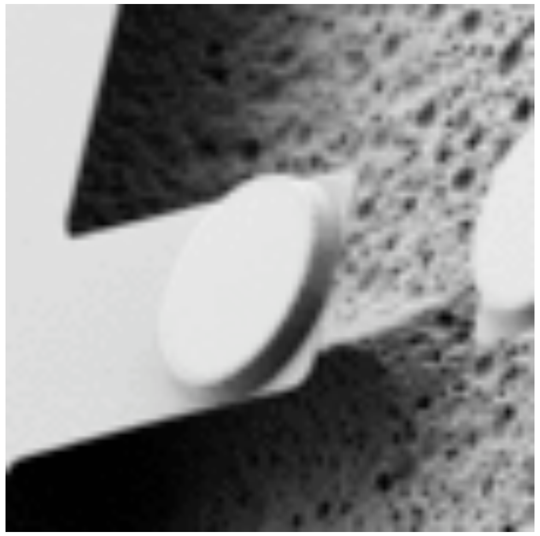
SiN dilatation resonator
(A. Cleland, Santa Barbara)

$$\omega_r \approx 6\,\mathrm{GHz}$$

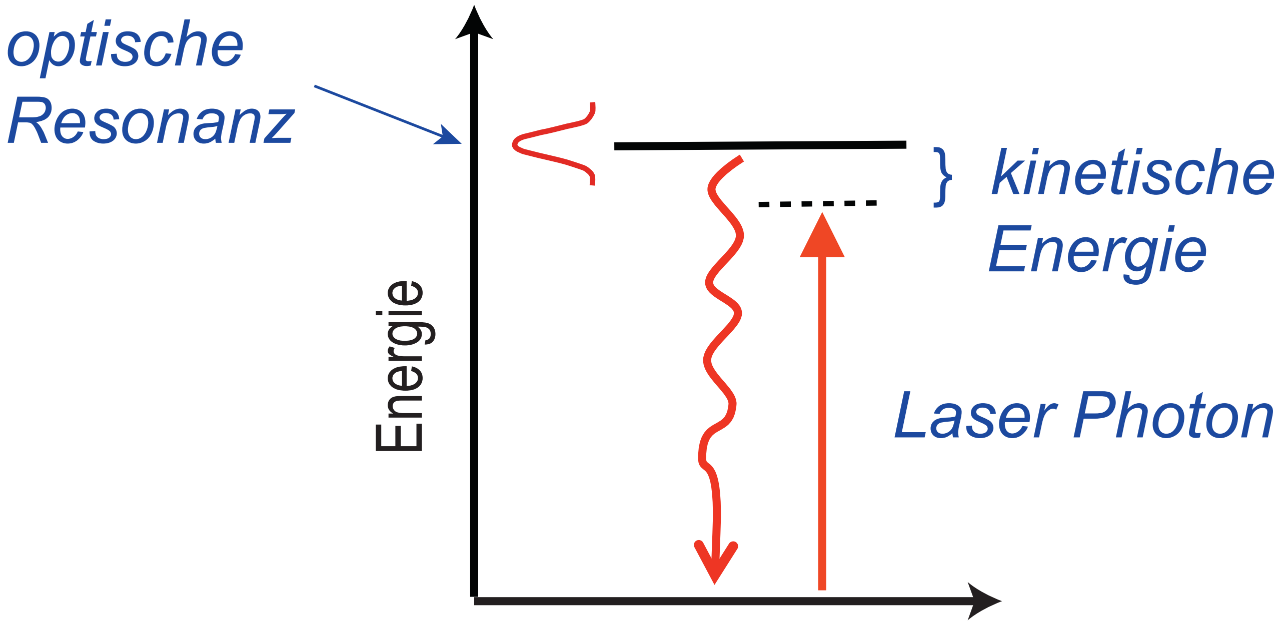
$$\langle n \rangle \approx 0$$

Laser cooling of macroscopic objects



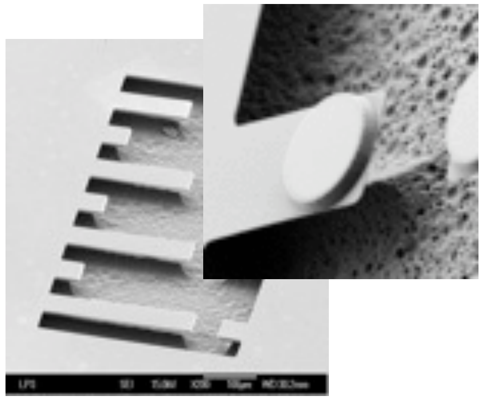


vibrating mirror
(~40x40 µm)



Optomechanical systems

optical ...



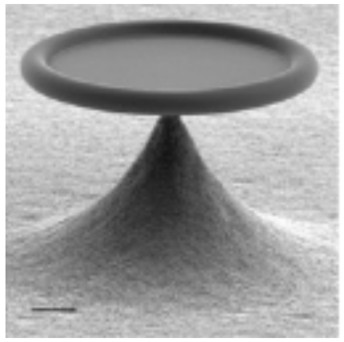
(Vienna)

(Yale)

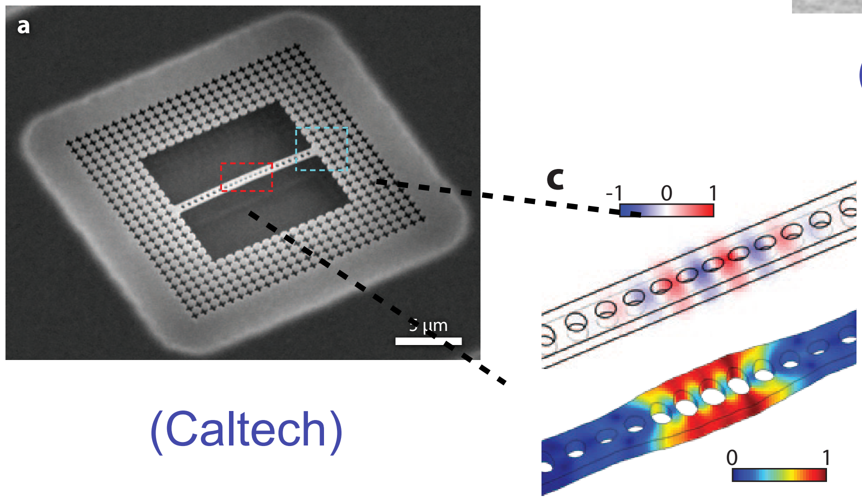
(Santa Barbara)

(MIT)

nano-photonic ...

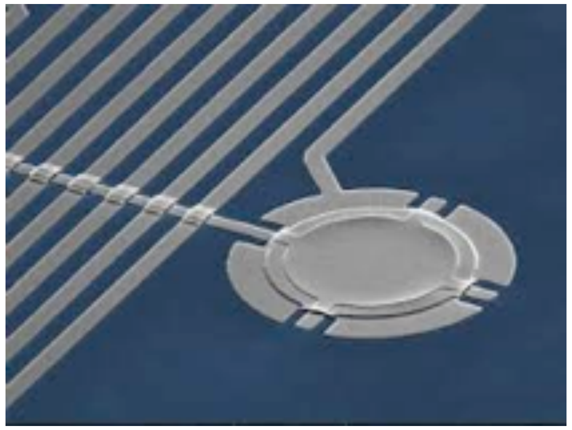


(Lausanne)



(Caltech)

microwave ...



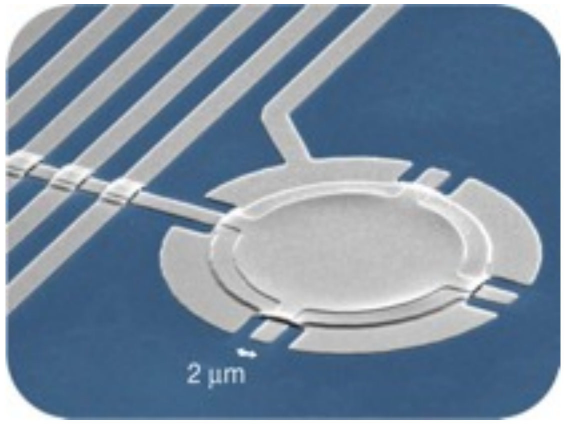
(NIST/Jila)

... and

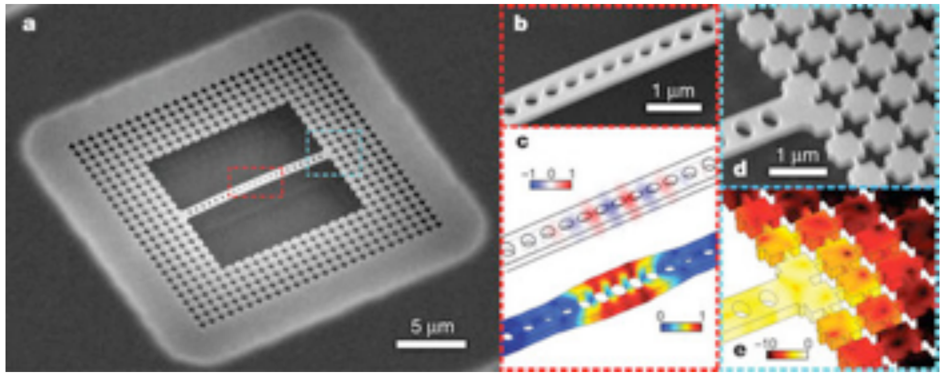
many more!

“Quantum” mechanical systems

O’Connell *et al.*,
Nature **464**, 697 (2010)



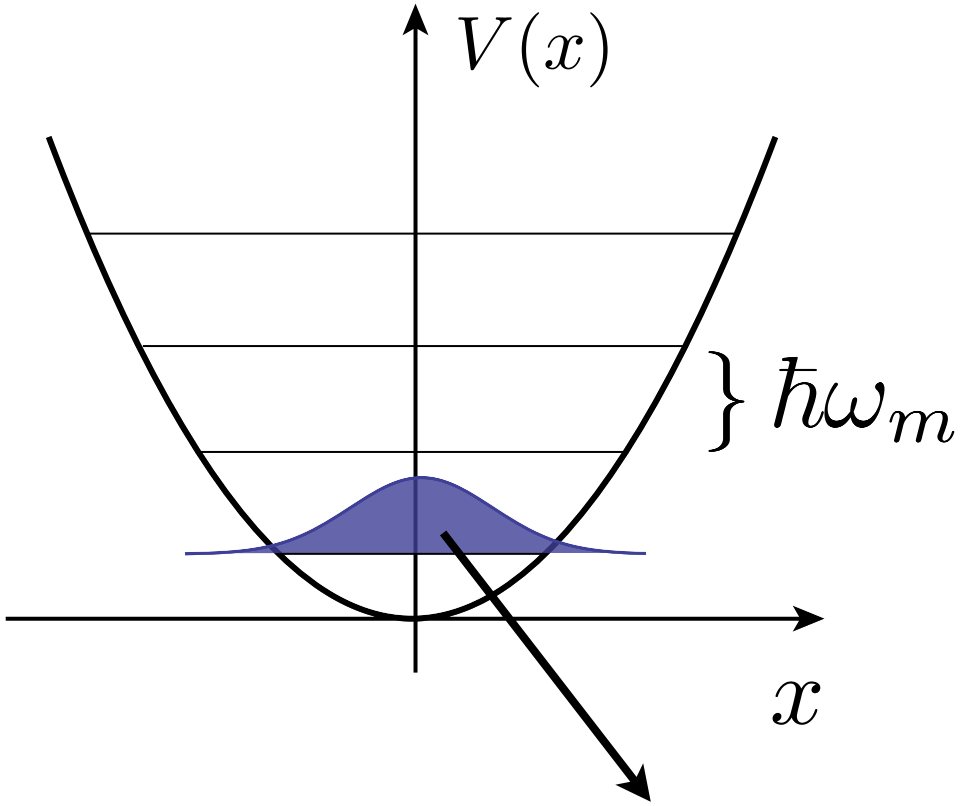
Teufel *et al.*,
Nature **471**, 204 (2011)



Chan *et al.*,
Nature **478**, 89 (2011)

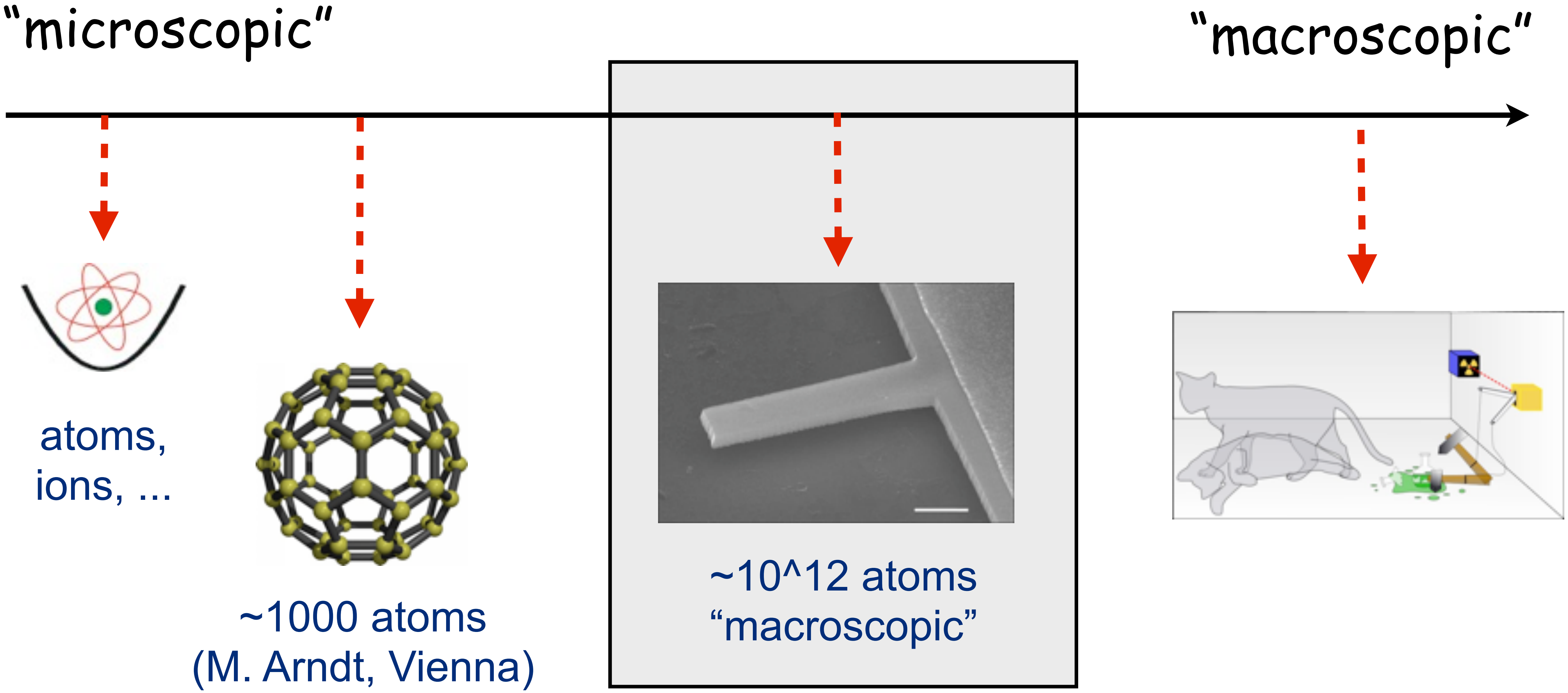
ground state cooling!

$$\boxed{\langle n \rangle < 1}$$



zero point oscillations

“Quantum” mechanical systems



“microscopic”

“macroscopic”

atoms,
ions, ...

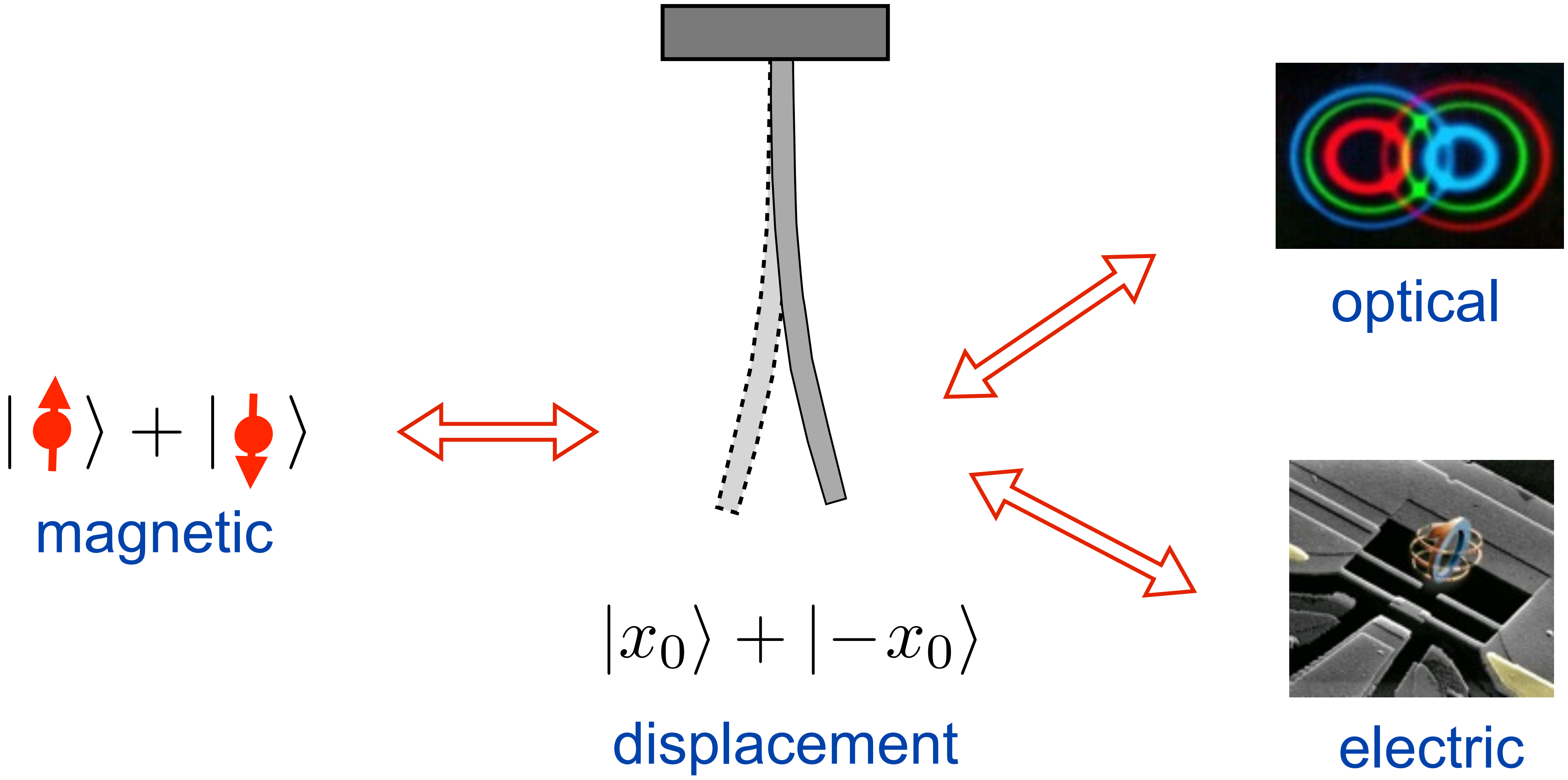
~1000 atoms
(M. Arndt, Vienna)

~10^12 atoms
“macroscopic”

Massive objects ⇒ new physics ??

- *Corrections to QM, wavefunction collapse, ...*
- *General relativity + quantum mechanics ????*

“Quantum” mechanical systems



$|\uparrow\rangle + |\downarrow\rangle$

magnetic

$|x_0\rangle + |-x_0\rangle$

displacement

optical

electric

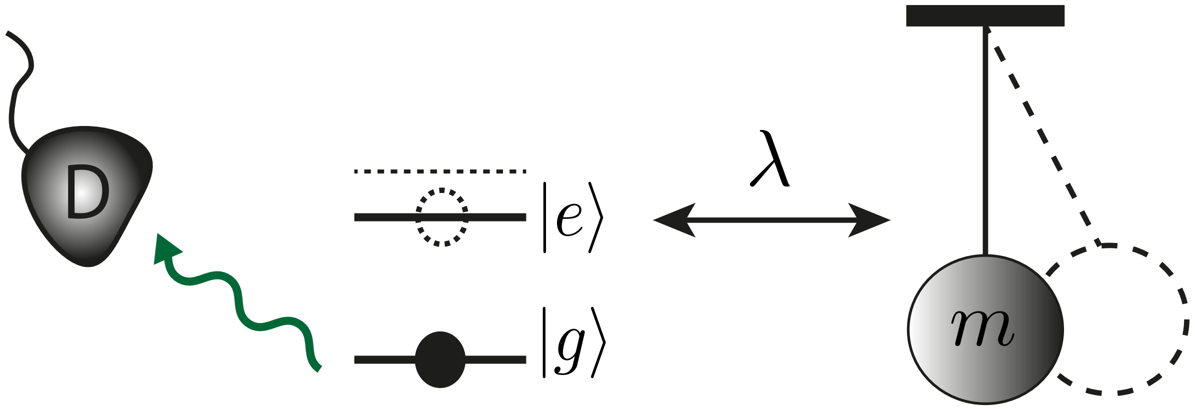
Macroscopic + quantum ⇒ new applications !

- *Mechanical quantum transducers & interfaces*
- *Q. information processing, mechanical sensing, ...*

This talk ...

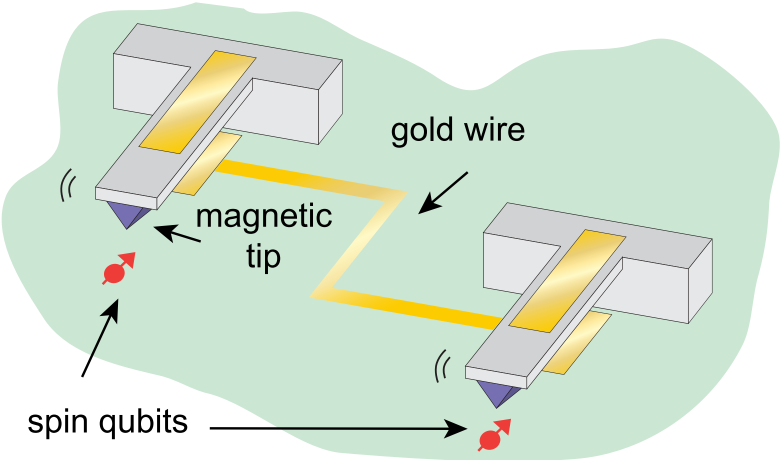
Part I:

“Probing macroscopic superpositions”



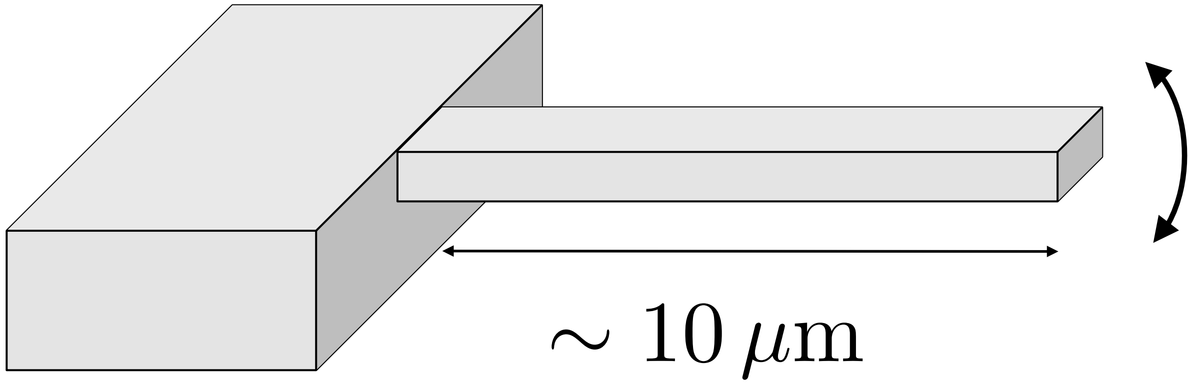
Part II:

“Electro-mechanical spin transducers”



Mechanical resonators: basics

Mechanical resonators



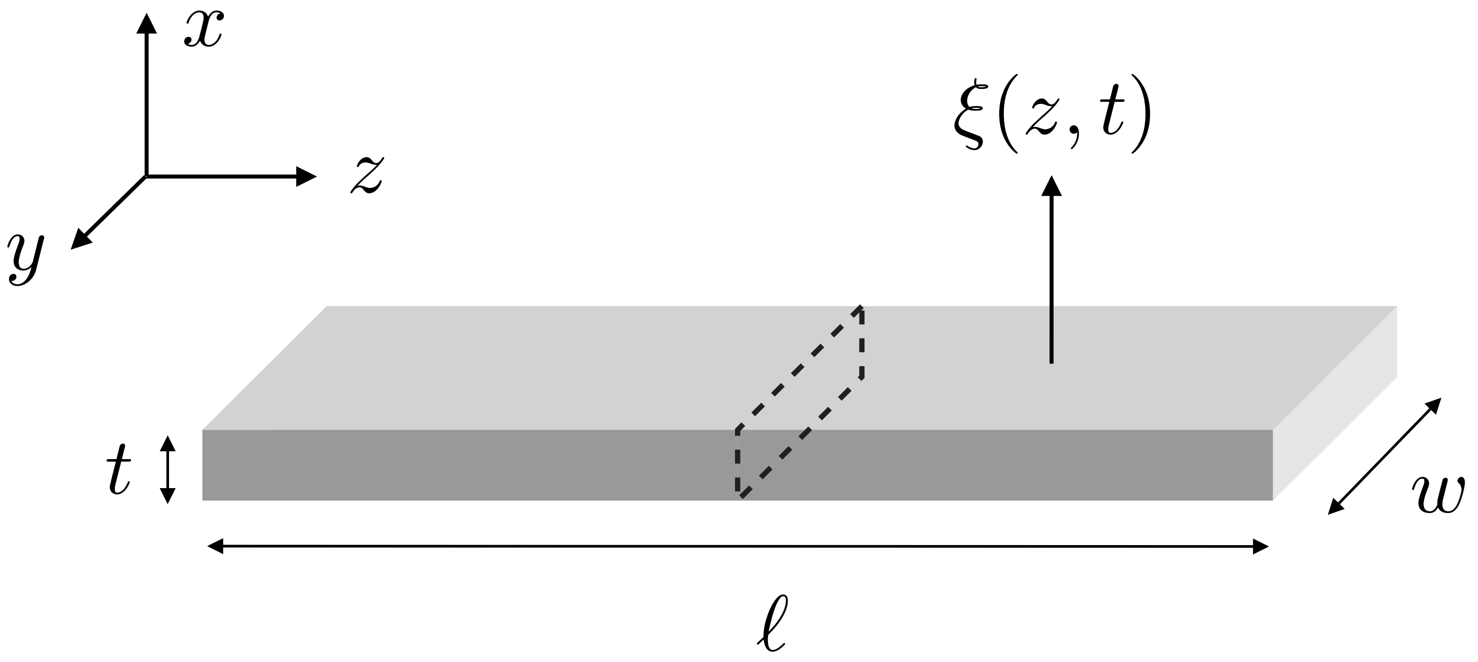
N~10^12 atoms:

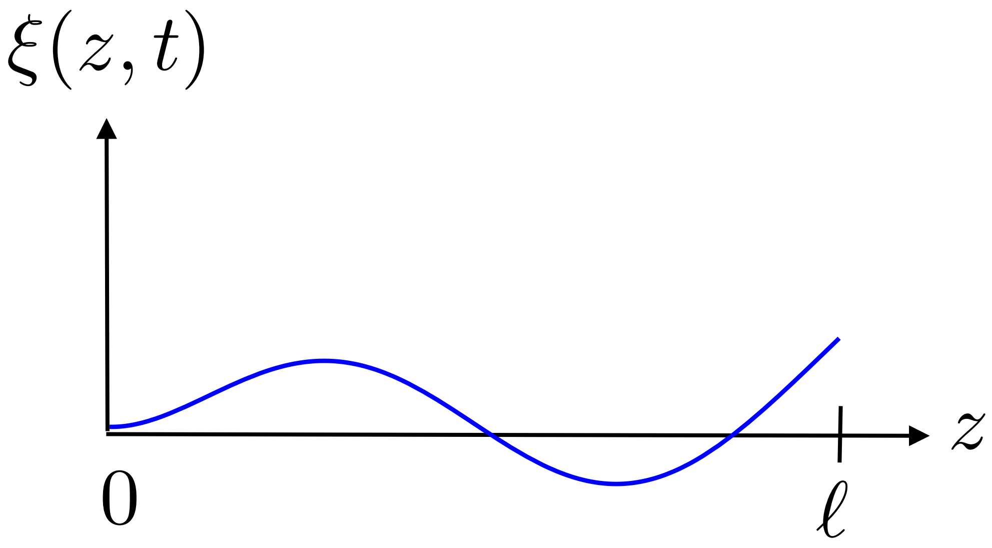
⇒ *N independent vibrational modes !*

⇒ *Quantum system with N independent degrees of freedom !*

We are interested in controlling only one of those modes !

Mechanical resonators





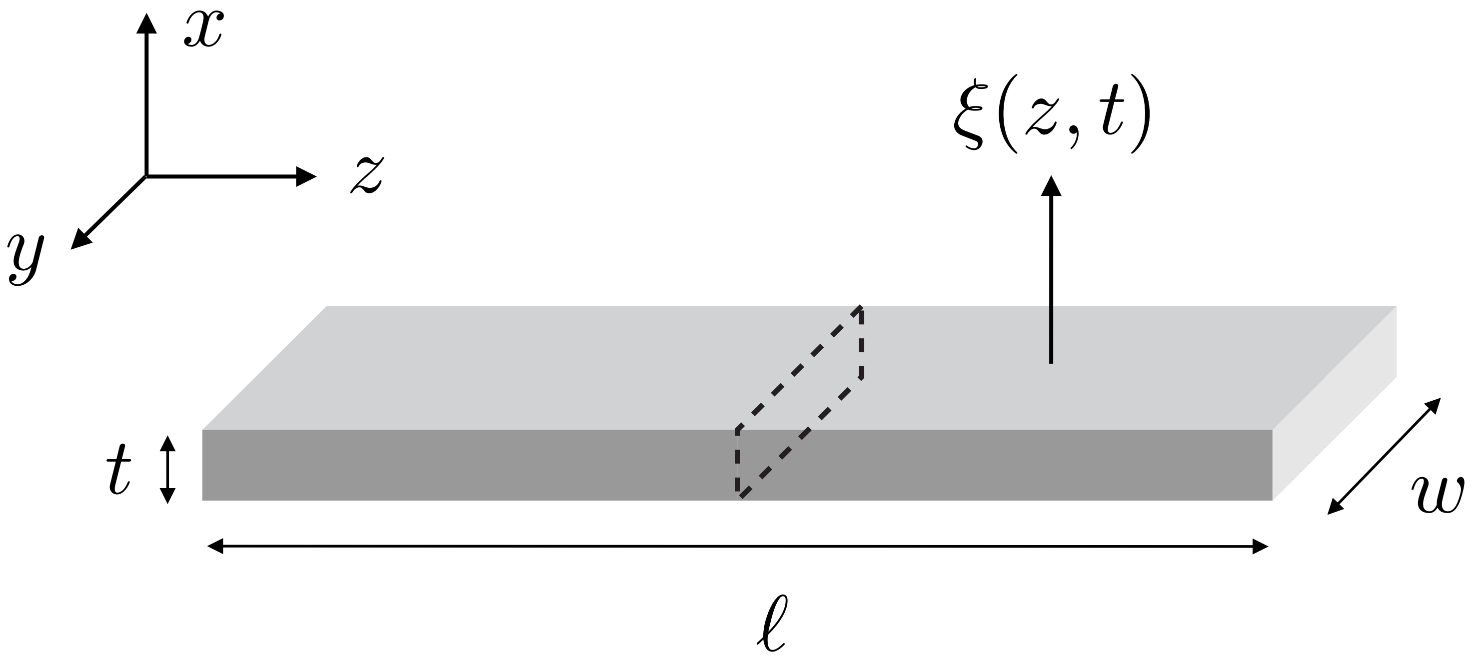
Elasticity theory (thin beam approximation):

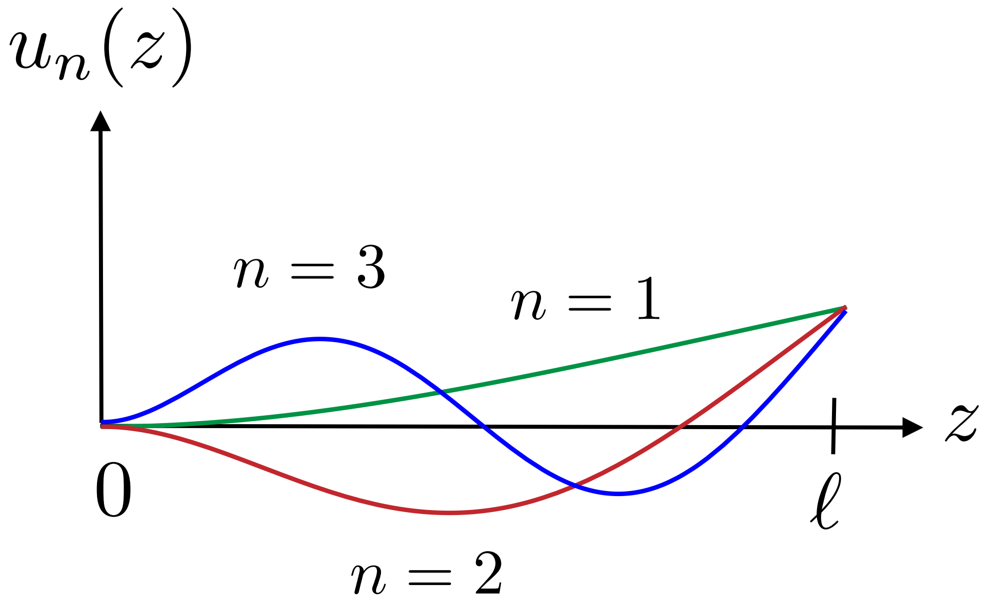
$$\rho A \frac{\partial^2}{\partial t^2}\xi(z,t) = -EI\frac{\partial^4}{\partial z^4}\xi(z,t)$$

$$I = \int x^2 dA = w\frac{t^3}{12}$$

(A ... beam cross section, ρ ... density, E ... Youngs modulus)

Mechanical resonators





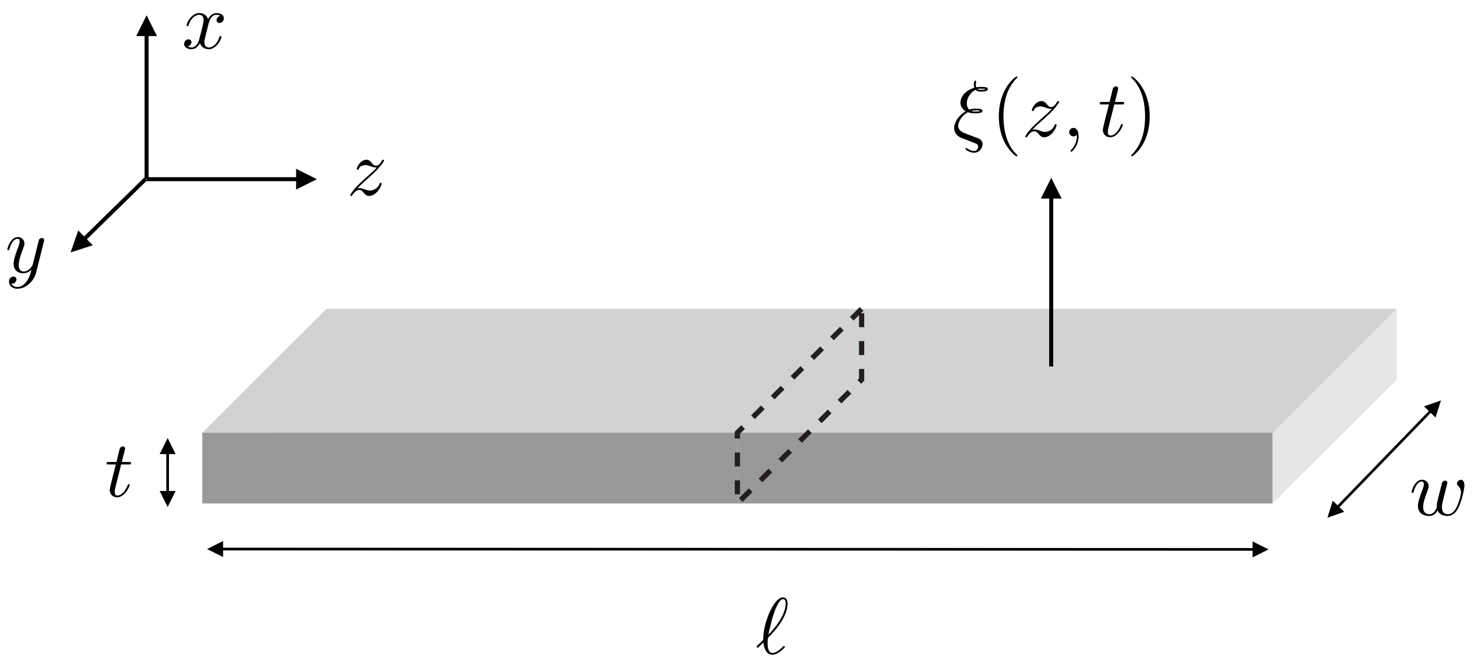
Solutions: $\xi(z,t) \sim u_n(z)e^{i\omega_n t}$

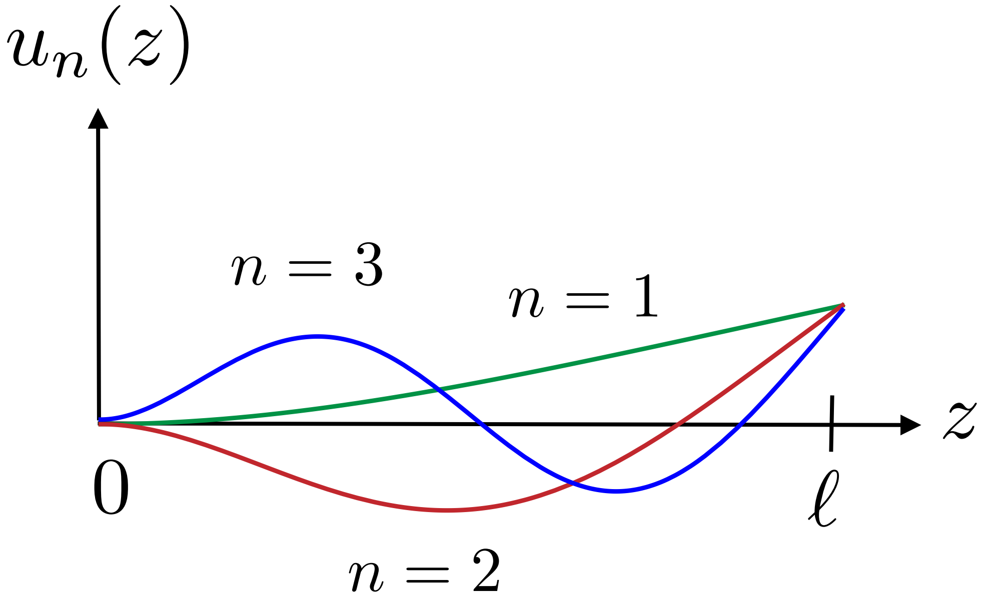
$$u_n(z) = A\cos(k_n z) + B\sin(k_n z) + C\cosh(k_n z) + D\sinh(k_n z)$$

mode frequencies:

$$\omega_n = k_n^2\sqrt{\frac{EI}{\rho A}}$$

Mechanical resonators





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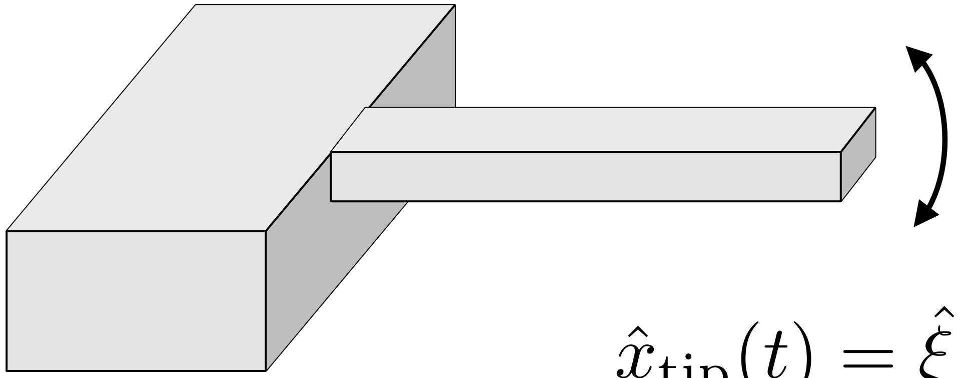
mode frequencies:

$$\omega_n = k_n^2 \sqrt{\frac{EI}{\rho A}}$$

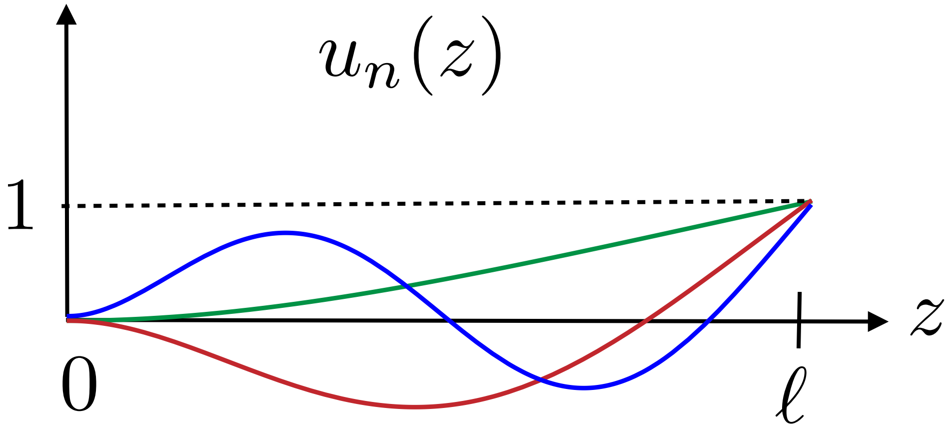
boundary conditions (singly clamped beam):

$$\Rightarrow \quad k_n \ell \simeq 1.875, 4.694, 7.855, \ldots$$

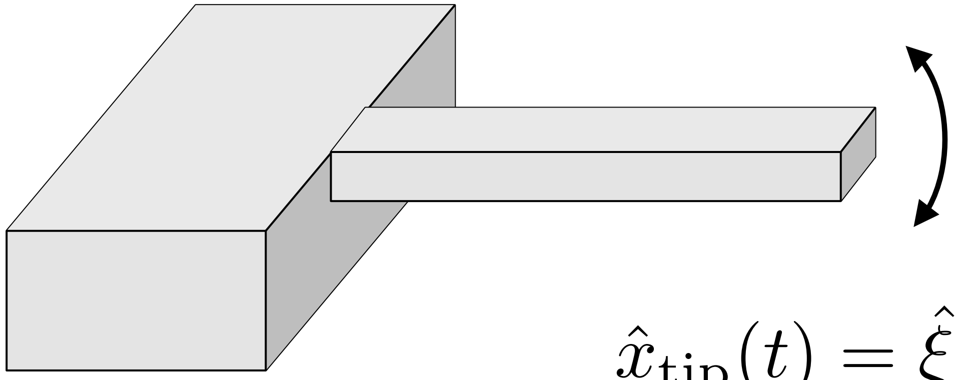
Quantized mechanical resonators



$$\hat{x}_{\mathrm{tip}}(t) = \hat{\xi}(z = \ell, t)$$



Quantized mechanical resonators



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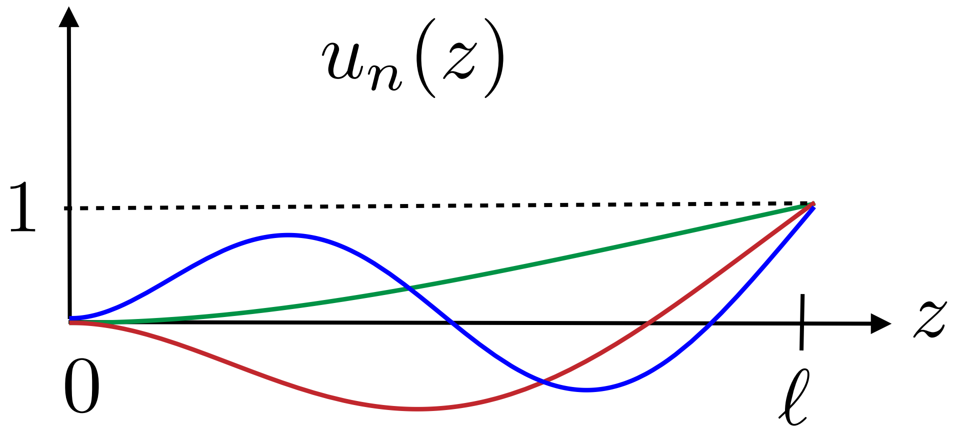
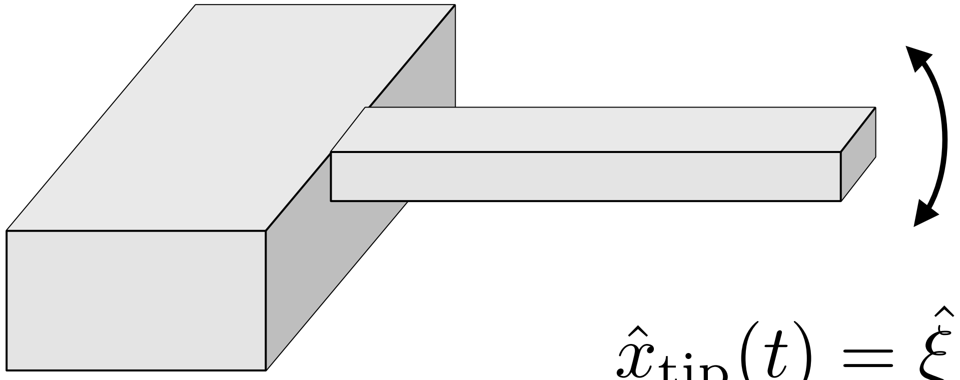


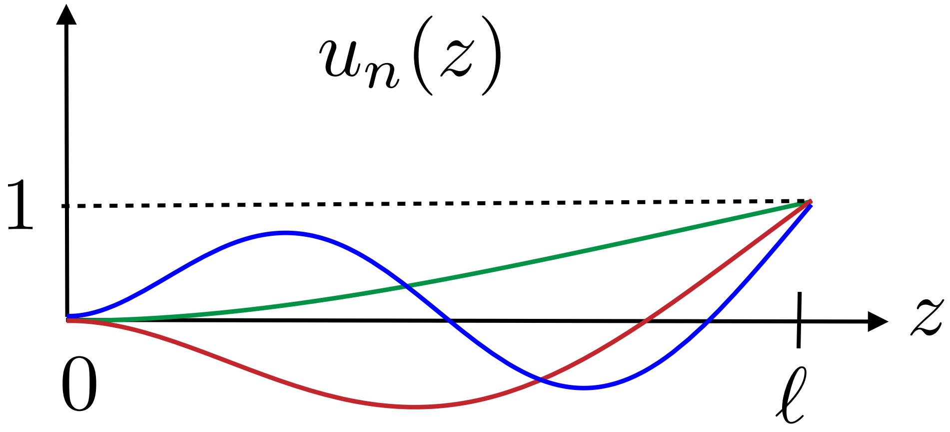
I) Lagrangian:

$$L = \int_0^l dz \left[\frac{\rho A}{2} \left(\frac{\partial \xi}{\partial t} \right)^2 - \frac{EI}{2} \left(\frac{\partial^2 \xi}{\partial z^2} \right)^2 \right]$$

Quantized mechanical resonators



$$\hat{x}_{\rm tip}(t) = \hat{\xi}(z=\ell,t)$$



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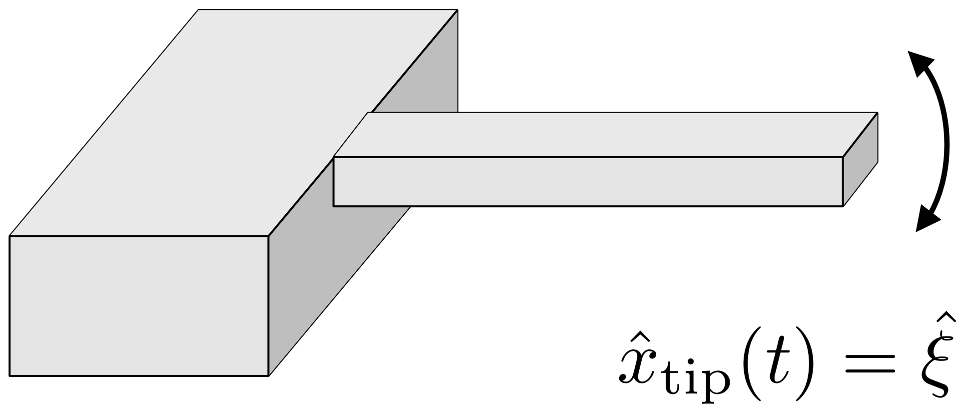
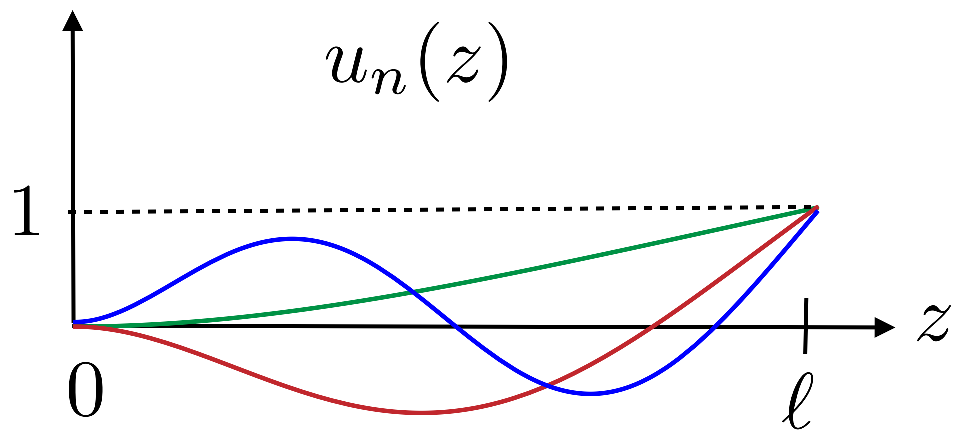
Eigenmode expansion: $\xi(z,t) = \sum_n q_n(t) u_n(z)$

$$\boxed{u_n(z=\ell) = 1}$$

Quantized mechanical resonators



$$\hat{x}_{\mathrm{tip}}(t) = \hat{\xi}(z = \ell, t)$$



I) Lagrangian:

$$L = \int_0^l dz \left[\frac{\rho A}{2} \left(\frac{\partial \xi}{\partial t} \right)^2 - \frac{EI}{2} \left(\frac{\partial^2 \xi}{\partial z^2} \right)^2 \right]$$

Eigenmode expansion: $\xi(z,t) = \sum_n q_n(t) u_n(z)$

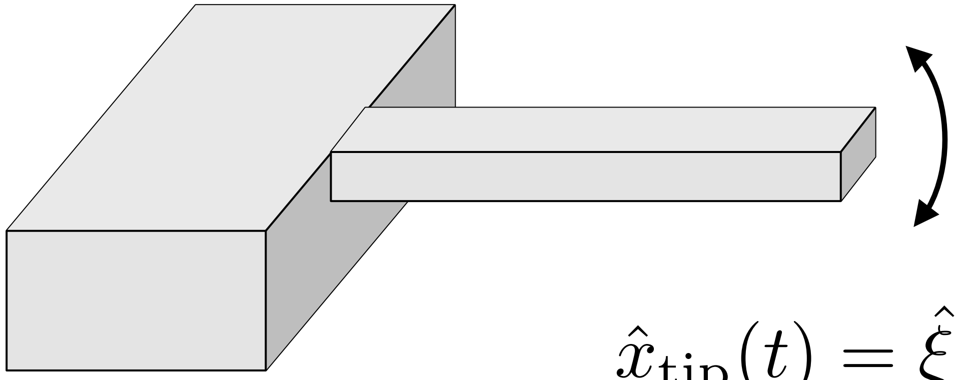
$$\boxed{u_n(z = \ell) = 1}$$

$$\boxed{L = \sum_n \frac{m_{\mathrm{eff}}}{2} \dot{q}_n^2 - \frac{m_{\mathrm{eff}} \omega_n^2}{2} q_n^2 \,,}$$

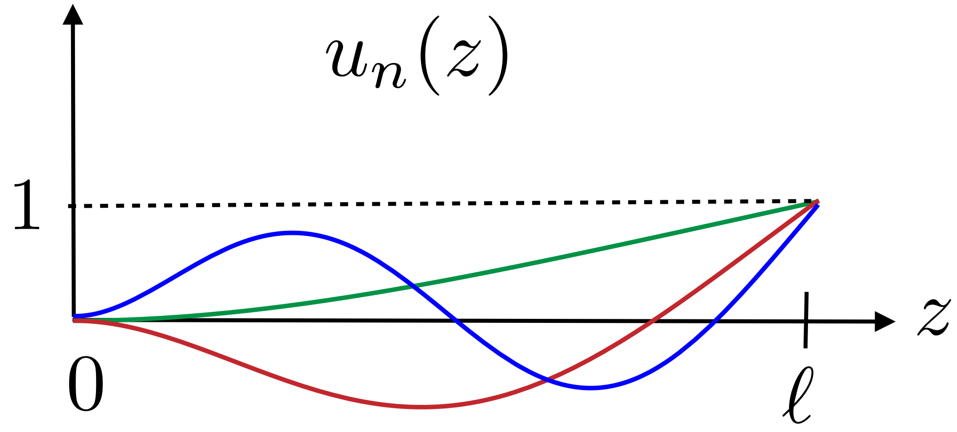
$$m_{\mathrm{eff}} = \rho A \int_0^\ell dz \, |u_n(z)^2| = \frac{m}{4}$$

(effective mass)

Quantized mechanical resonators



$$\hat{x}_{\text{tip}}(t) = \hat{\xi}(z = \ell, t)$$



I) Lagrangian:

$$L = \sum_n \frac{m_{\text{eff}}}{2}\dot{q}_n^2 - \frac{m_{\text{eff}}\omega_n^2}{2}q_n^2$$

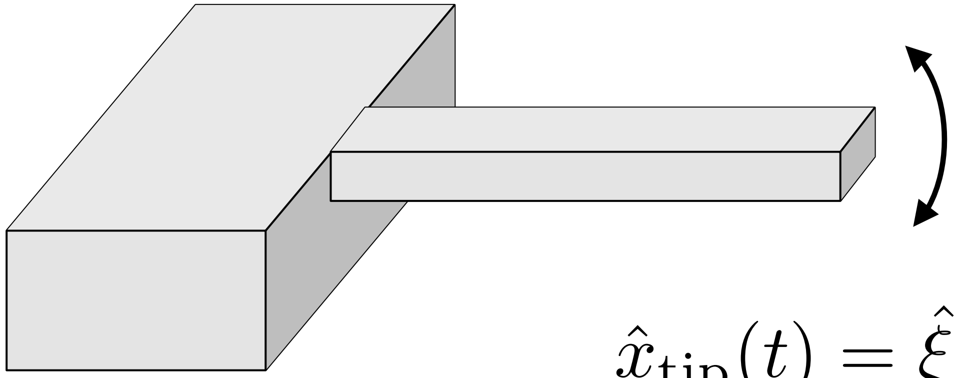
independent harmonic oscillators !

II) Canonical quantization / Hamiltonian:

$$q_n \;\rightarrow\; \hat{q}_n, \qquad p_n = m_{\text{eff}}\dot{q}_n \;\rightarrow\; \hat{p}_n, \qquad [\hat{q}_n, \hat{p}_n] = i\hbar$$

$$H = \sum_n \frac{\hat{p}_n^2}{2m_{\text{eff}}} + \frac{1}{2}m_{\text{eff}}\omega_n^2\hat{q}_n^2 = \sum_n \hbar\omega_n a_n^\dagger a_n \qquad [a_n, a_m^\dagger] = \delta_{n,m}$$

Quantized mechanical resonators



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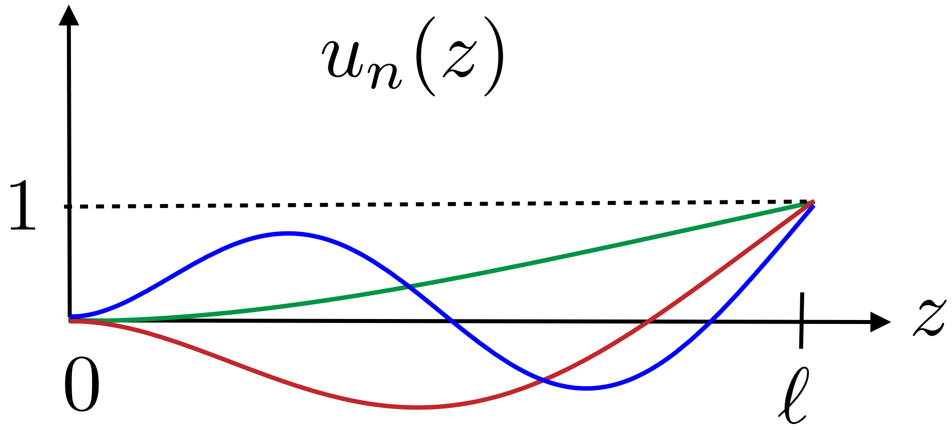
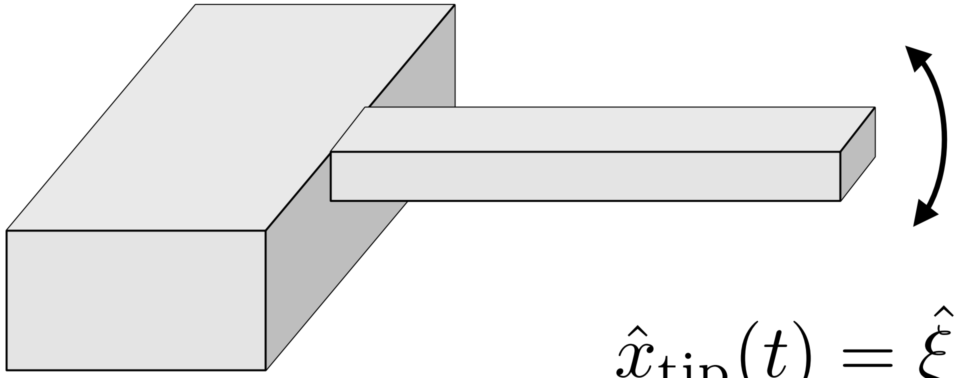


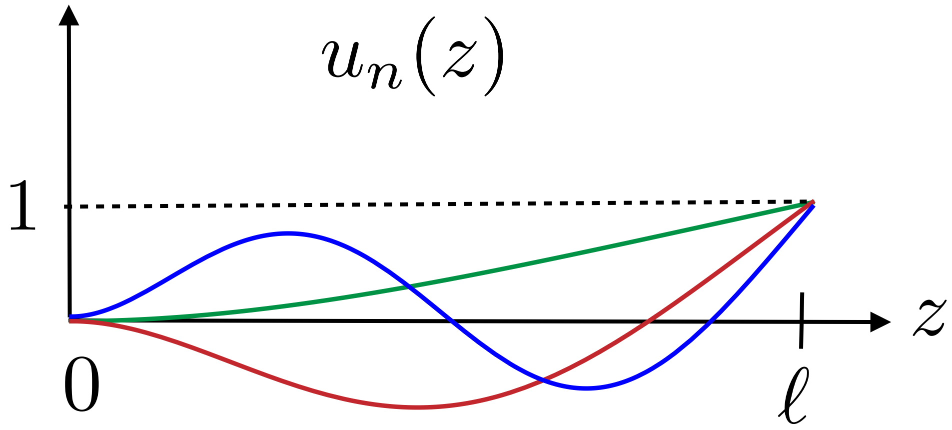
- ***quantized displacement of the tip:***

$$\hat{x}_{\text{tip}}(t) = \sum_n u_n(\ell)\hat{q}_n(t) = \sum_n \sqrt{\frac{\hbar}{2m_{\text{eff}}\omega_n}}\left(a_n e^{-i\omega_n t} + a_n^\dagger e^{i\omega_n t}\right)$$

Quantized mechanical resonators



$$\hat{x}_{\rm tip}(t) = \hat{\xi}(z=\ell,t)$$



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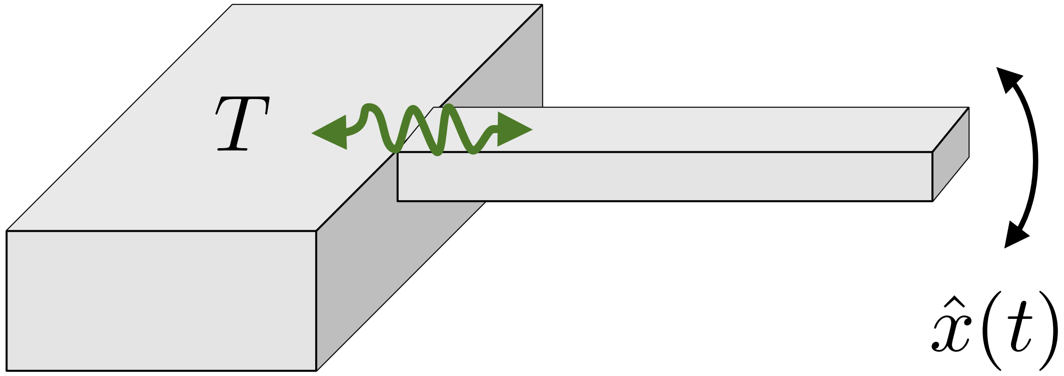
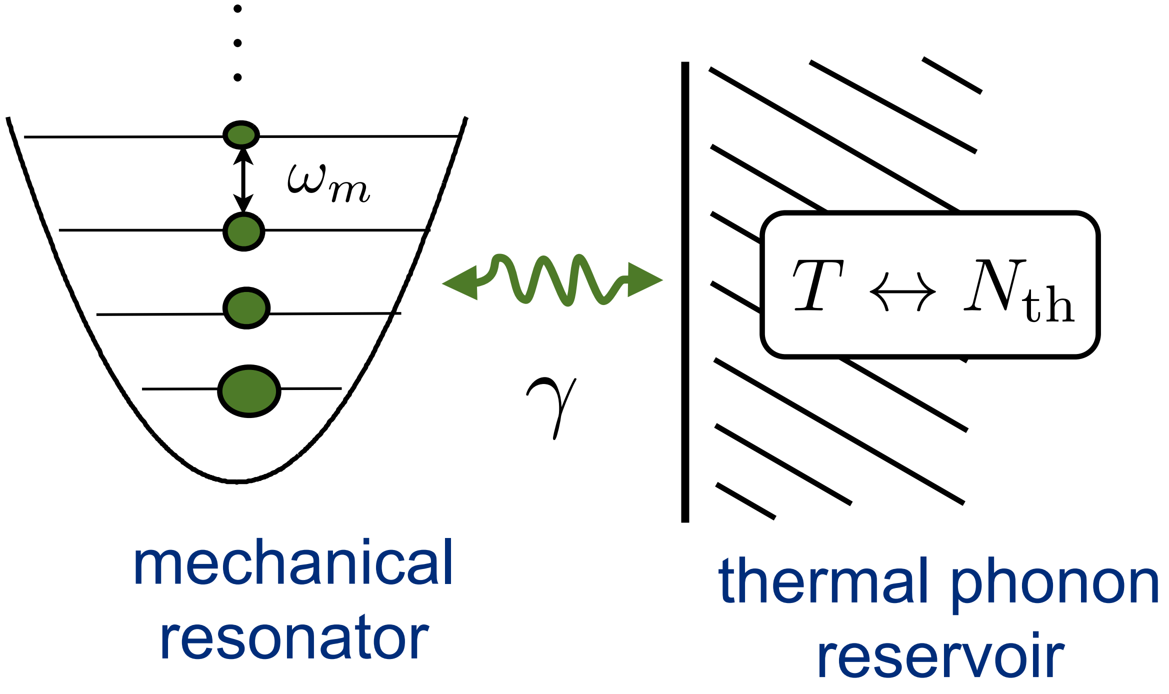
- ***single mode approximation:*** $(\omega_m \equiv \omega_1,\ \omega_{n>1} > \omega_1)$

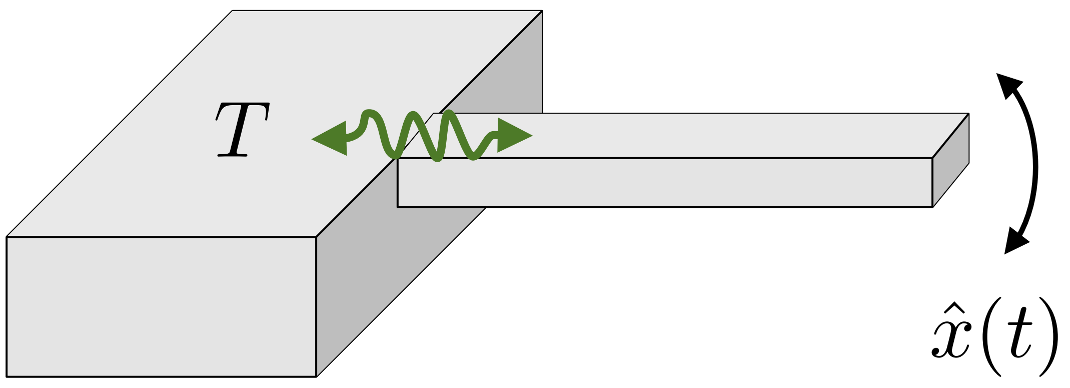
$$H = \hbar\omega_m a^\dagger a, \qquad \hat{x}_{\rm dip} = \sqrt{\frac{\hbar}{2m_{\rm eff}\omega_m}}\left(a + a^\dagger\right)$$

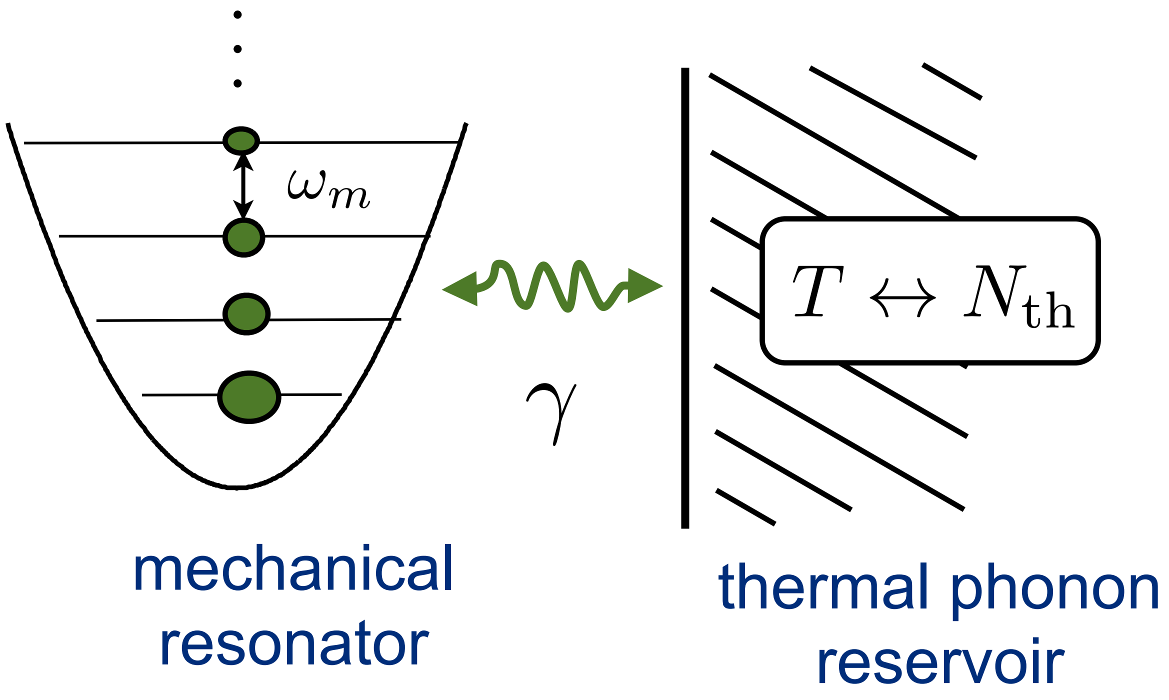
Coupling to other modes





Coupling to other modes





The (weak) coupling to all other modes can be taken into account by a master equation:

$$\dot{\rho} = \frac{\gamma}{2}(N_{\text{th}}+1)\left(2a\rho a^\dagger + a^\dagger a\rho + \rho a^\dagger a\right) + \frac{\gamma}{2}N_{\text{th}}\left(2a^\dagger\rho a + aa^\dagger\rho + \rho aa^\dagger\right)$$

$$\gamma = \frac{\omega_m}{Q}$$

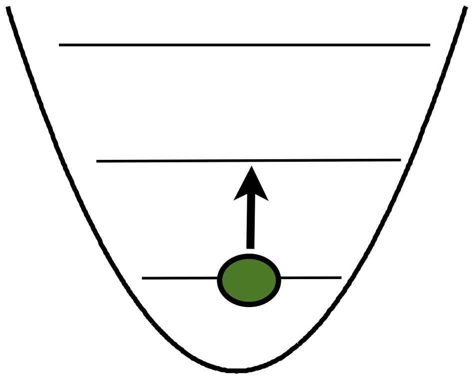
(mechanical damping rate)

$$N_{\text{th}} = \frac{1}{e^{\hbar\omega_m/k_B T} - 1}$$

(thermal occupation number)

Thermalization & mechanical decoherence

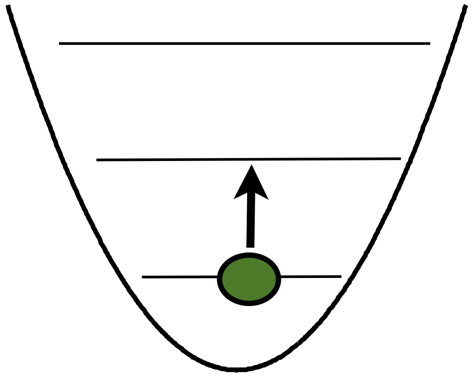
Average phonon occupation number:



$$\partial_t \langle a^\dagger a \rangle = -\gamma \langle a^\dagger a \rangle + \gamma N_{th}$$

Thermalization & mechanical decoherence

Average phonon occupation number:



$$\partial_t \langle a^\dagger a\rangle = -\gamma\langle a^\dagger a\rangle + \gamma N_{th}$$

When the mechanical mode is cooled to the ground state, it takes a time $\tau_{\rm th} \sim (\gamma N_{\rm th})^{-1}$ *to populate it again with 1 phonon.*

⇒ *mechanical decoherence rate:* $\Gamma_m := \gamma(N_{\rm th}+1) \approx k_B T/\hbar Q$

Thermalization & mechanical decoherence

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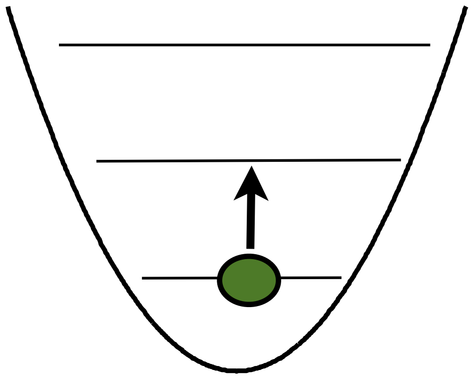

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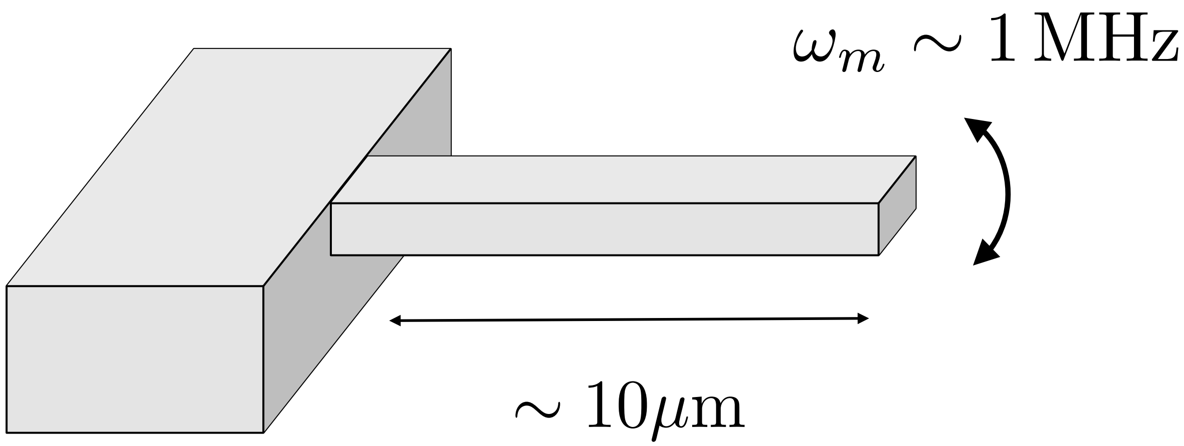
⇒ *mechanical decoherence rate:* $\Gamma_m := \gamma(N_{\rm th}+1) \approx k_B T/\hbar Q$

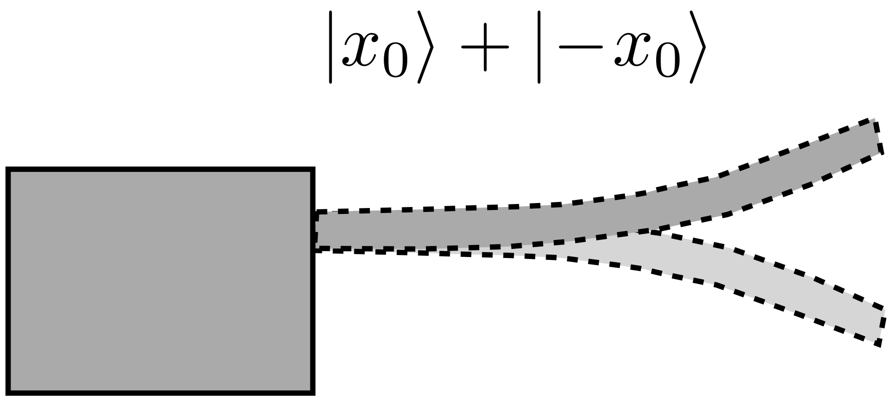
$\Gamma_m/(2\pi)$	$Q = 10^5$	$Q = 10^6$
$T = 4\,\mathrm{K}$	$\sim 1\,\mathrm{MHz}$	$\sim 100\,\mathrm{kHz}$
$T = 100\,\mathrm{mK}$	$\sim 20\,\mathrm{kHz}$	$\sim 2\,\mathrm{kHz}$

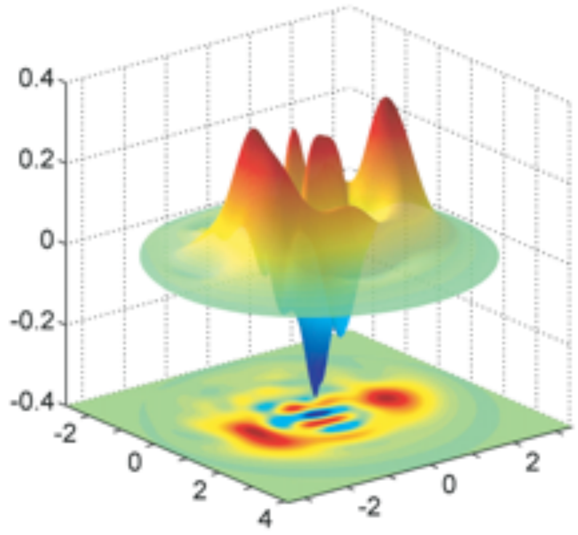
Resonators & spin qubits

Quantum control of macroscopic objects



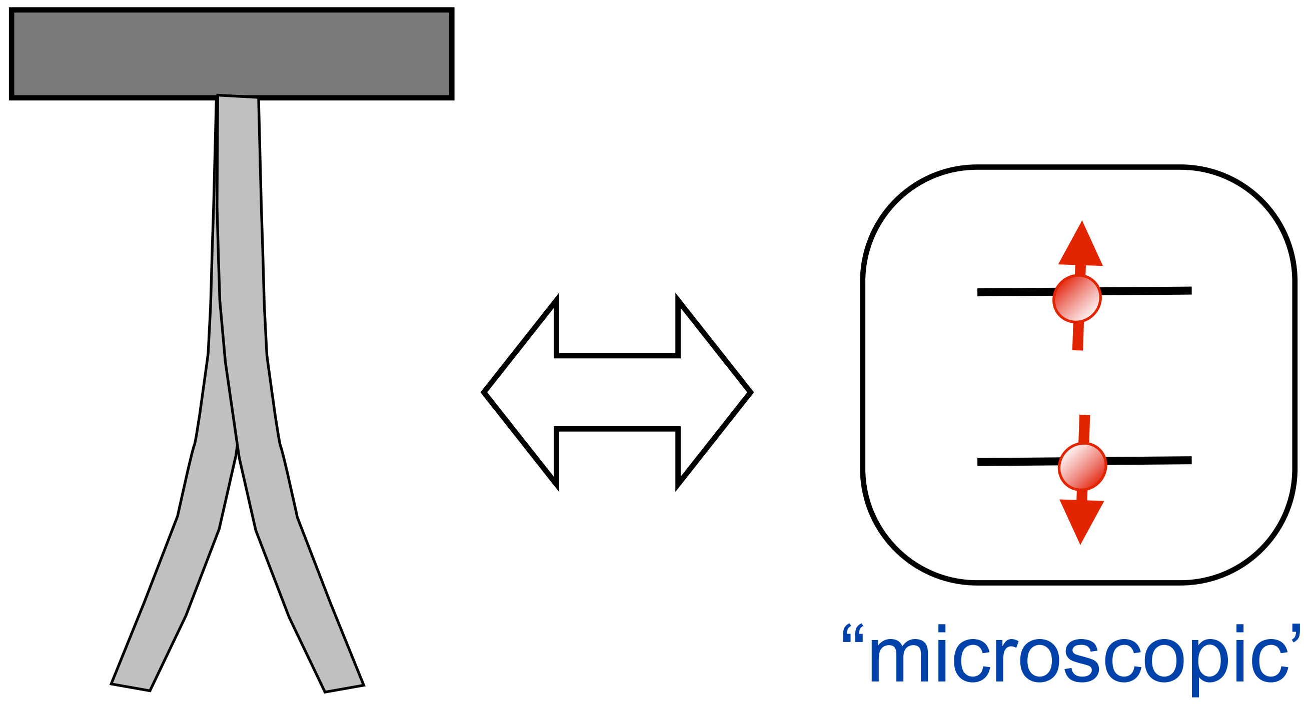
?





Quantum control of macroscopic objects

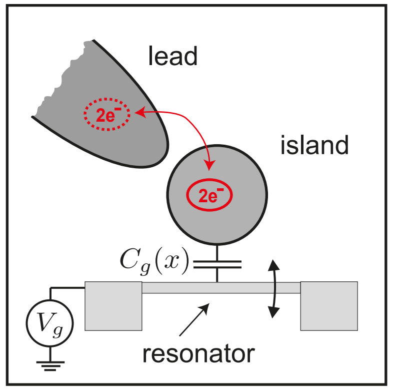
Idea:



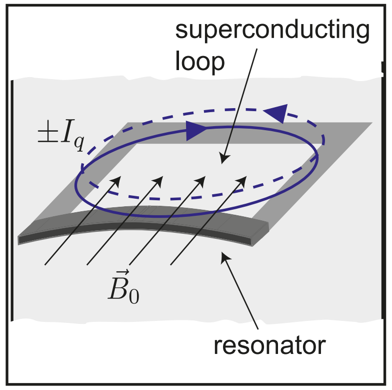
“microscopic”

“macroscopic”

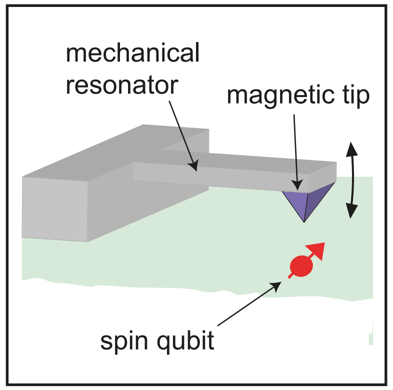
Mechanical resonators & solid state qubits



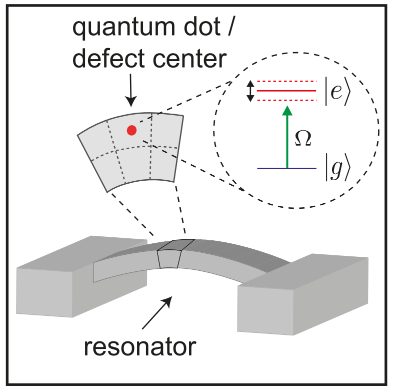
- *Electrostatic coupling to charge qubits.*



- *Magnetic (Lorentz force) coupling to flux qubits.*



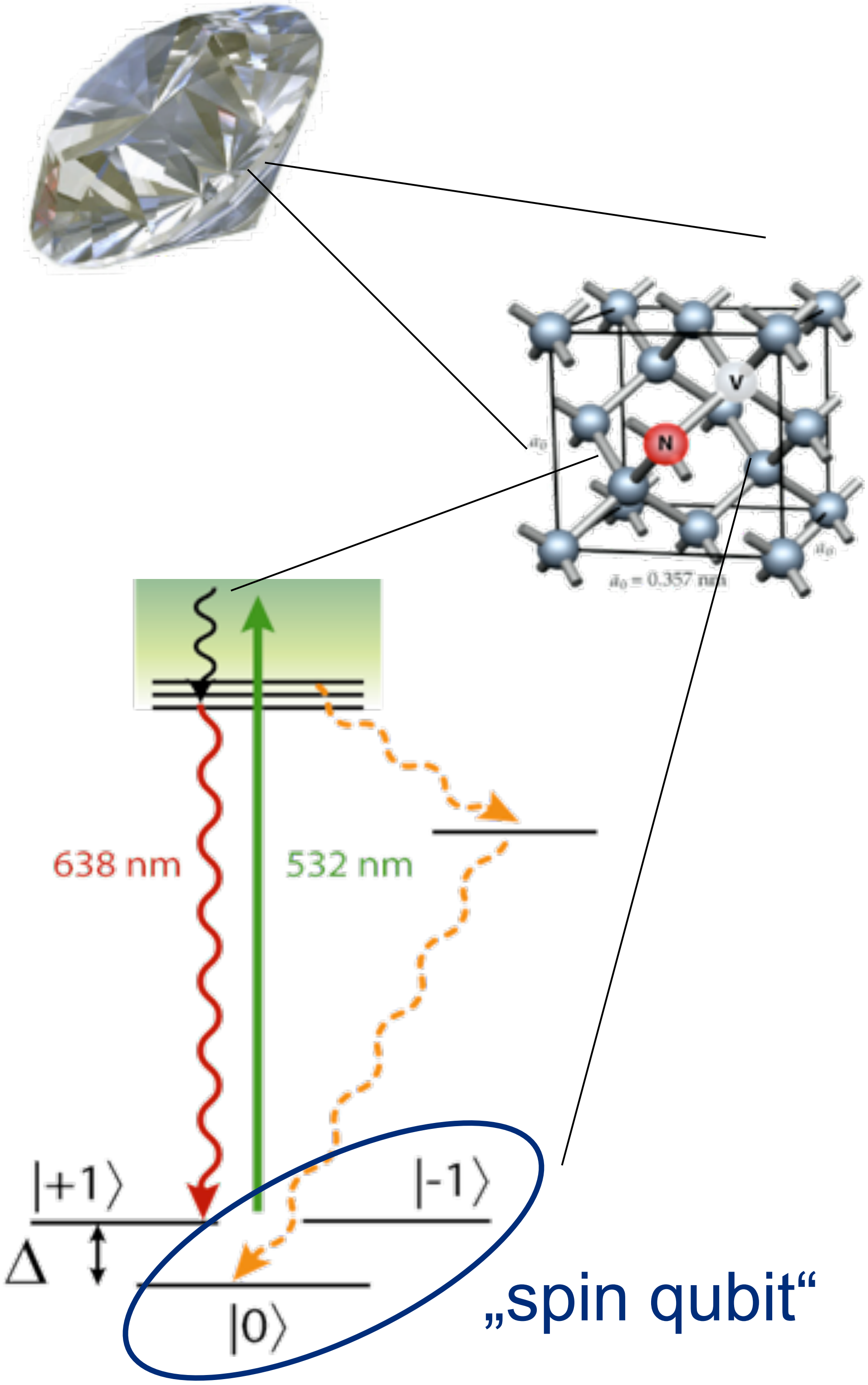
- *Magnetic gradient coupling to spin qubits.*



- *Strain coupling to quantum dots / defect centers.*

see e.g.: *P. Treutlein, C. Genes, K. Hammerer, M. Poggio, PR, arXiv:1210.4151*

NV centers in diamond



„Spin qubit“:

- long coherence (T_2 ~10ms @ T=300 K)
- ESR (=microwave) control

„Quantum optical qubit“:

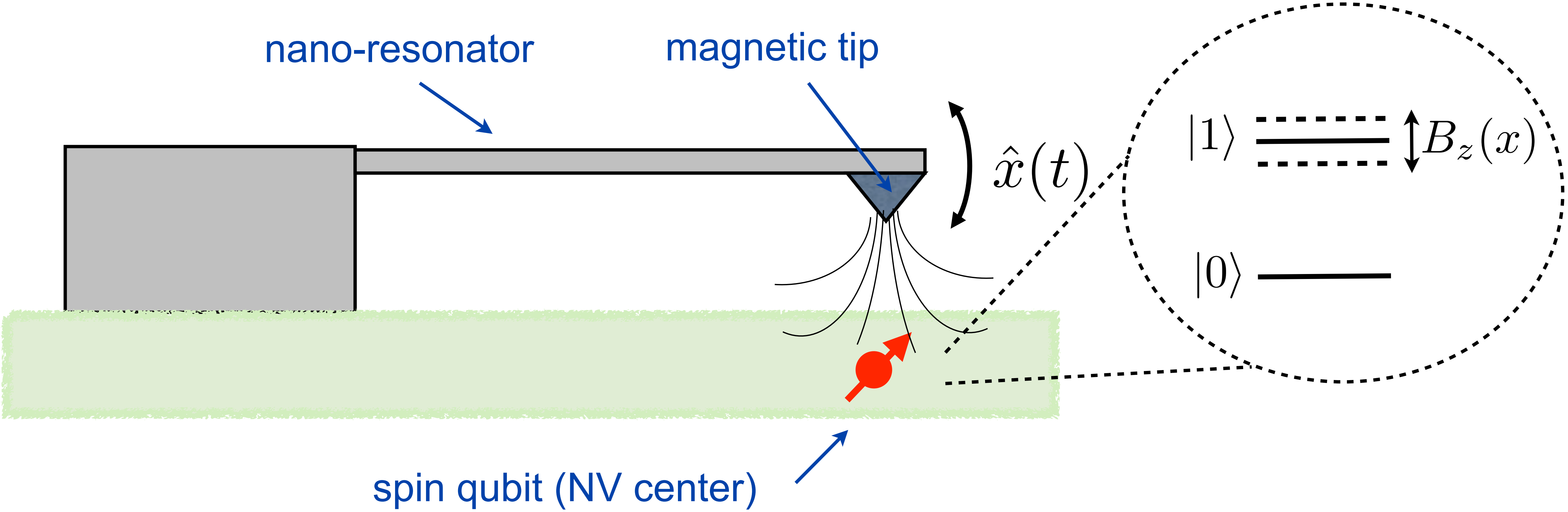
- state preparation (optical pumping)
- state detection (cycling transitions)

„Solid state qubit“:

- stable / no trapping requirements
- localized < 50 nm

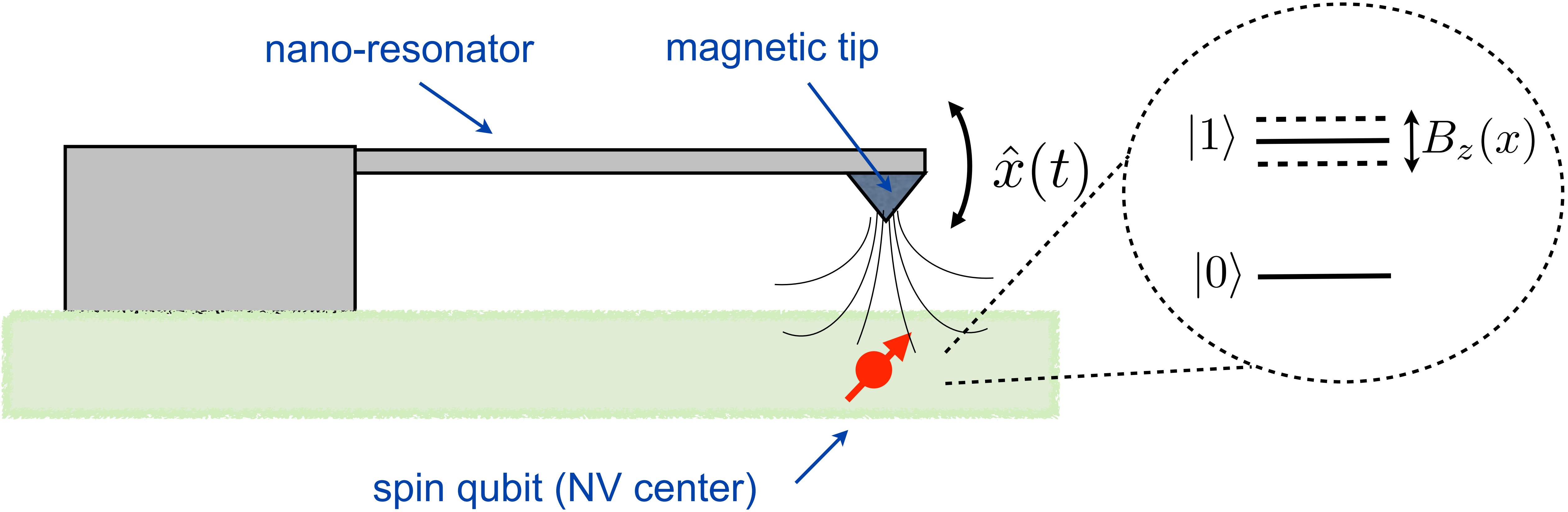
⇒ ***“Nature’s own trapped ion”***

Magnetic spin-resonator coupling



MRFM (D. Rugar, IBM), BEC-resonator coupling (P. Treutlein), ...

Magnetic spin-resonator coupling



magnetic coupling: $H_{\text{int}} = \lambda(a + a^\dagger)|1\rangle\langle 1|$

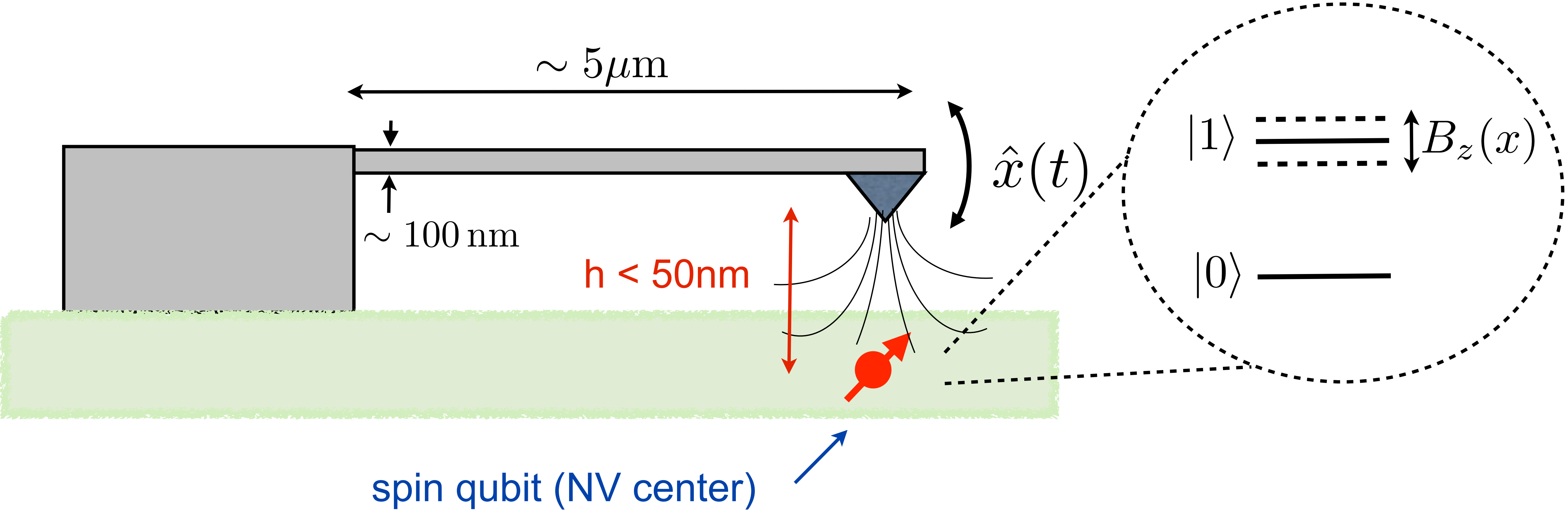
$$\lambda = g_s \mu_B a_0 \nabla B / \hbar$$

zero point motion

magnetic field gradient

"Zeeman shift per vibrational quanta"

Magnetic spin-resonator coupling



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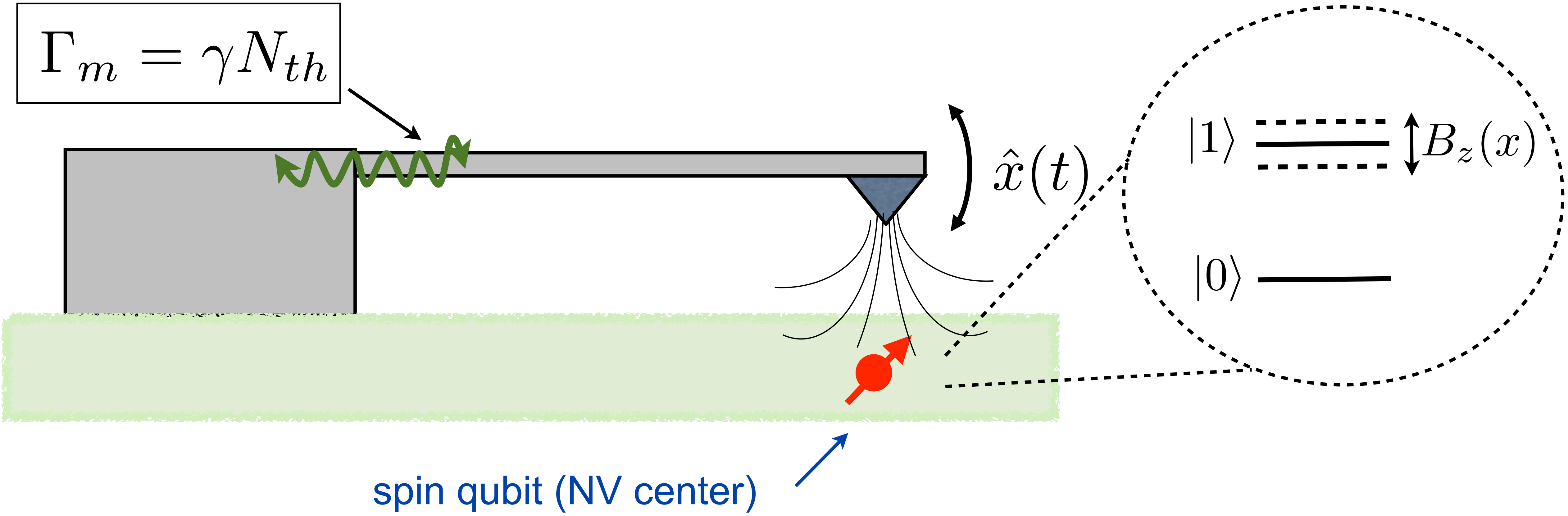
zero point motion

magnetic field gradient

$$\lambda \approx 100\,\text{kHz}$$

“Zeeman shift per vibrational quanta”

Magnetic spin-resonator coupling



coherent coupling

$$\lambda \approx 100\,\mathrm{kHz} \gg$$

spin dephasing

$$1/T_2,$$

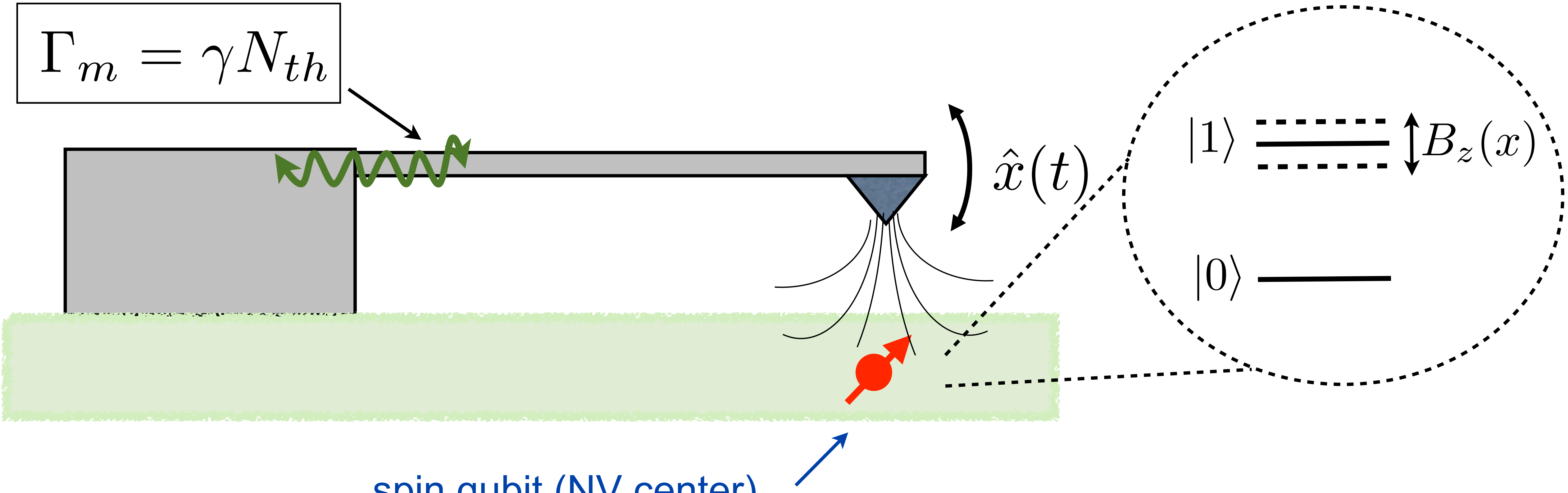
$$(<1\,\mathrm{kHz})$$

motional dephasing

$$\Gamma_m = k_B T/\hbar Q$$

$$(1-10\,\mathrm{kHz})$$

Magnetic spin-resonator coupling



spin qubit (NV center)

coherent coupling → $\lambda \approx 100\,\mathrm{kHz}$

$\gg$

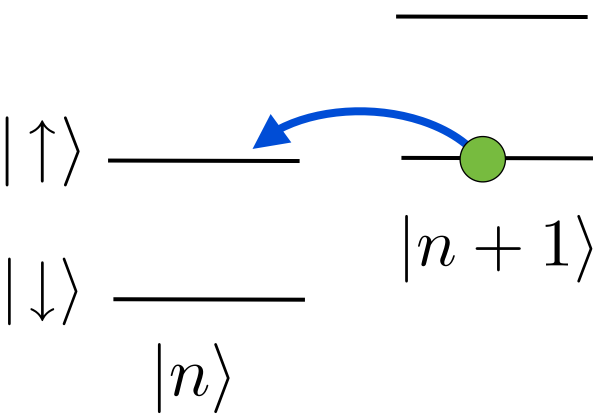
spin dephasing → $1/T_2,$ $(<1\,\mathrm{kHz})$

motional dephasing → $\Gamma_m = k_B T/\hbar Q$ $(1-10\,\mathrm{kHz})$

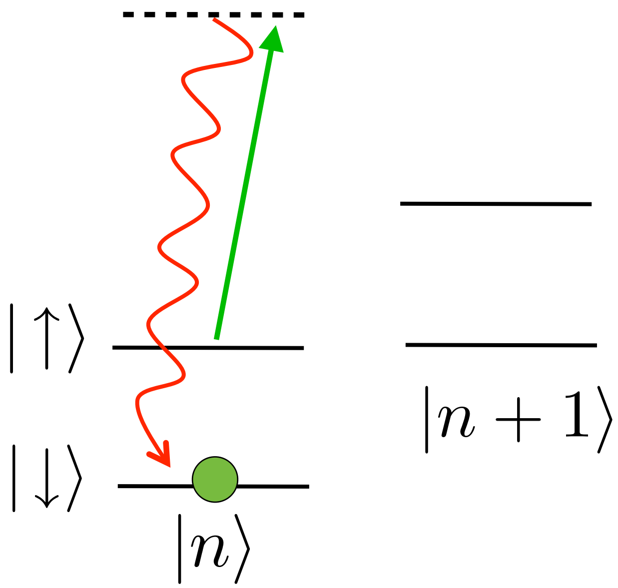
⇒ *strong coupling conditions !*

Quantum control of mechanical motion

- **Cooling / state preparation:**



motional quanta → spin excitation

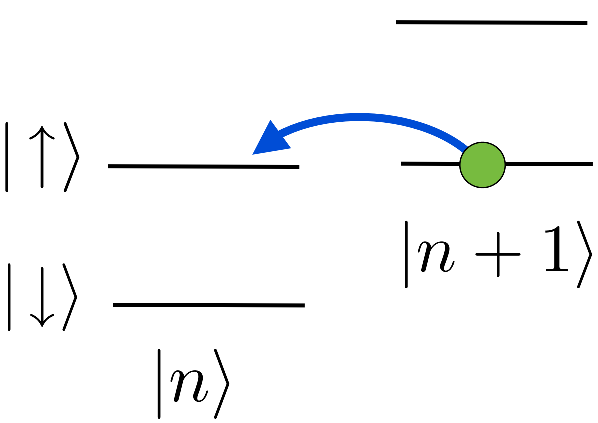


optical pumping

PR, P. Cappellaro, Gurudev Dutt, L. Jiang, J. Maze, M. Lukin, Phys. Rev. B **79**, 041302 (2009).

Quantum control of mechanical motion

- **Cooling / state preparation:**



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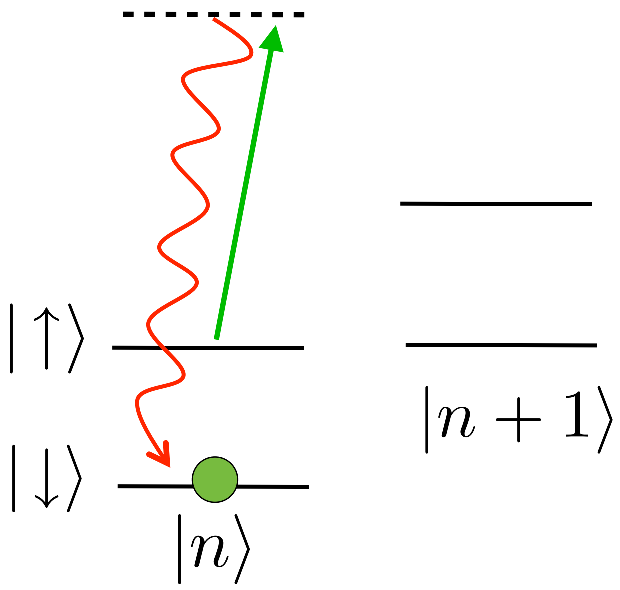
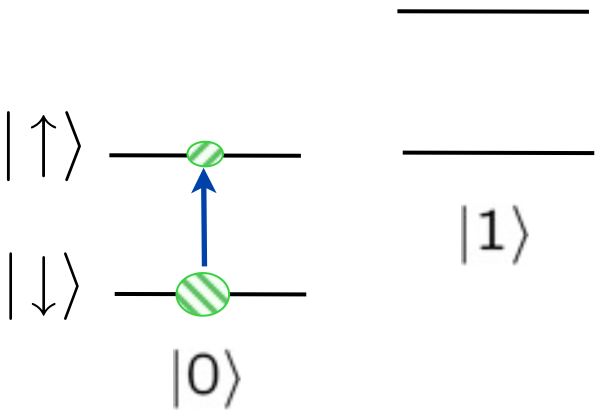
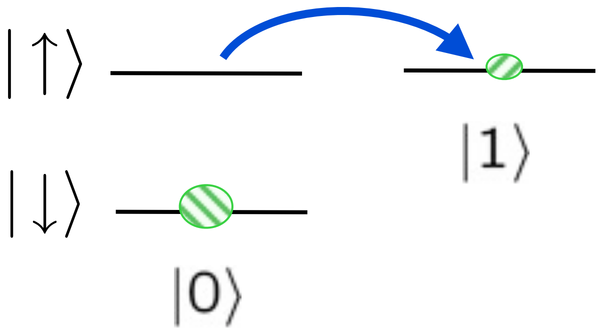


optical pumping

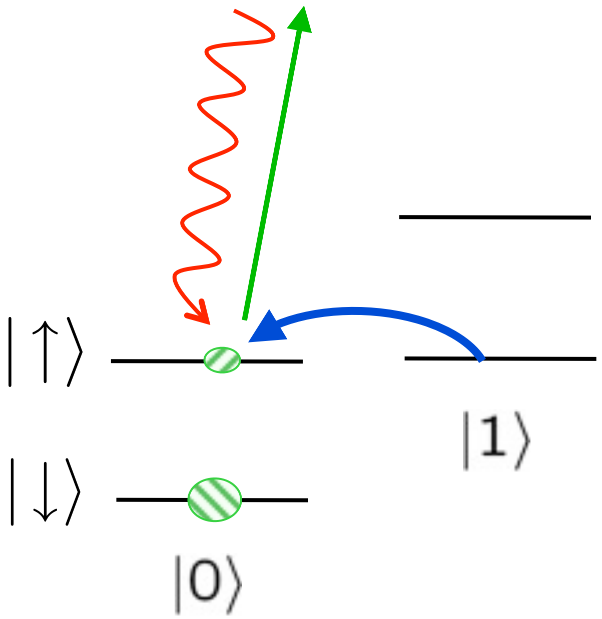
- **Quantum control & readout:**



spin superposition



motional superposition

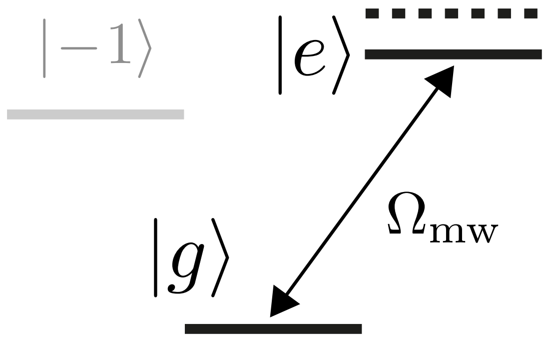


map back and
detect spin state

PR, P. Cappellaro, Gurudev Dutt, L. Jiang, J. Maze, M. Lukin, Phys. Rev. B **79**, 041302 (2009).

Probing macroscopic superpositions

Magnetometry

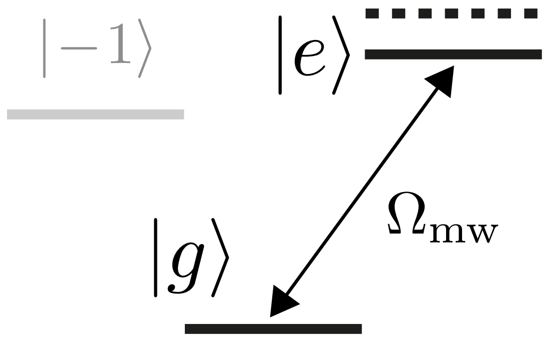


$$H = \Delta_B |e\rangle\langle e| + H_{\mathrm{ESR}}(t)$$

$$\Delta_B = (g_s \mu_B/\hbar) \times \delta B_z$$

How can I measure a small magnetic field δB_z *?*

Magnetometry



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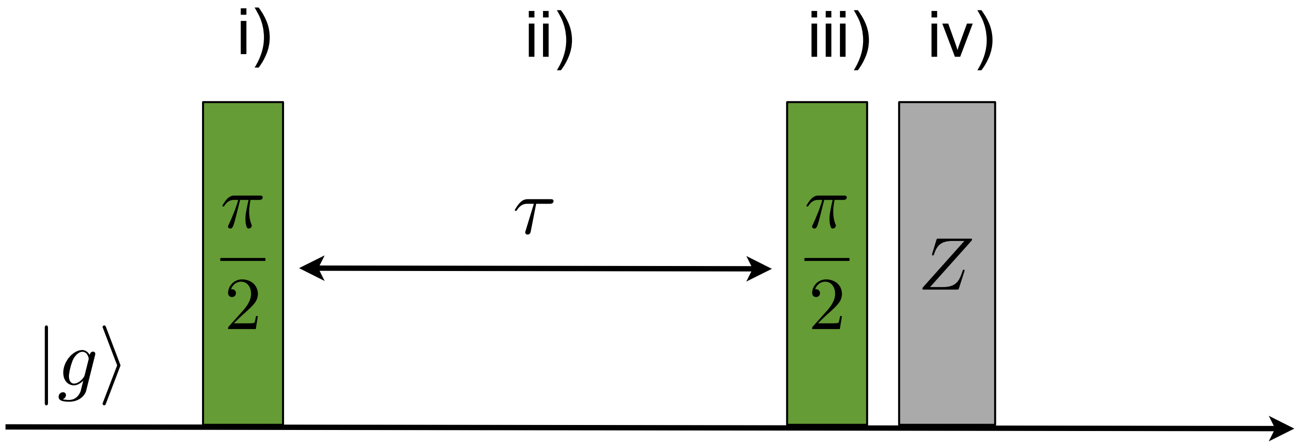
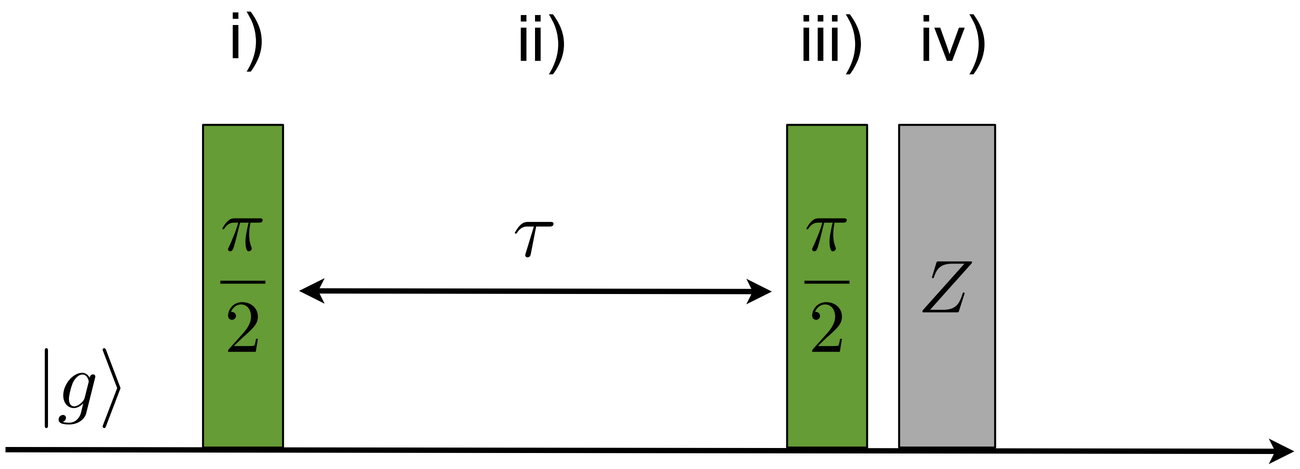
How can I measure a small magnetic field δB_z *?*

⇒ *Ramsey interference measurement:*



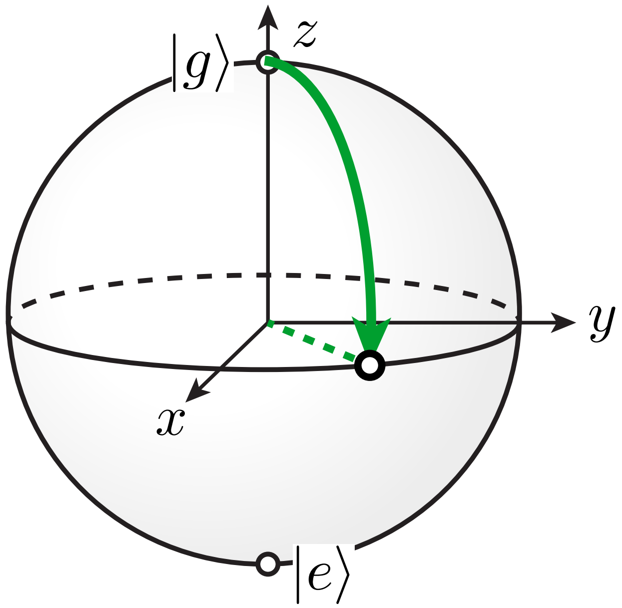
i) apply pi/2 pulse
ii) wait
iii) apply another pi/2 pulse
iv) measure spin populations

Magnetometry

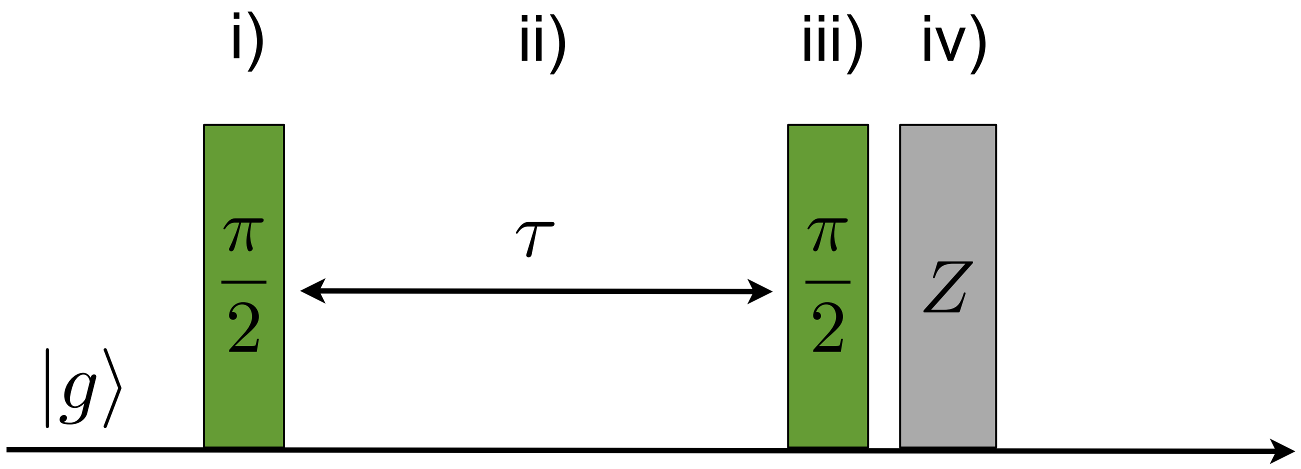


i) $|g\rangle \rightarrow \frac{1}{\sqrt{2}}\left(|g\rangle + e^{i\varphi}|e\rangle\right)$

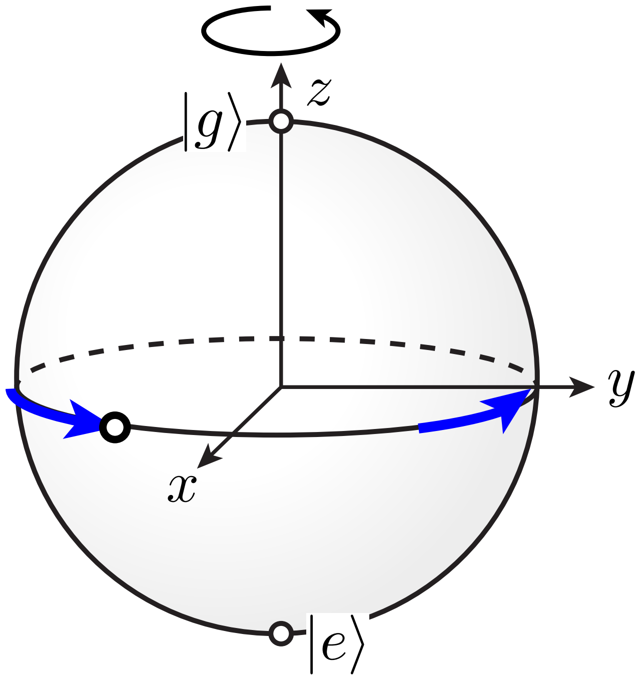
$R_{\pi/2}(\varphi)$



Magnetometry



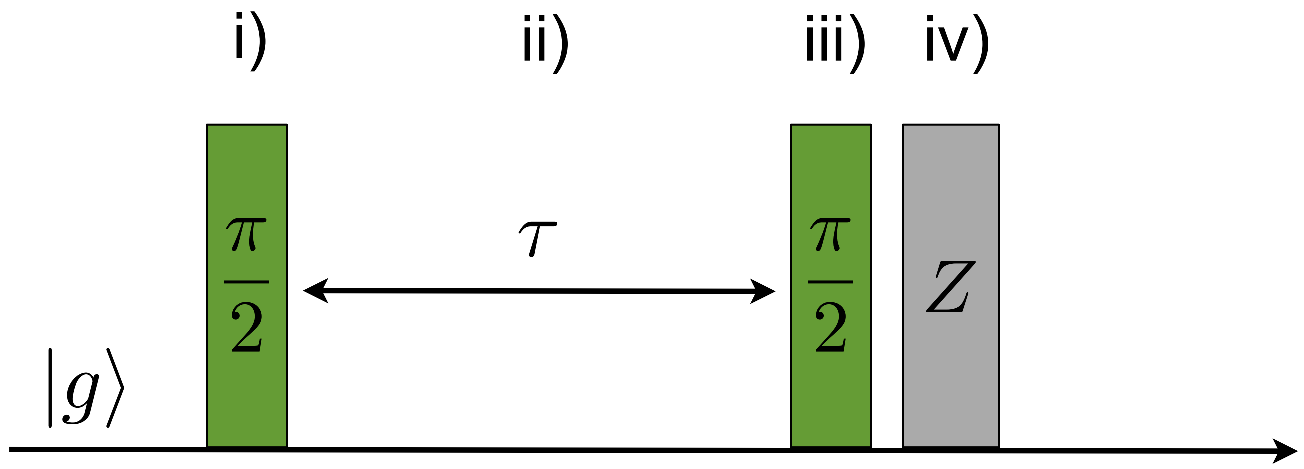
$$H = \Delta_B |e\rangle\langle e|$$



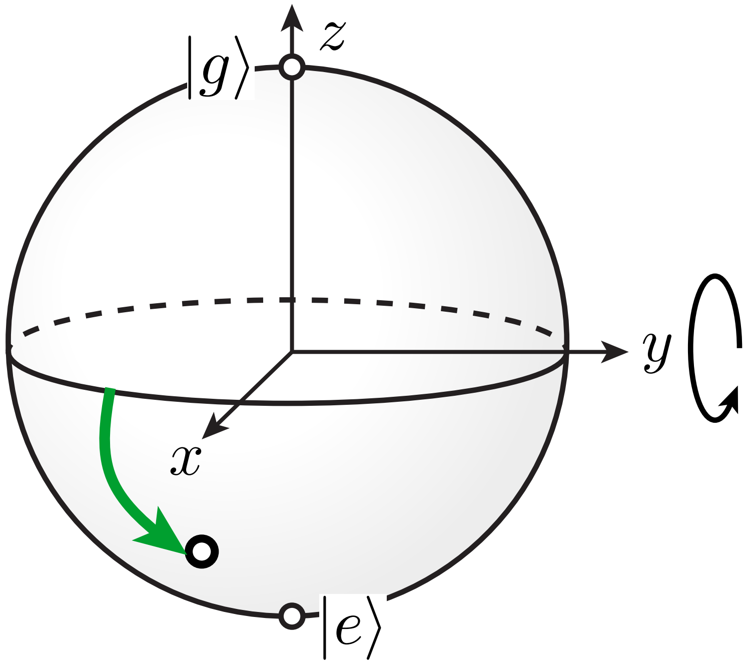
i) $|g\rangle \rightarrow \frac{1}{\sqrt{2}}\left(|g\rangle + e^{i\varphi}|e\rangle\right)$

ii) $\rightarrow \frac{1}{\sqrt{2}}\left(|g\rangle + e^{i\varphi}e^{-i\Delta_B\tau}|e\rangle\right)$

Magnetometry



$$R_{\pi/2}(\varphi = 0)$$



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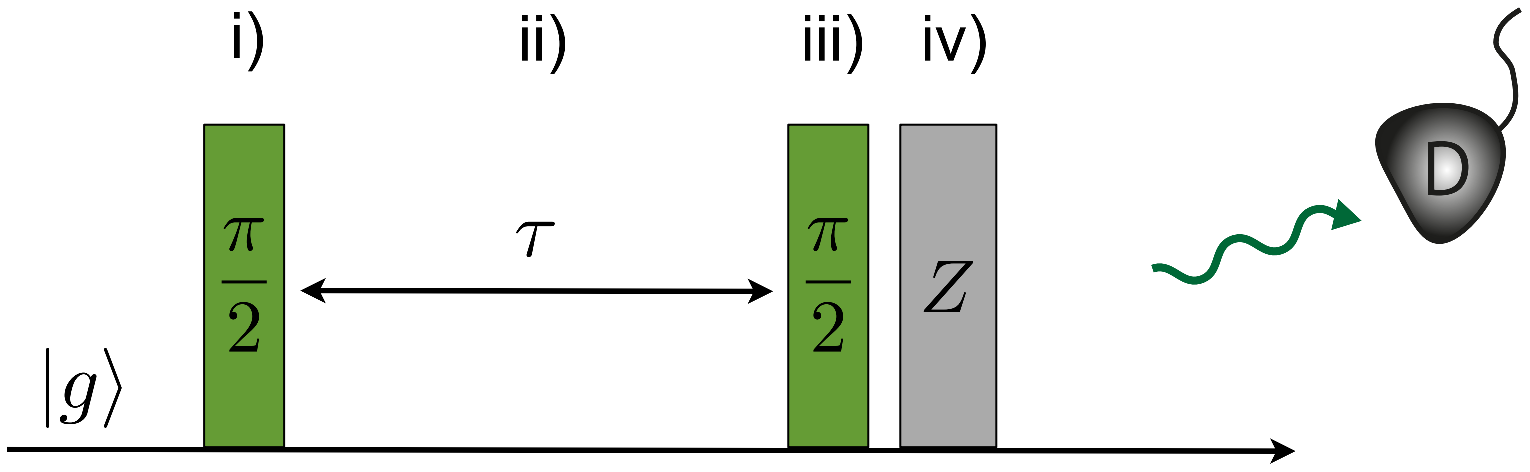
ii) $\rightarrow\; \frac{1}{\sqrt{2}}\left(|g\rangle + e^{i\varphi}e^{-i\Delta_B\tau}|e\rangle\right)$

iii) $\rightarrow\; \frac{1}{2}\left(1 + e^{i(\varphi-\Delta_B\tau)}\right)|g\rangle + \frac{1}{2}\left(1 + e^{i(\varphi-\Delta_B\tau)}\right)|e\rangle$

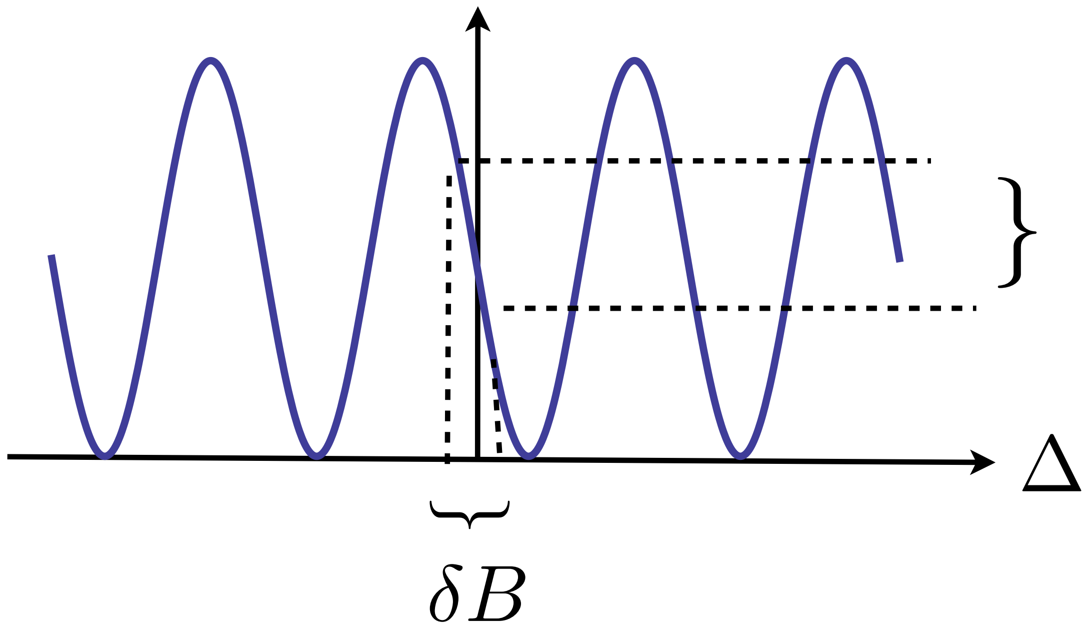
Magnetometry



$|e\rangle : \; Z = +1$

$|g\rangle : \; Z = -1$

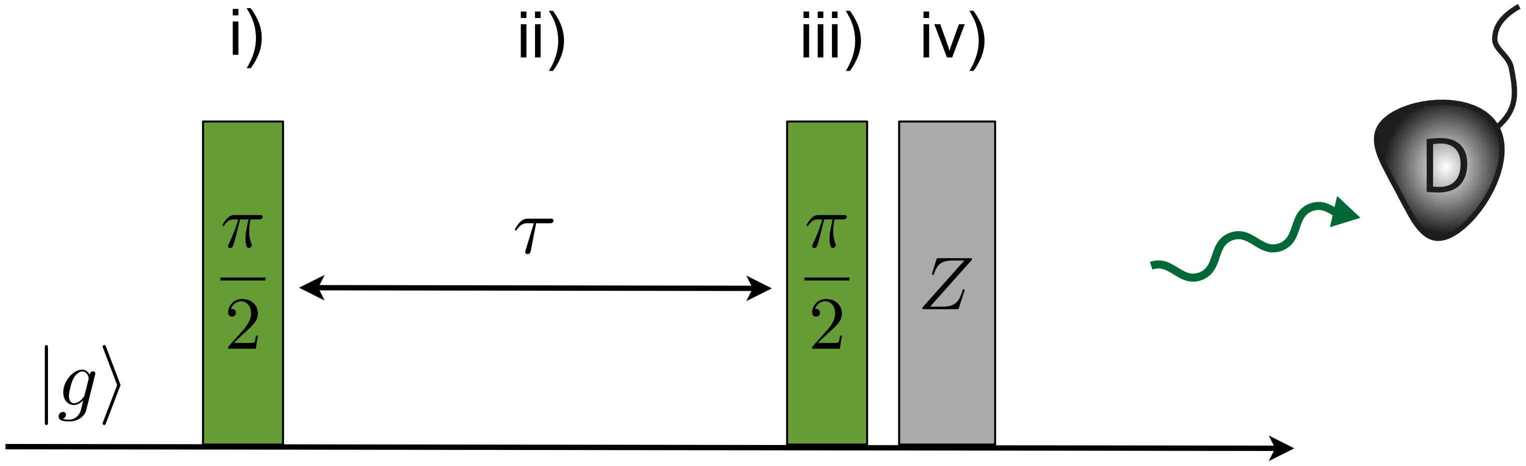
iv) $$p_{\pm} = \frac{1}{2}\left(1 \pm \cos(\varphi - \Delta_B \tau)\right)$$



$$\Big\} \; \frac{g_s \mu_B \tau}{\hbar} \times \delta B$$

$$\tau \approx T_2$$

Magnetometry



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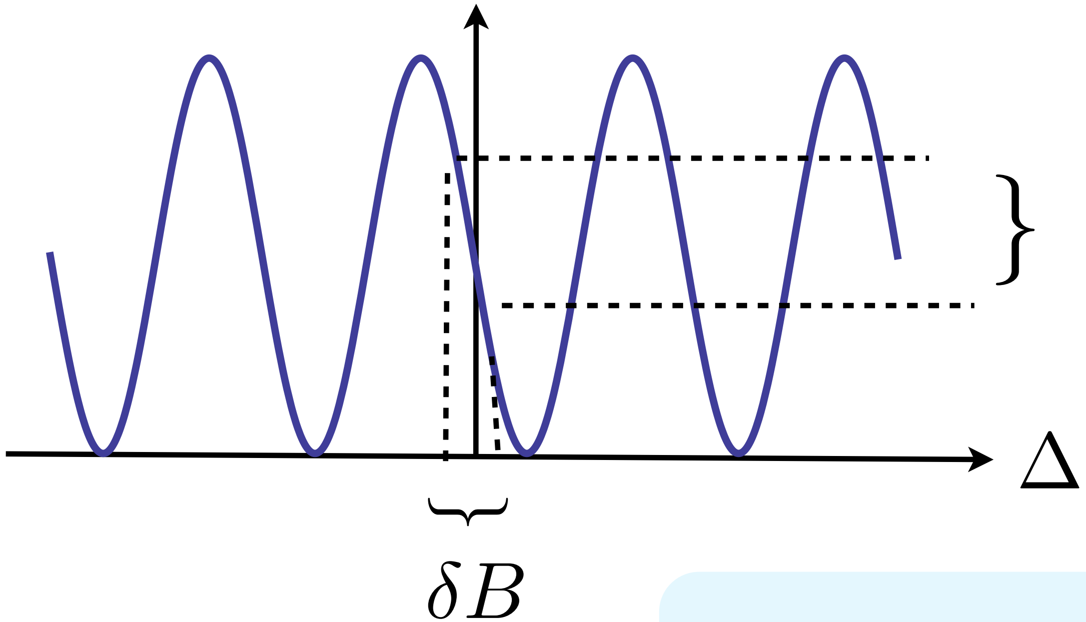
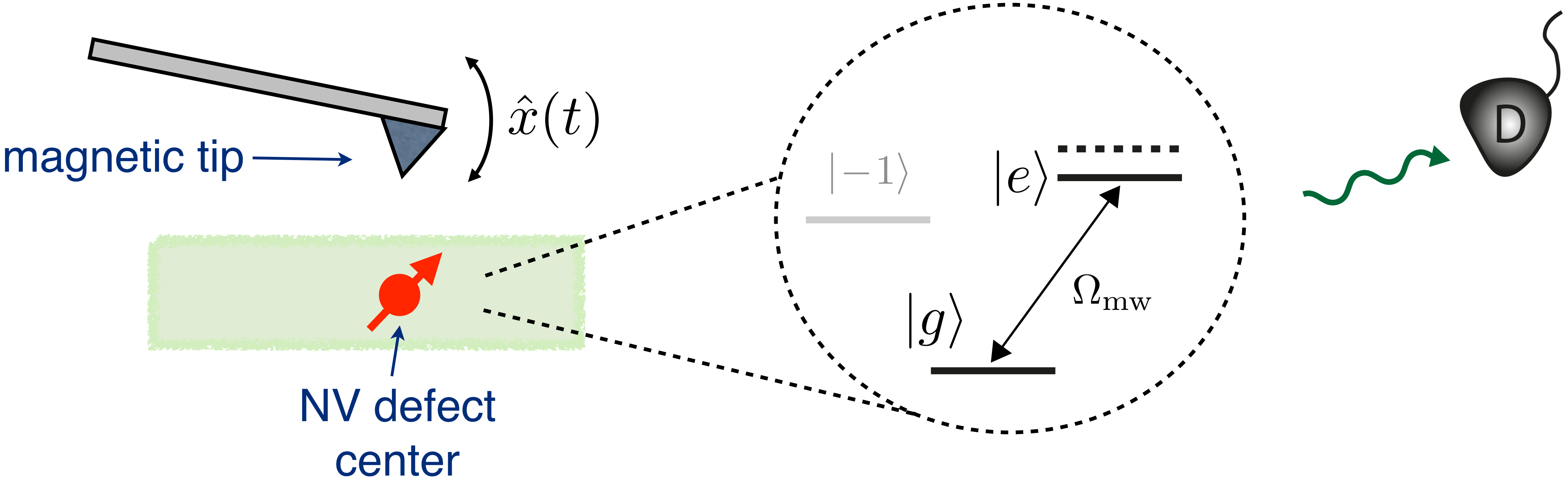


$$\Big\} \; \frac{g_s \mu_B \tau}{\hbar} \times \delta B$$

$$\tau \approx T_2$$

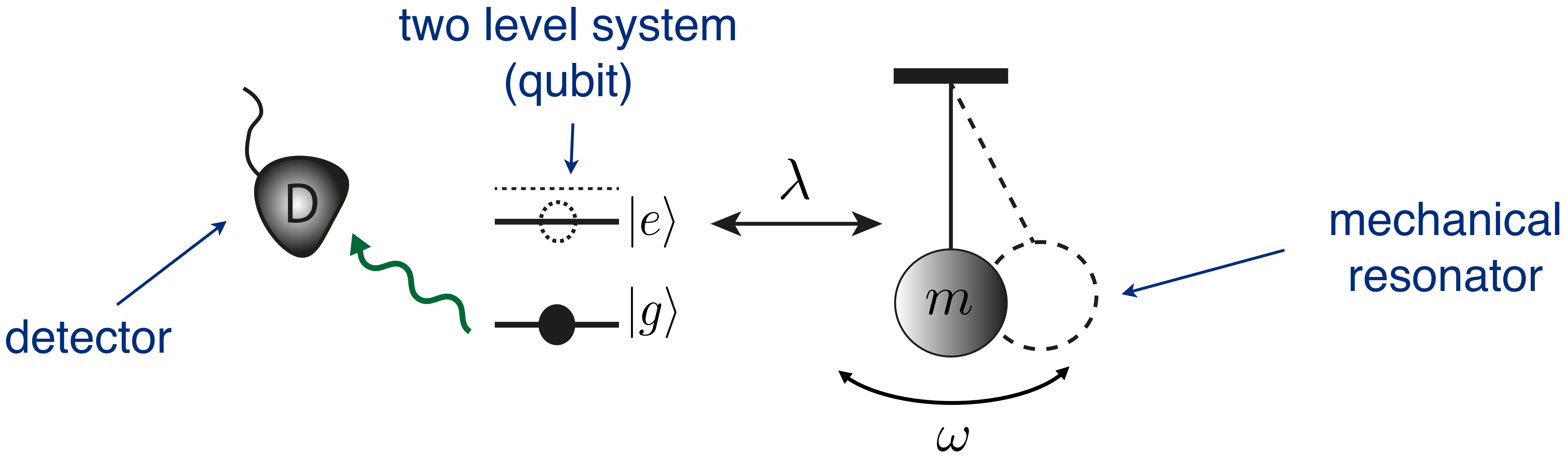
The long coherence times of NV are used for highly sensitive magnetometers with nanoscale resolution !

Magnetometry for “quantum” signals ?



⇒ *Use Ramsey method to detect “quantum field”* $\hat{x}(t)$ *?*

Setup



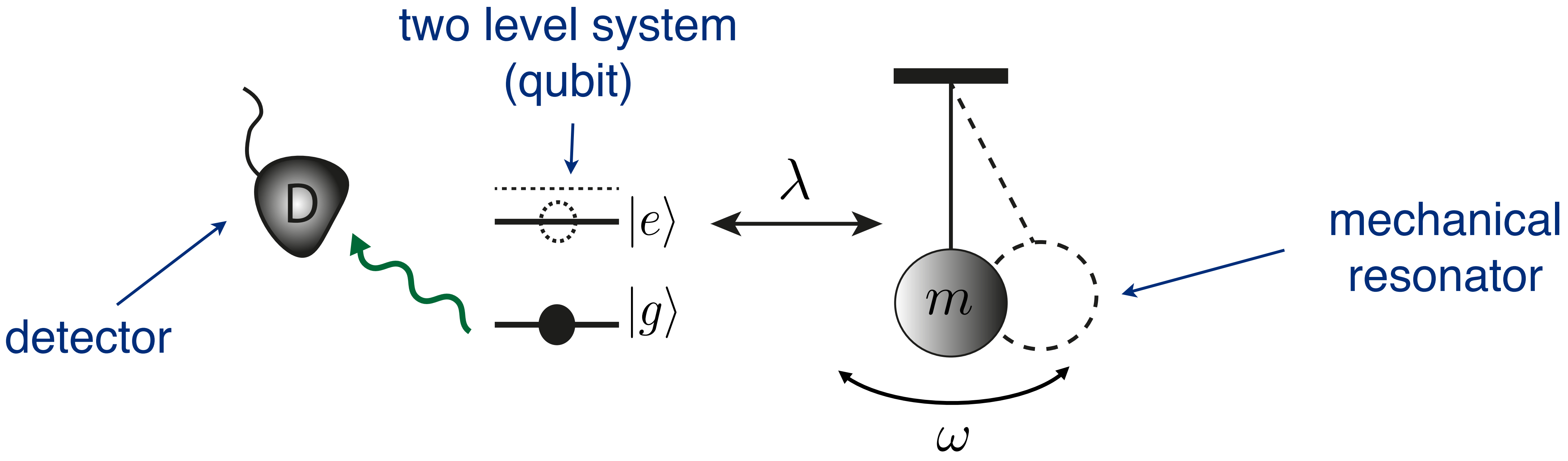
Qubit-resonator coupling:

$$H = \omega a^\dagger a + \underline{\lambda(a + a^\dagger)|e\rangle\langle e|}$$

- *Frequency shift ~ to resonator displacement*

$\Longrightarrow$ ***Ramsey / magnetometry !***

Setup



Qubit-resonator coupling:

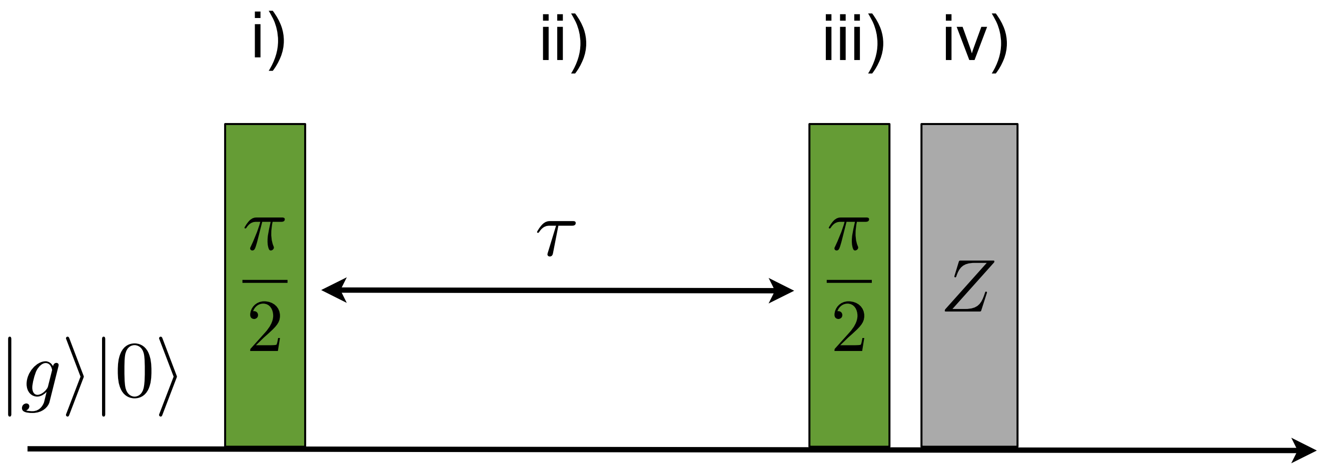
$$H = \omega a^\dagger a + \underline{\lambda(a + a^\dagger)|e\rangle\langle e|}$$

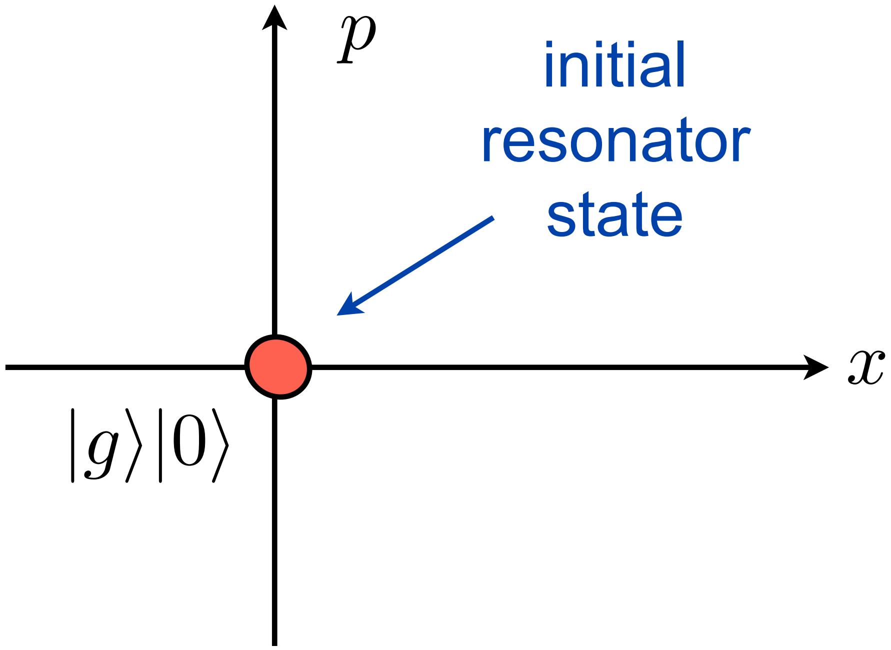
- *Frequency shift ~ to resonator displacement*

  $\Longrightarrow$ ***Ramsey / magnetometry !***

- *State dependent force:* $\Longrightarrow$ ***quantum backaction !***

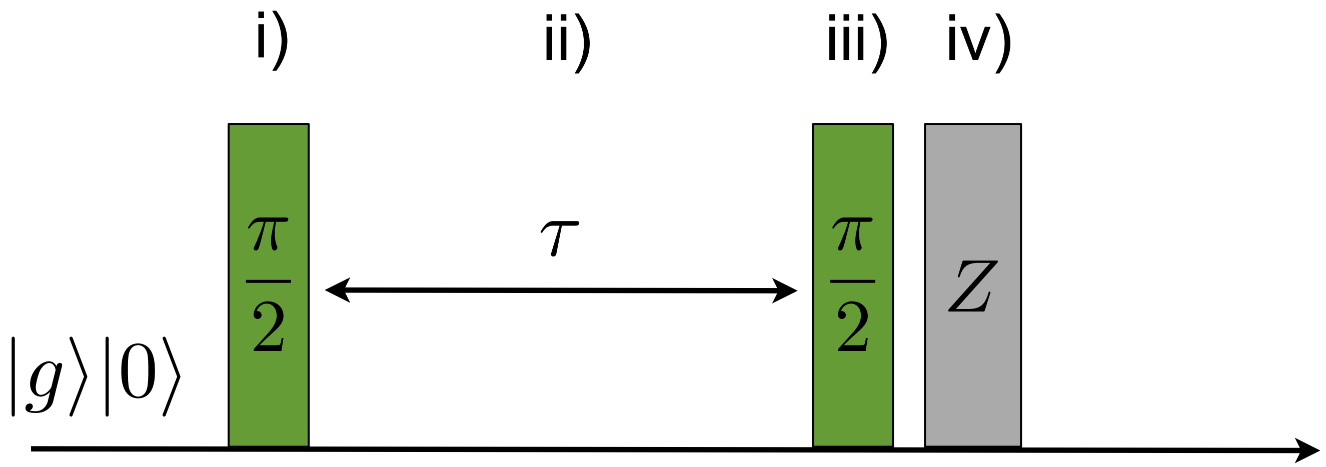
Quantum magnetometry

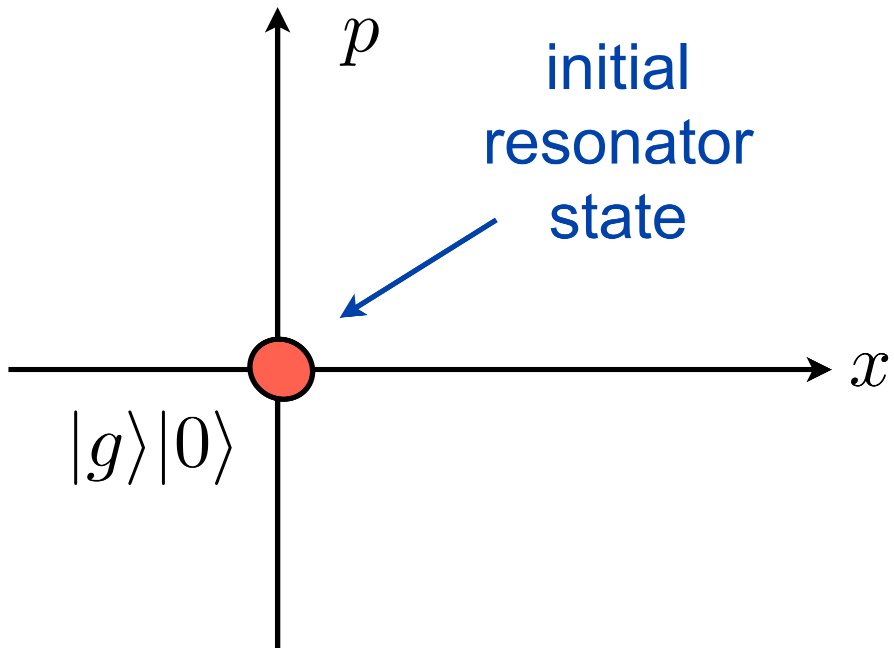




initial resonator state

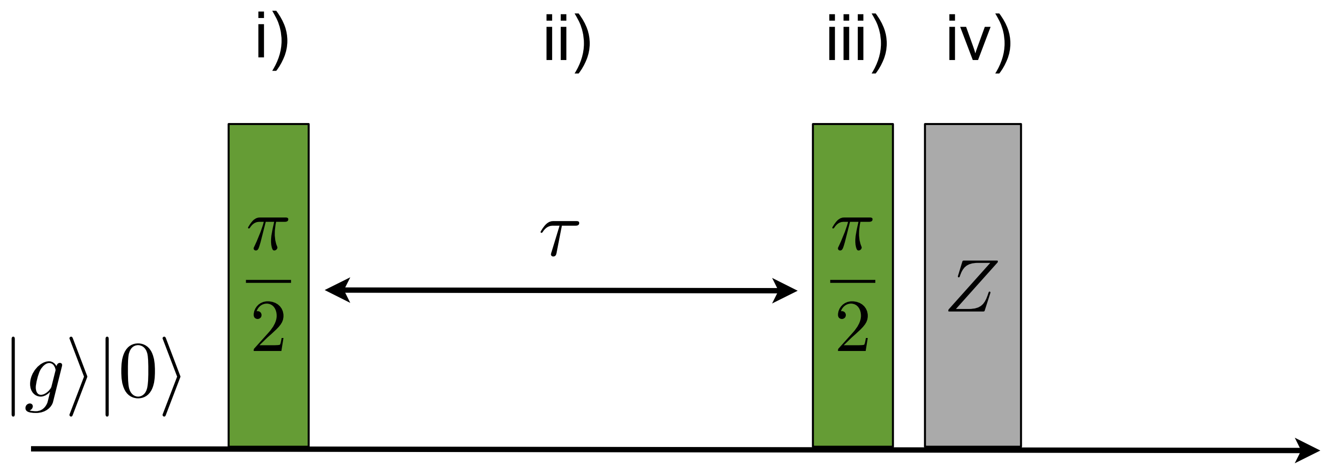
Quantum magnetometry

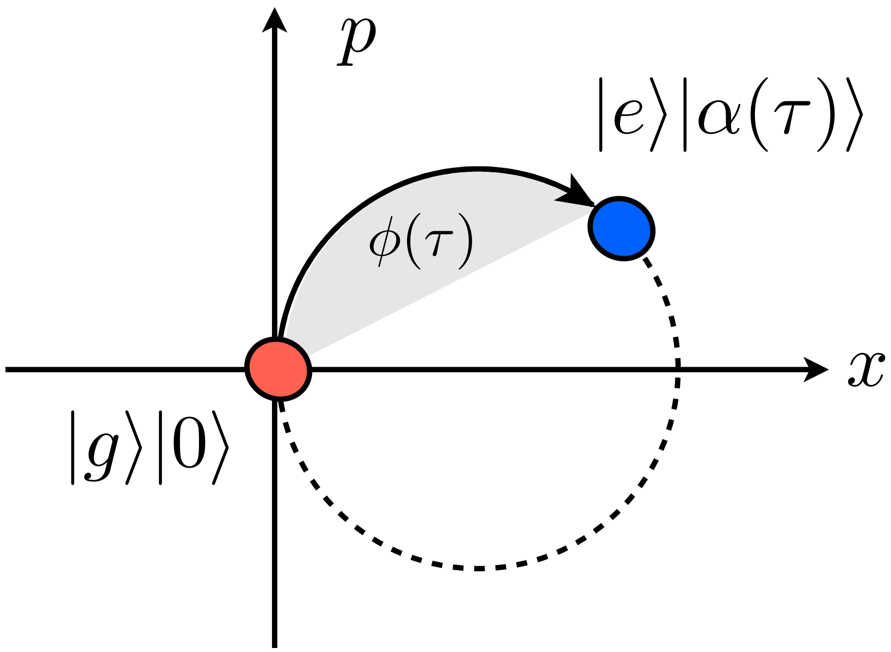




i) $|g\rangle|0\rangle \rightarrow \frac{1}{\sqrt{2}}\left(|g\rangle + e^{i\varphi}|e\rangle\right)|0\rangle$

Quantum magnetometry





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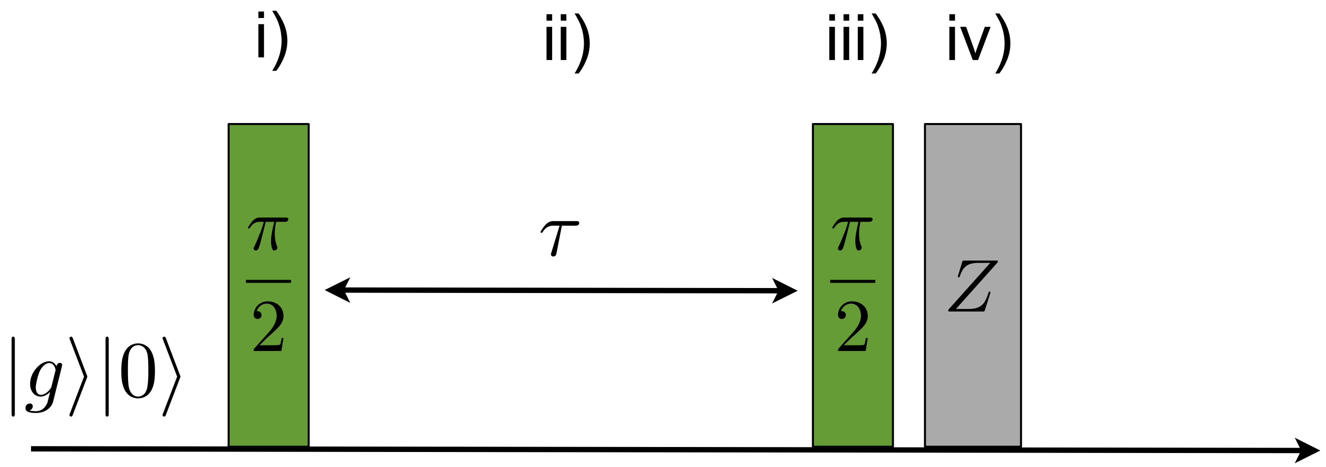
ii) $$\to \frac{1}{\sqrt{2}}(|g\rangle|0\rangle + e^{i\bar{\varphi}(\tau)}|e\rangle|\alpha(\tau)\rangle)$$

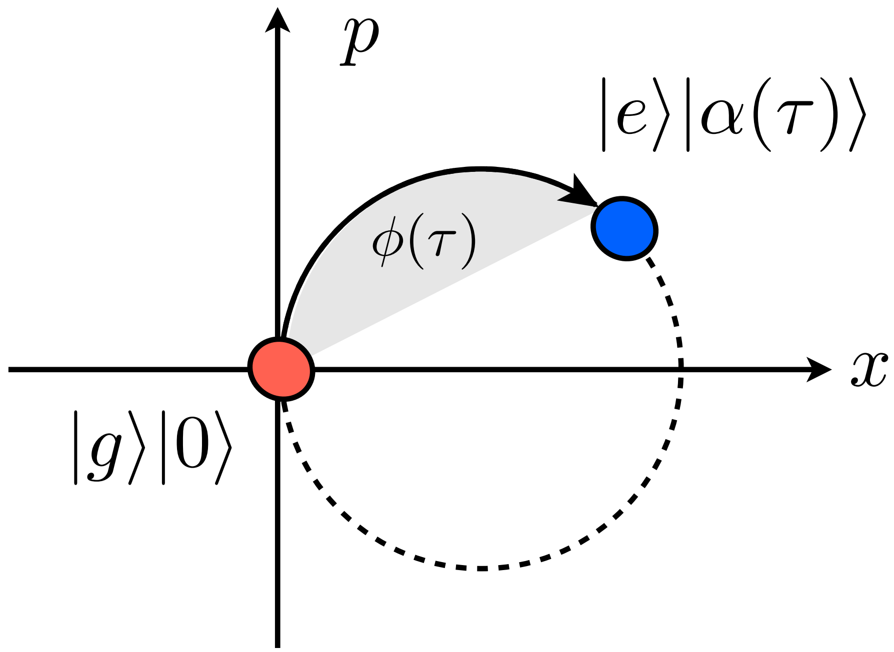
$$\bar{\varphi}(\tau) = \varphi + \phi(\tau)$$

"geometric phase"

$$\alpha(\tau) = \frac{\lambda}{\omega_m}(e^{-i\omega_m\tau} - 1)$$

Quantum magnetometry





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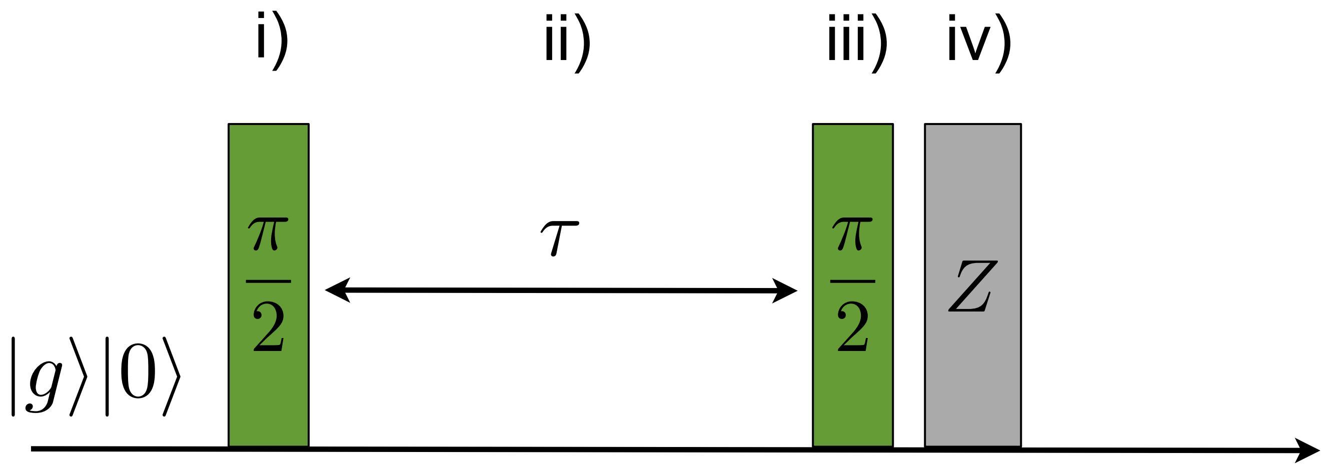

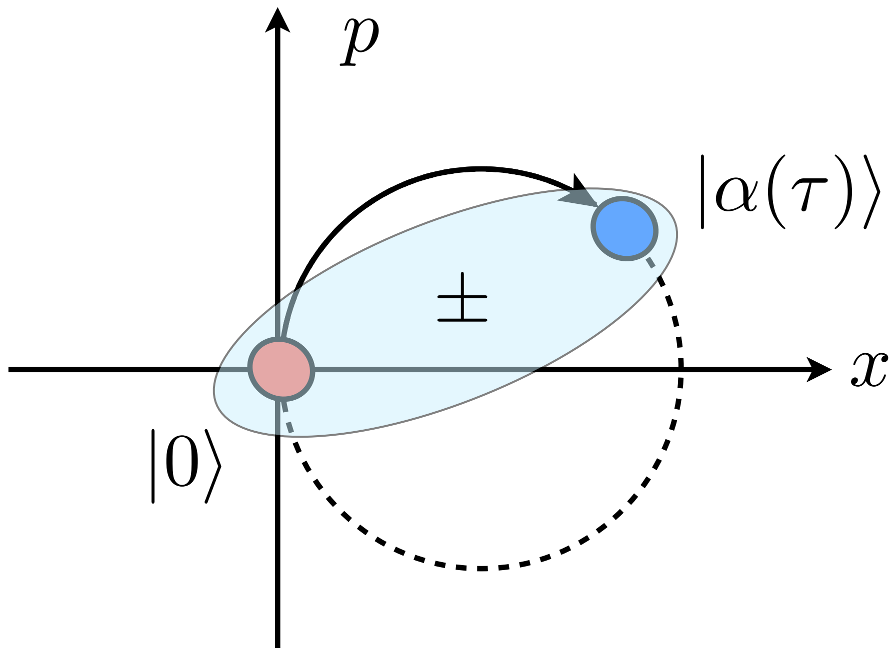

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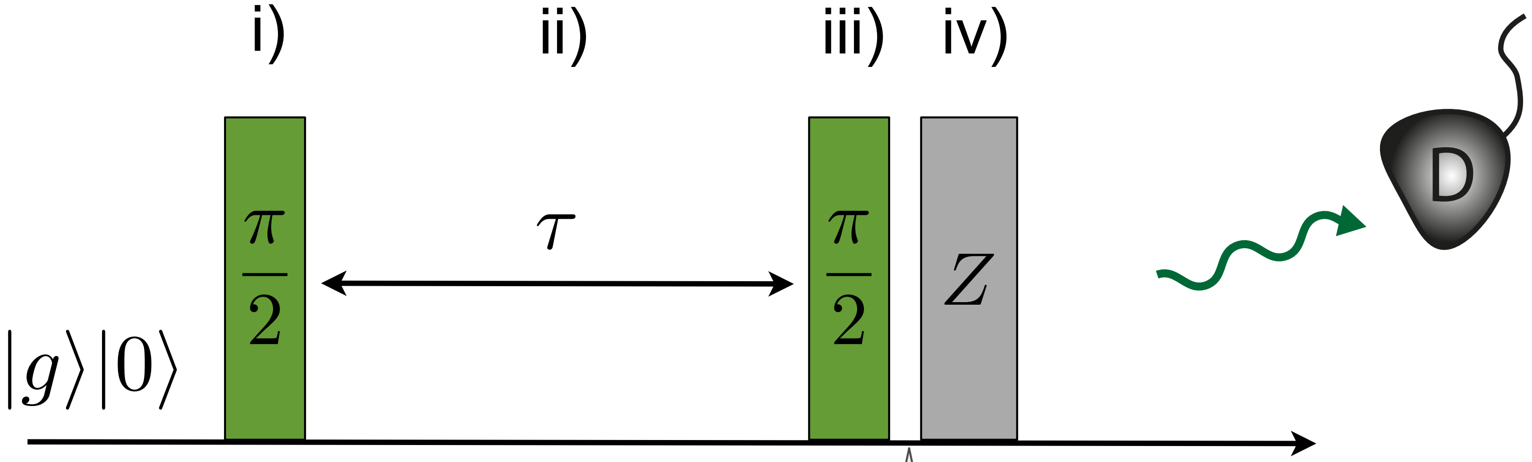
Quantum magnetometry



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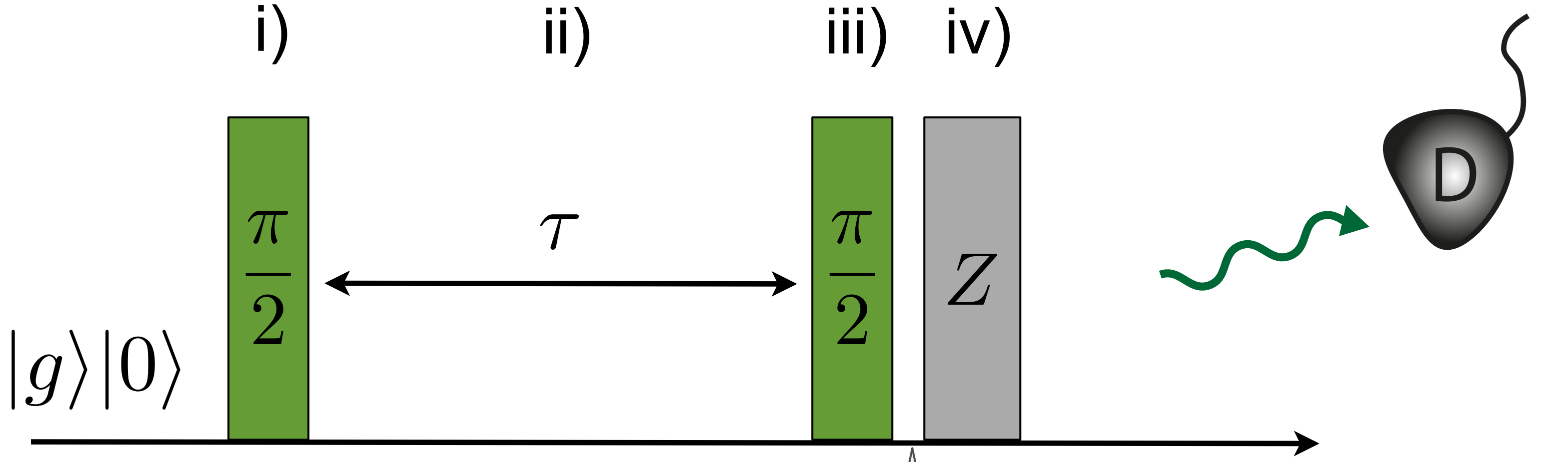
Quantum magnetometry



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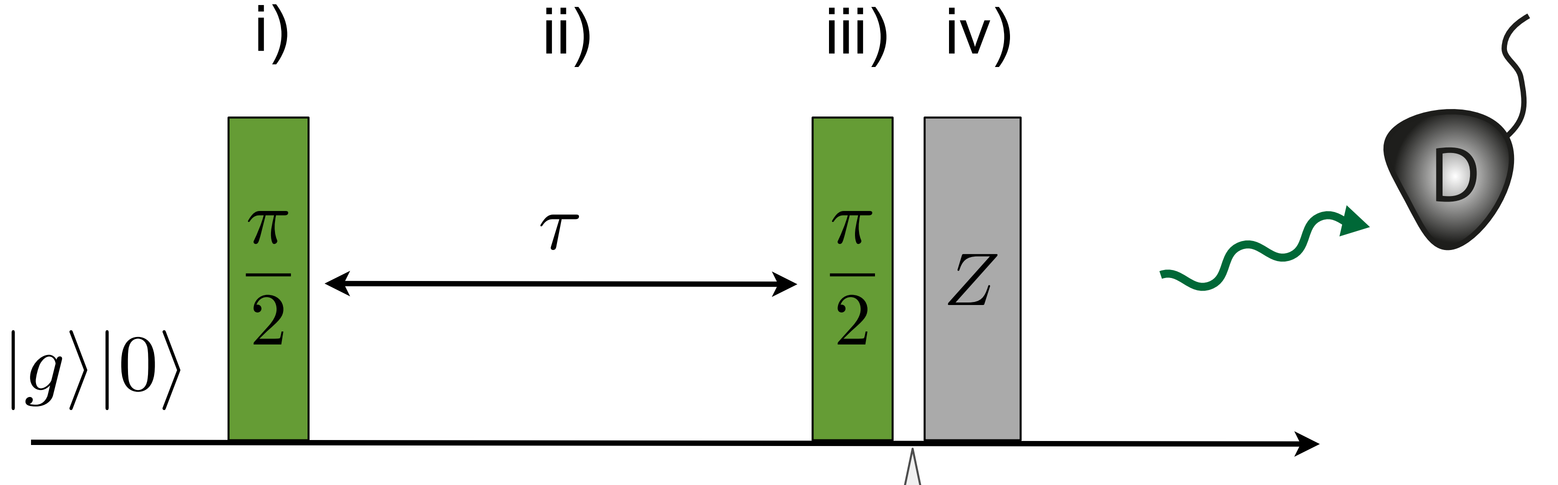
Quantum magnetometry



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- ***Conditioned resonator state after the measurement:*** $$|\psi\rangle_{\pm} = \frac{|0\rangle \pm e^{i\bar{\varphi}}|\alpha(\tau)\rangle}{2\sqrt{p_{\pm}}}$$

Collapse and revivals ...

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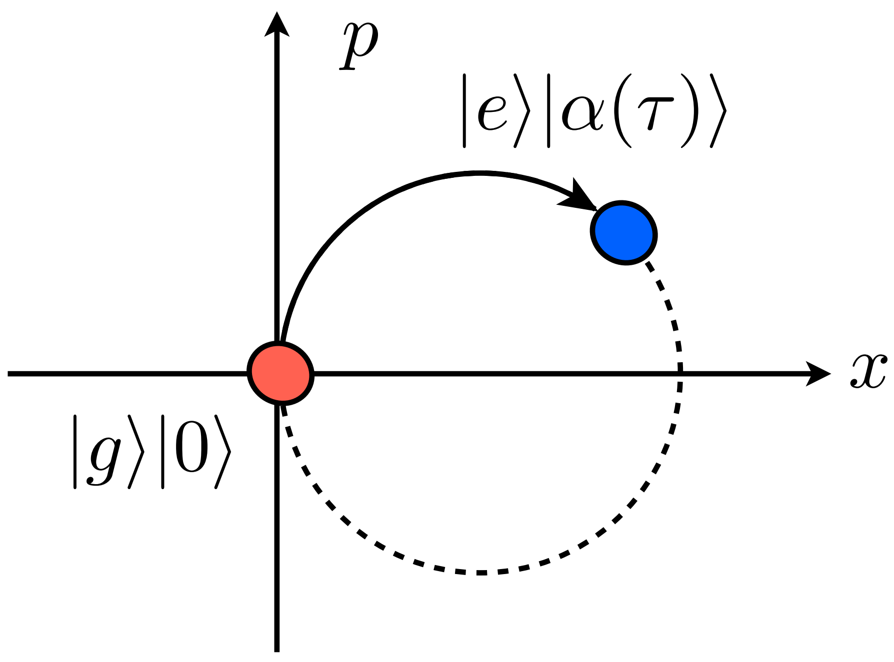
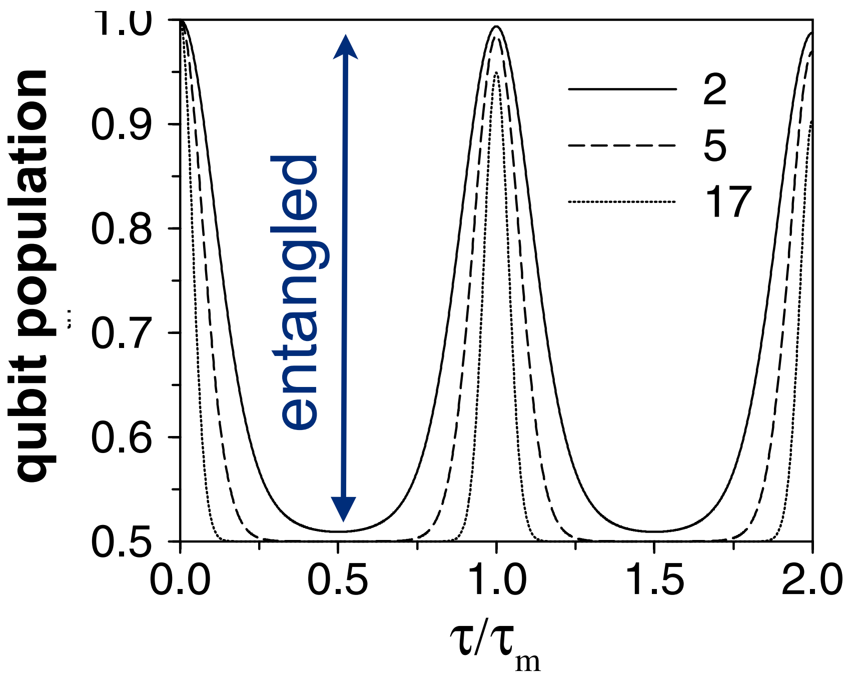


coherent rephasing



A. Armour, M. Blencowe, K. Schwab, PRL (2002);
W. Marshall, C. Simon, R. Penrose, D. Bouwmeester, PRL (2003); ....

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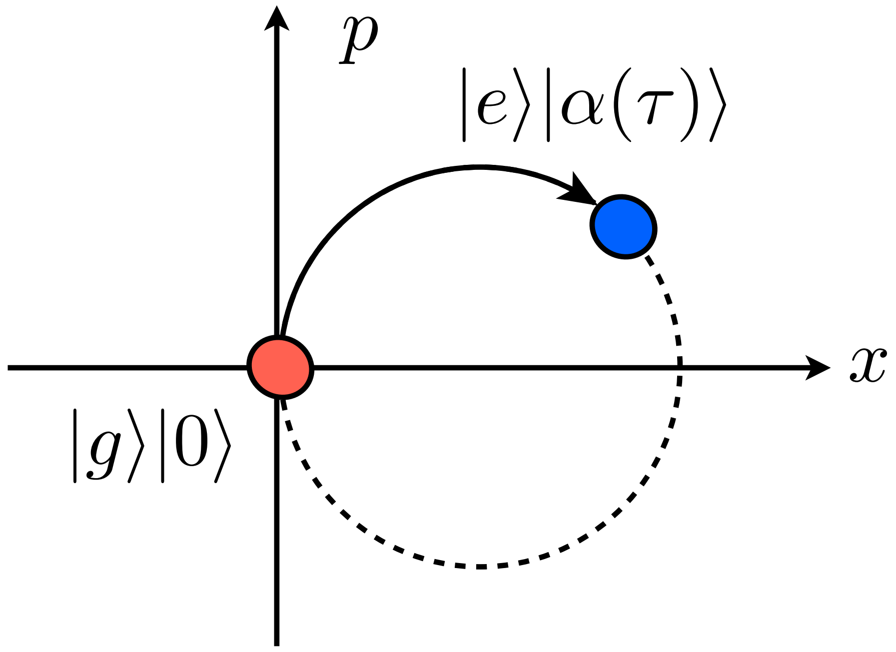

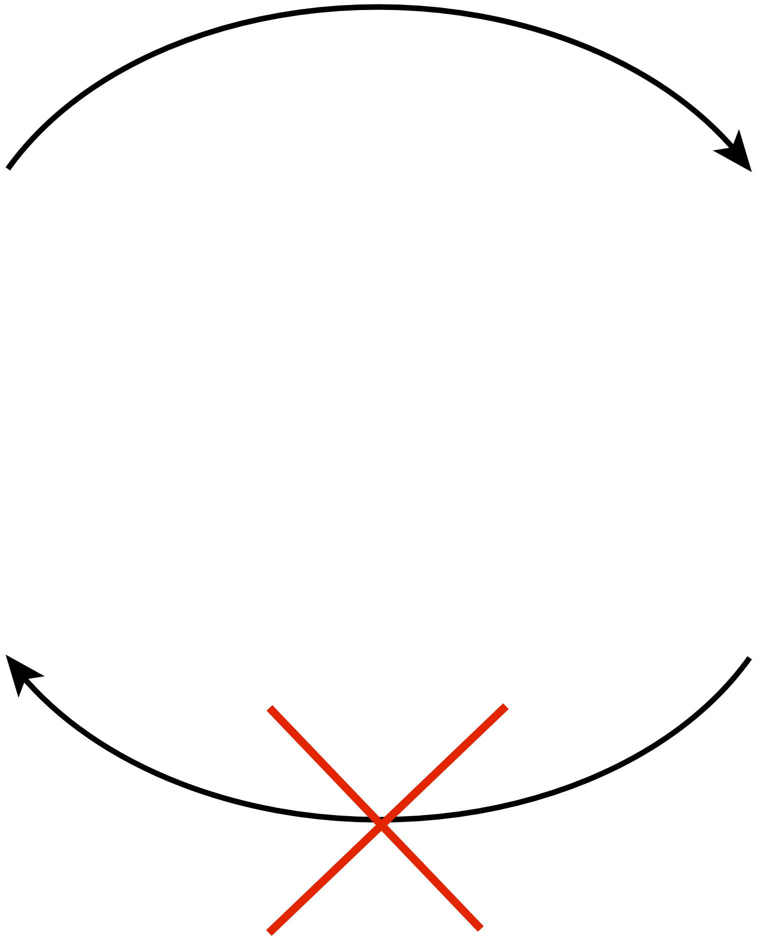

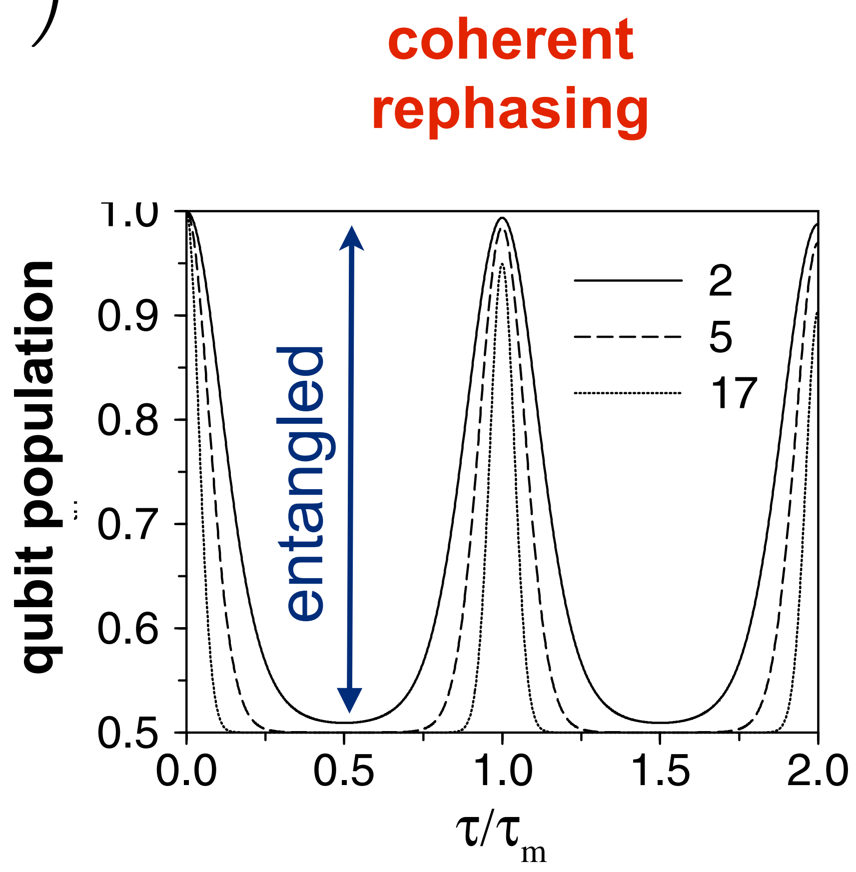

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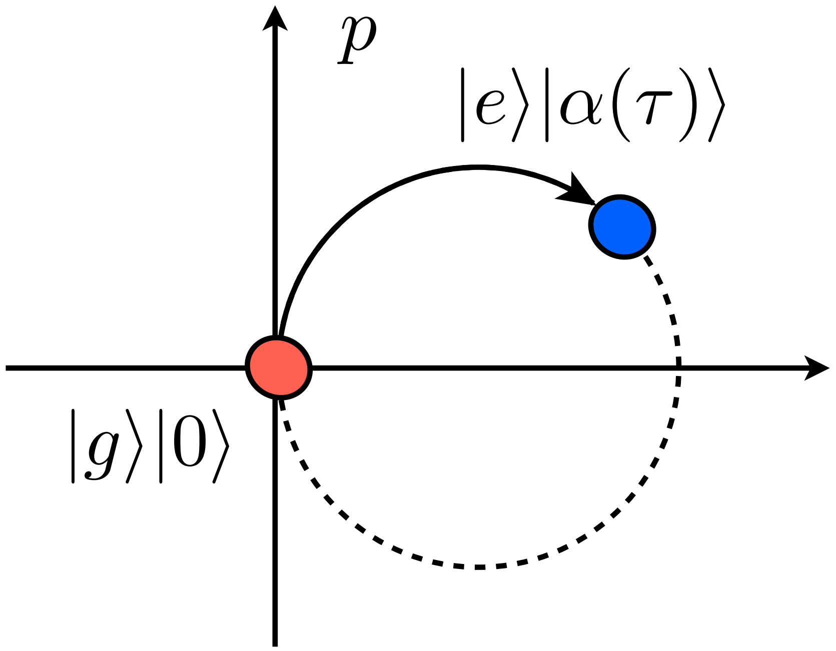

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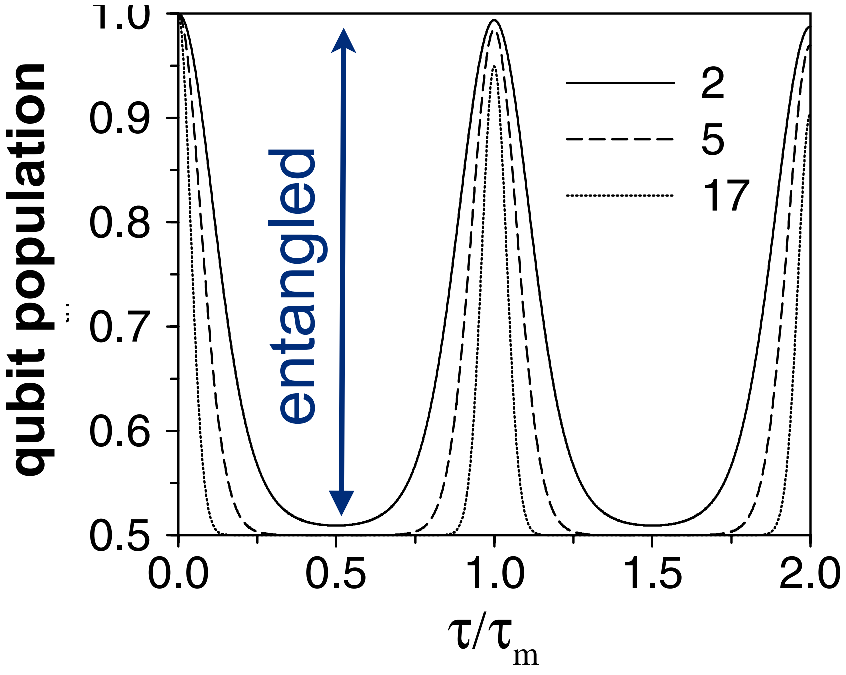
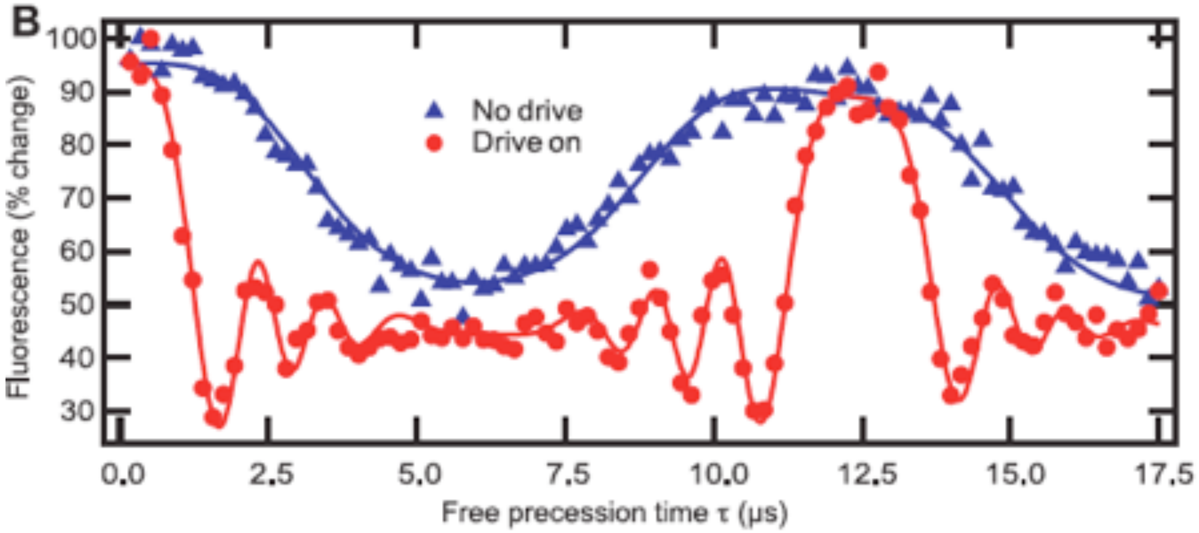
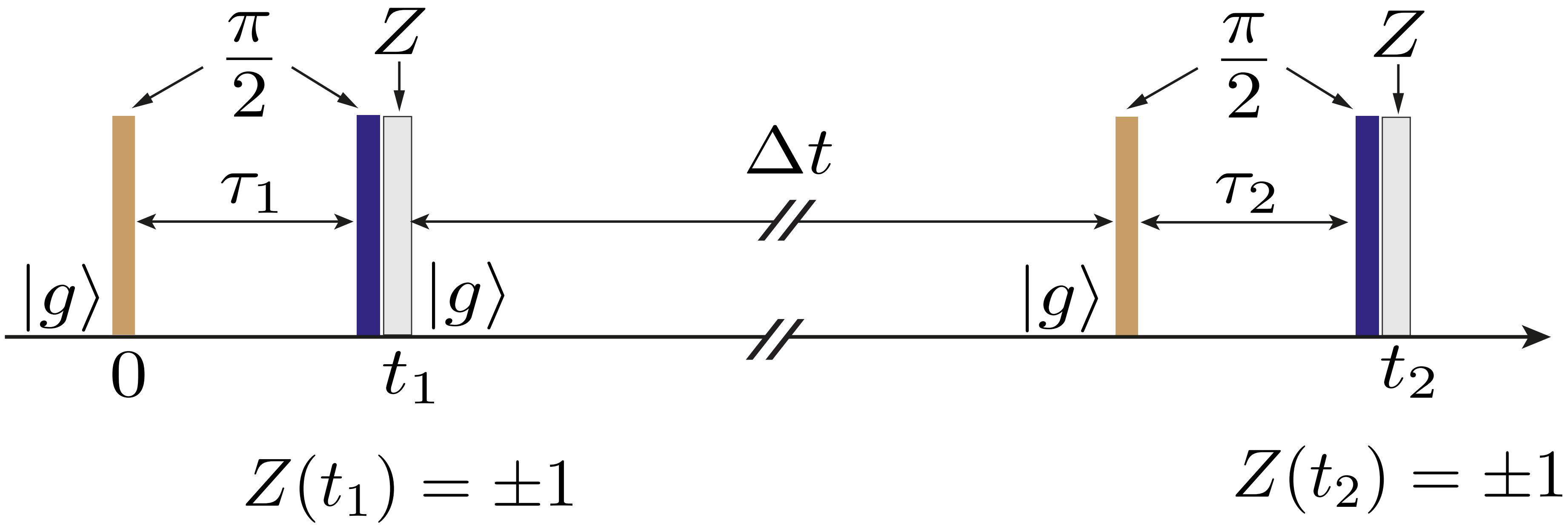




driven, thermal (T=300 K) resonator !

[1] S. Kolkowitz et al, Science (2012); S. Bennett et al. NJP (2012)

Idea: Correlations !



- *Conditioned on the outcome of the first measurement the resonator is projected into one of the superposition states*

$$|\psi^{\pm}\rangle = \frac{|0\rangle \pm e^{i\bar{\varphi}}|\alpha(\tau)\rangle}{2\sqrt{p_{\pm}}}$$

- *Use **correlations** between the first and second measurement to probe quantum superpositions over a time* Δt *.*

$$\langle Z(t_2) Z(t_1)\rangle = (p_{+|+} - p_{-|+})p_{+} - (p_{+|-} - p_{-|-})p_{-}$$

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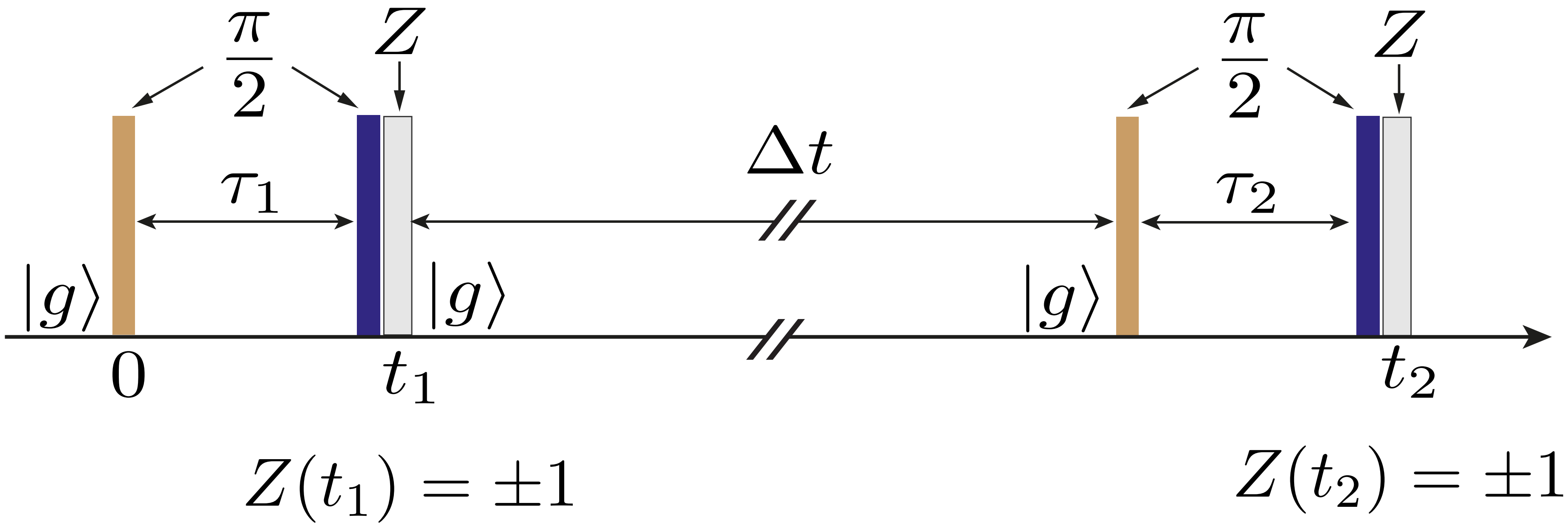

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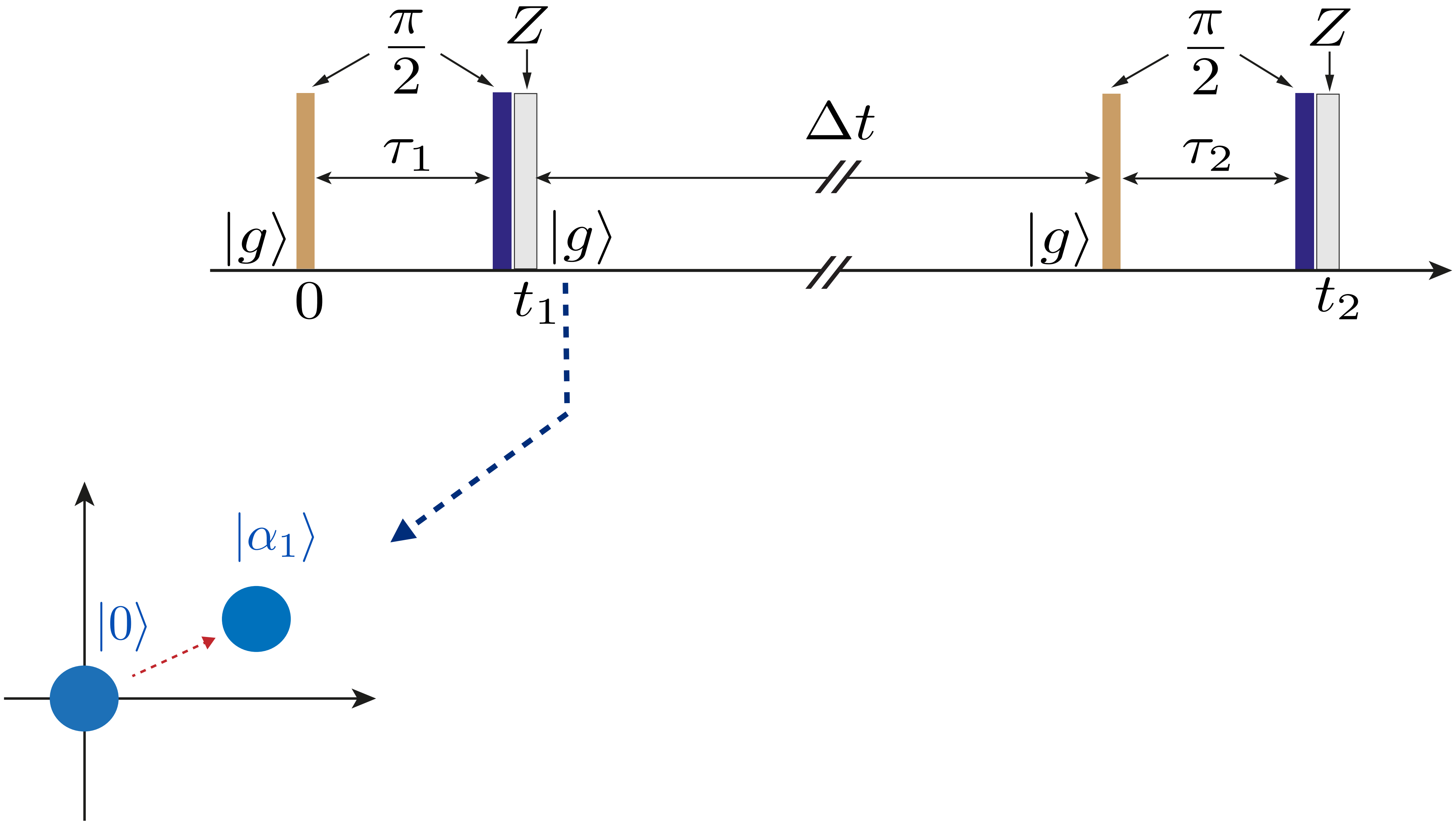
Ramsey correlation measurements



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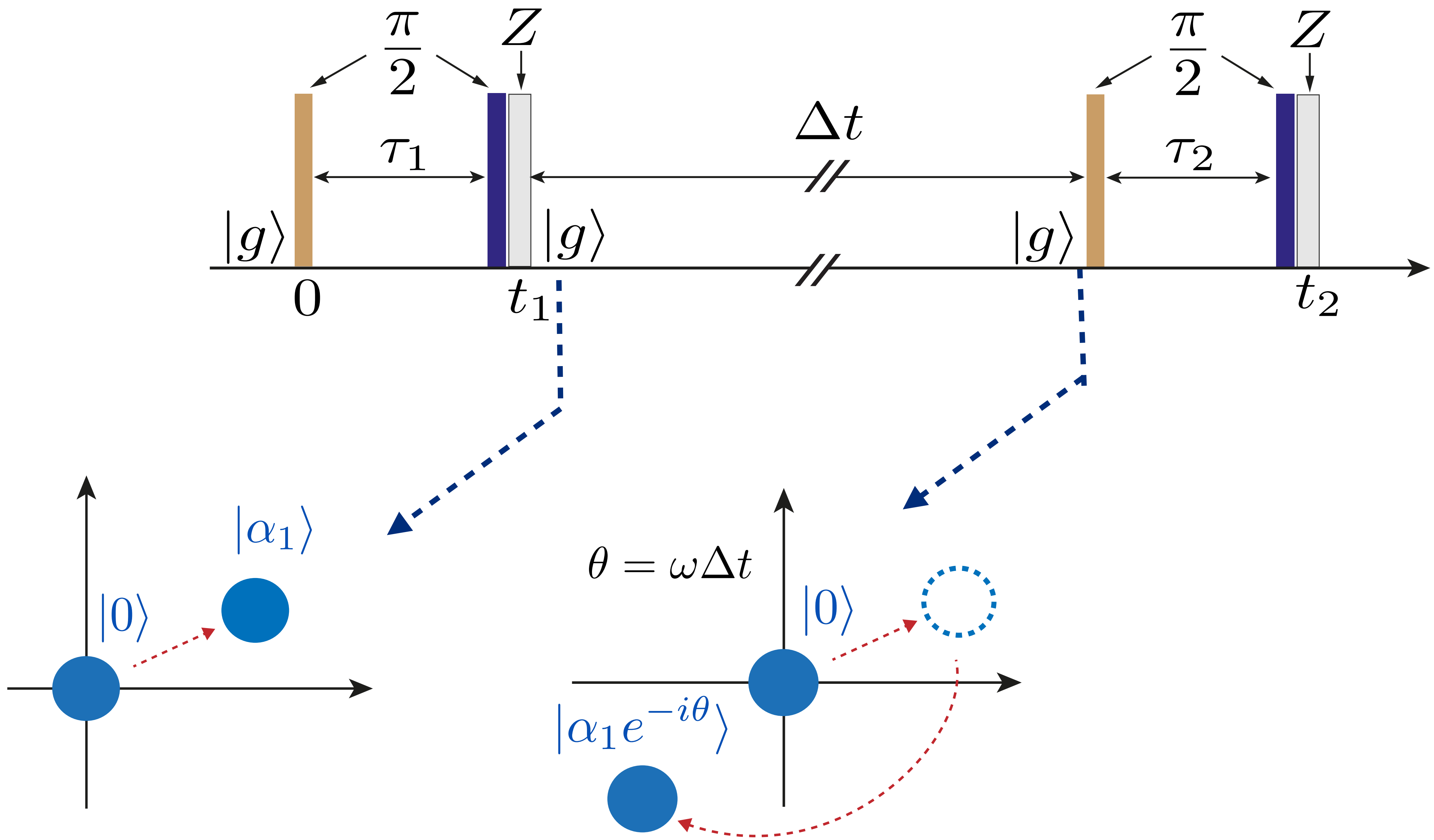

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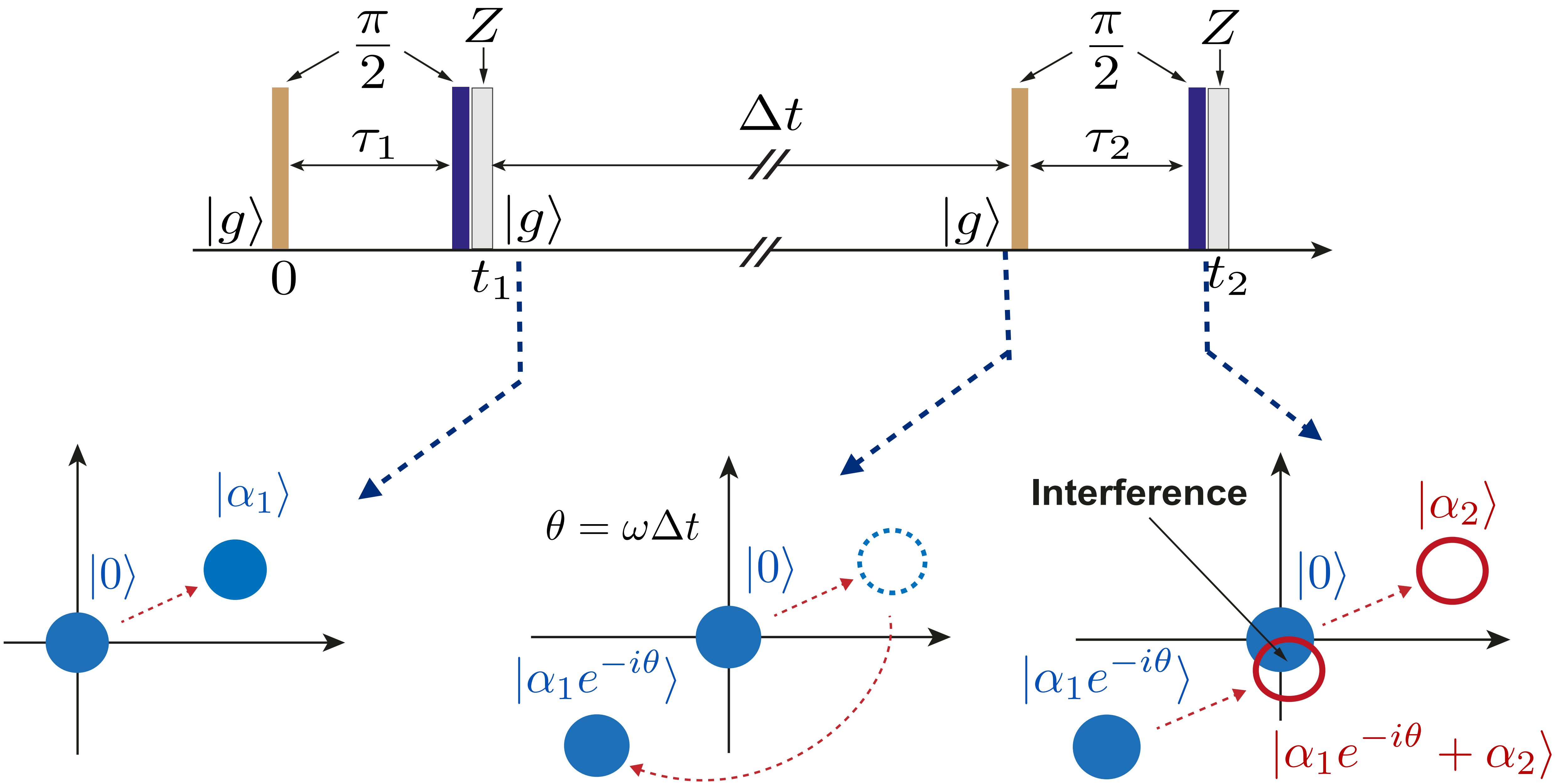
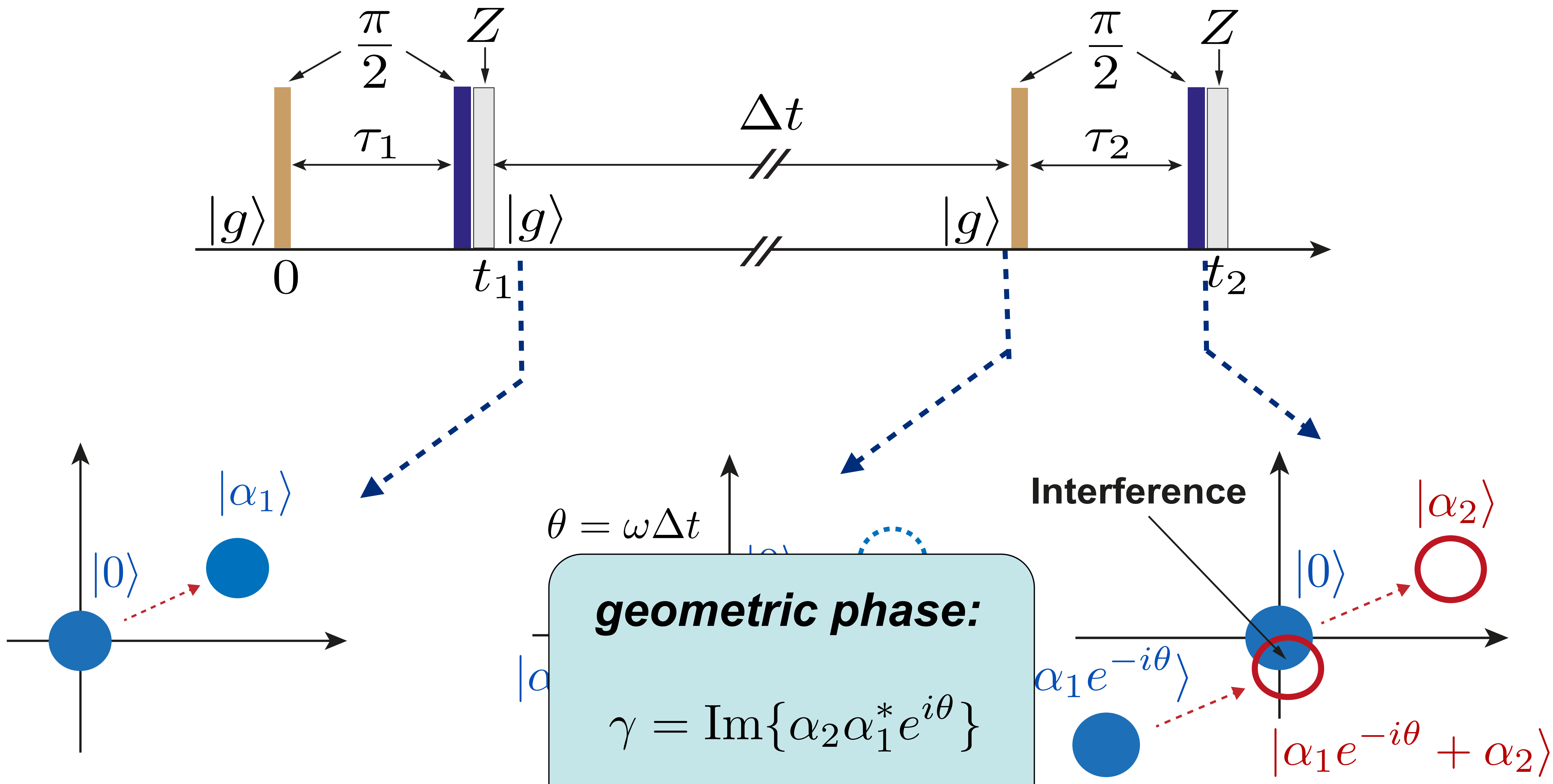


$t = t_2$:

$$|\psi^{\pm|+}\rangle = \frac{|0\rangle + e^{i\bar{\varphi}_1}|\alpha_1 e^{-i\theta}\rangle \pm e^{i\bar{\varphi}_2}|\alpha_2\rangle \pm e^{i(\bar{\varphi}_1+\bar{\varphi}_1+\gamma)}|\alpha_1 e^{-i\theta}+\alpha_2\rangle}{4\sqrt{p_+}}$$

Ramsey correlation measurements



geometric phase:

$$\gamma = \mathrm{Im}\{\alpha_2 \alpha_1^* e^{i\theta}\}$$

$t = t_2$:

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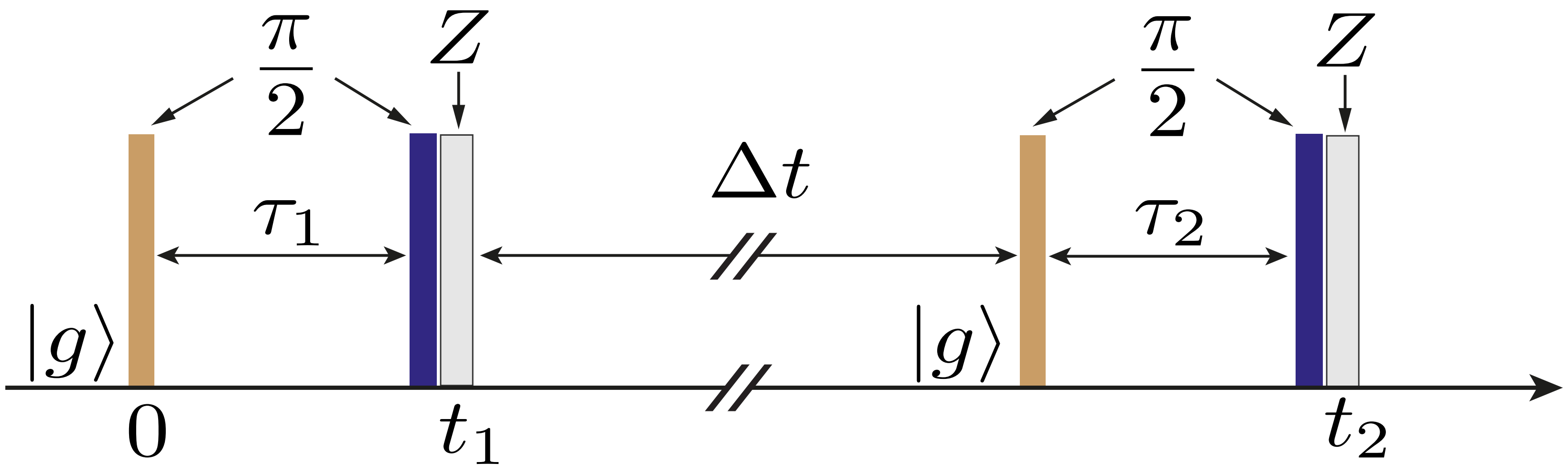
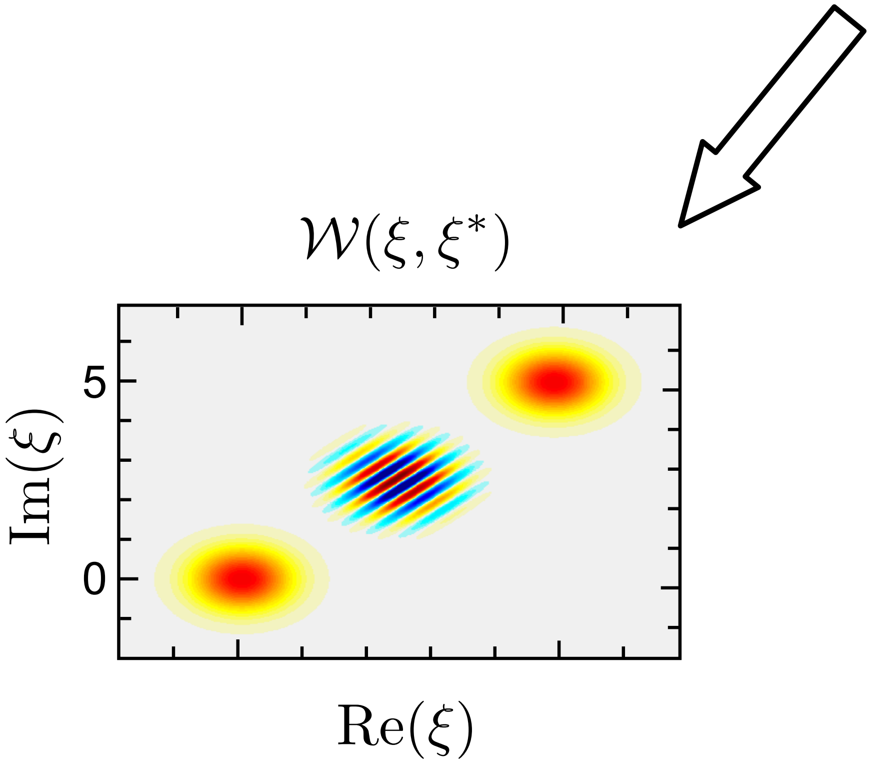
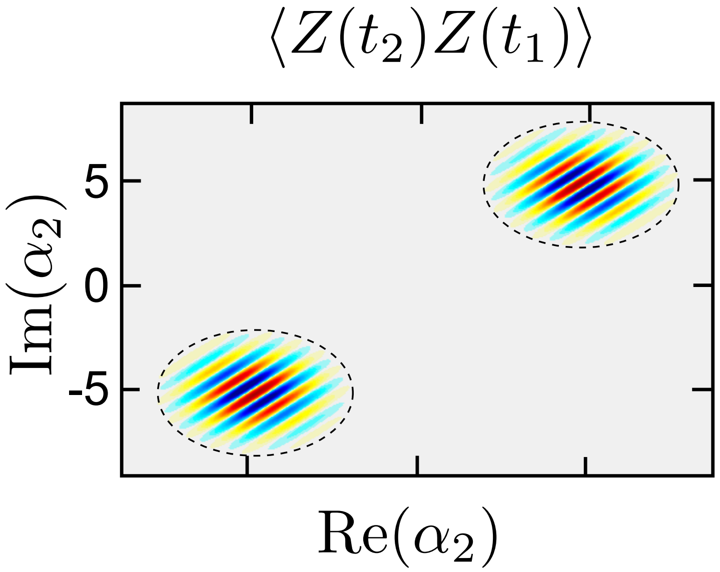


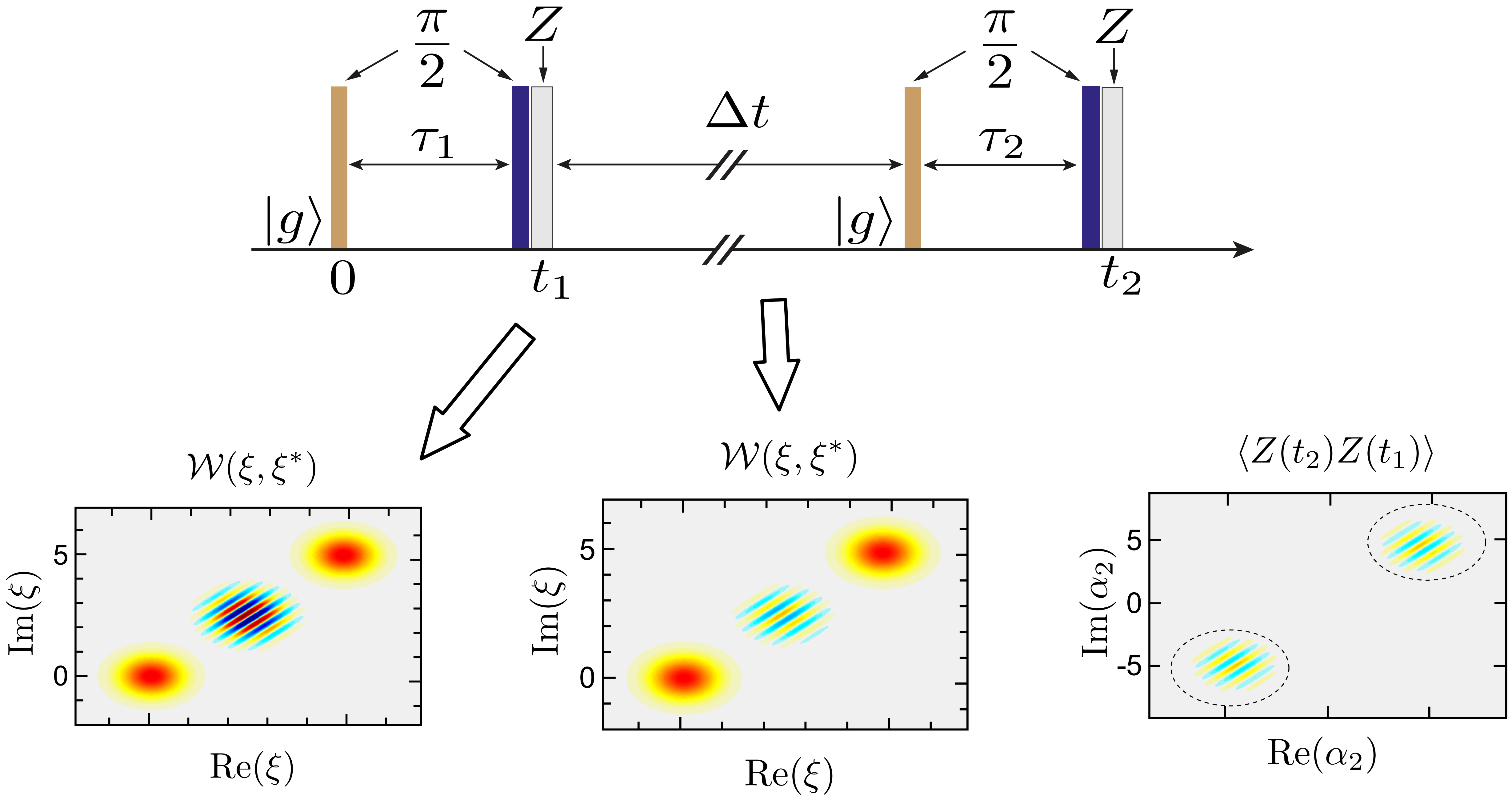


Wigner function of the conditioned state



correlations between the two measurements

Ramsey correlation measurements



Wigner function of the conditioned state

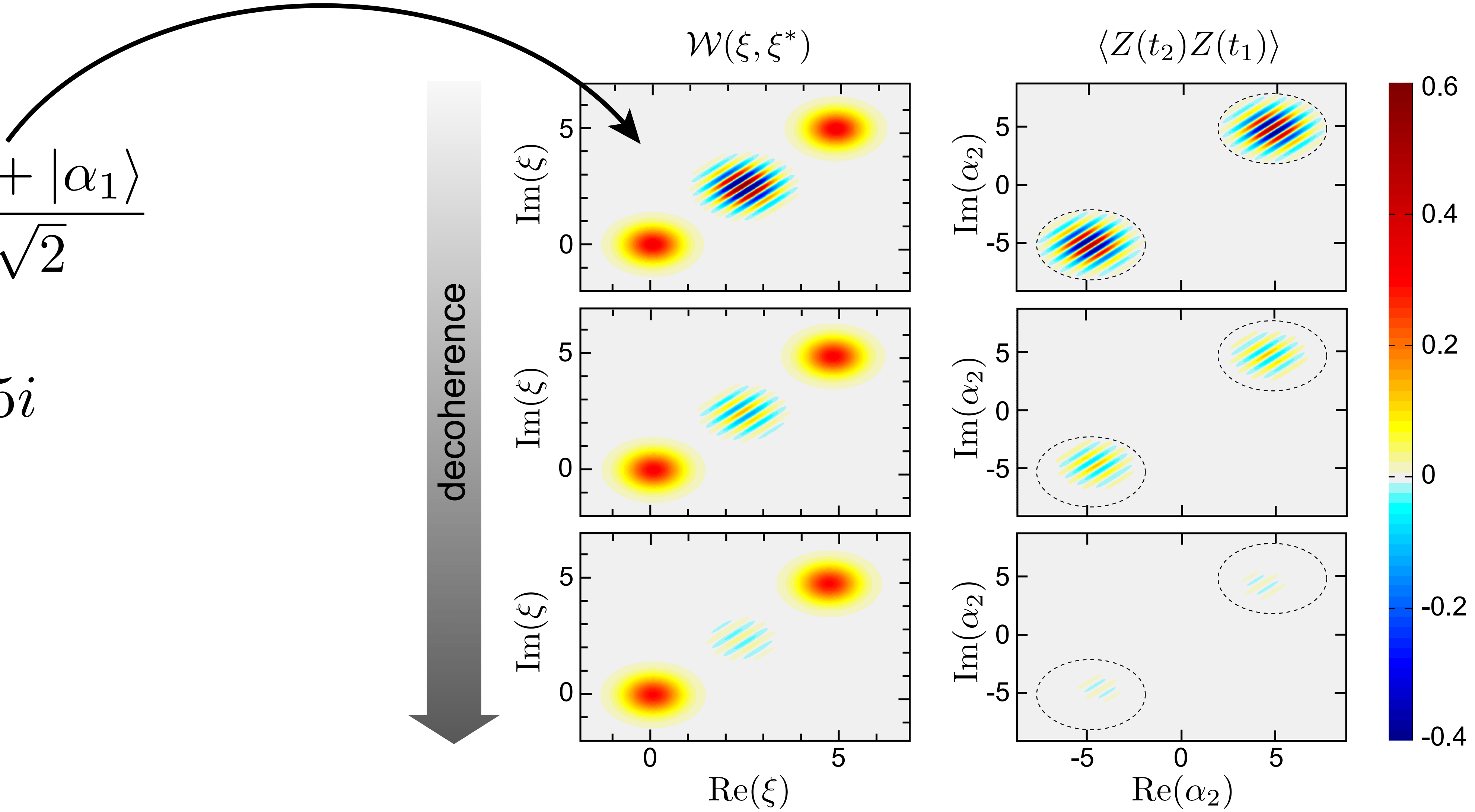
decoherence

correlations between the two measurements

Ramsey correlation measurements

$$|\psi\rangle_+ \simeq \frac{|0\rangle + |\alpha_1\rangle}{\sqrt{2}}$$

$$\alpha_1 = 5 + 5i$$



⇒ *Correlations* $C(t_1,t_2) = \langle Z(t_2)Z(t_1)\rangle$ *directly probe survival/decay of macroscopic superposition states !*

⇒ *During the waiting time the resonator is* ***decoupled*** *from the qubit ! (high-Q resonators, levitated objects!!)*

Ramsey correlation measurements

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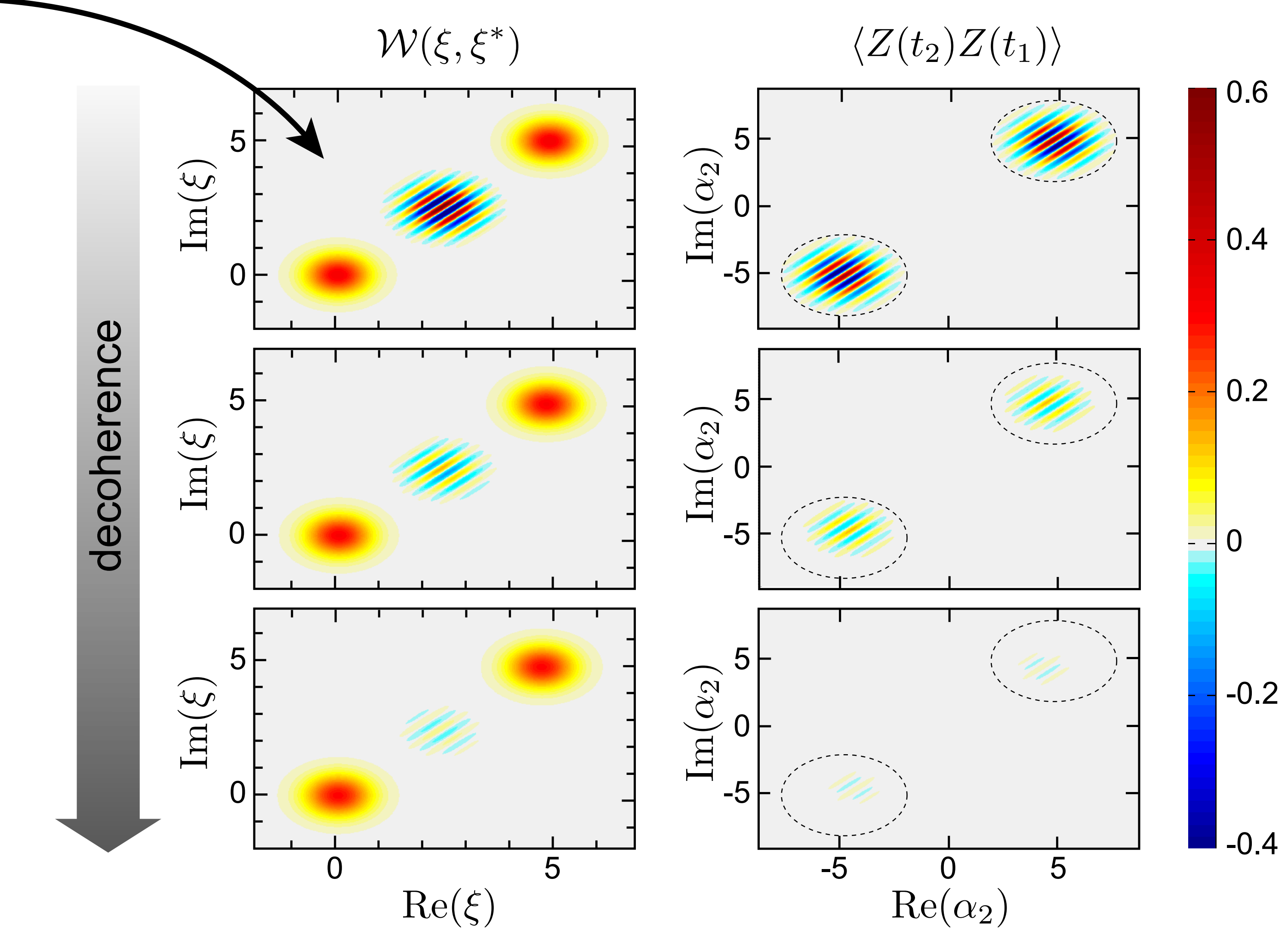
Do we really see quantum features ?



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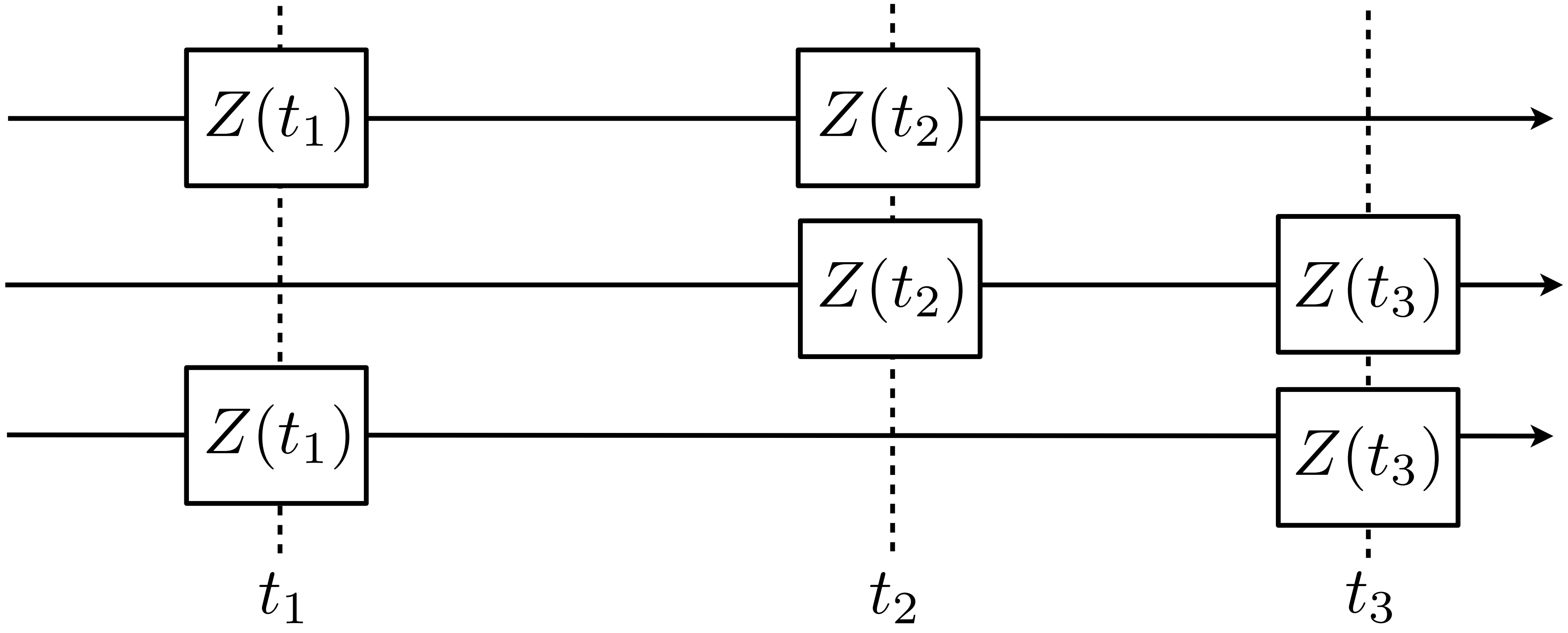
Leggett-Garg inequality



$$|Z(t_i)| \leq 1$$

$$C(t_i, t_j) = \langle Z(t_j) Z(t_i) \rangle$$

$$t_i < t_j$$

⇒ *For a **“macro-realistic”** [1] model these correlations are bound by the (Wigner type) Leggett-Garg inequality:*

$$C(t_1, t_2) + C(t_2, t_3) - C(t_1, t_3) \leq 1$$

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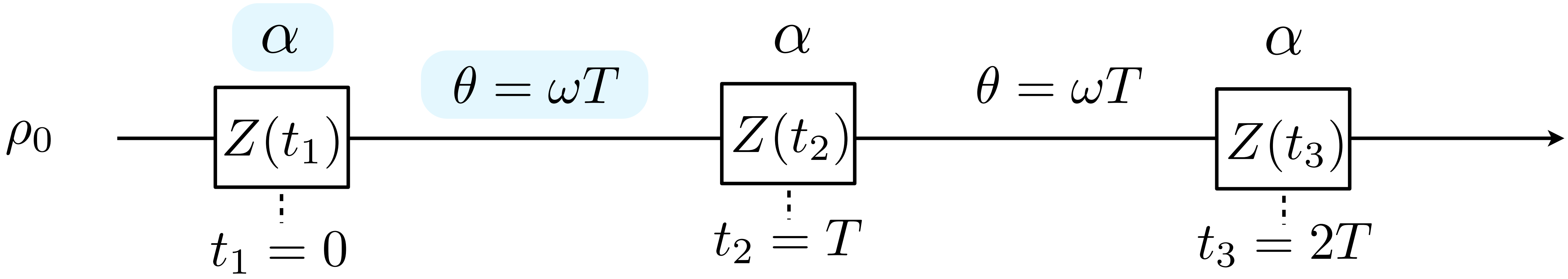
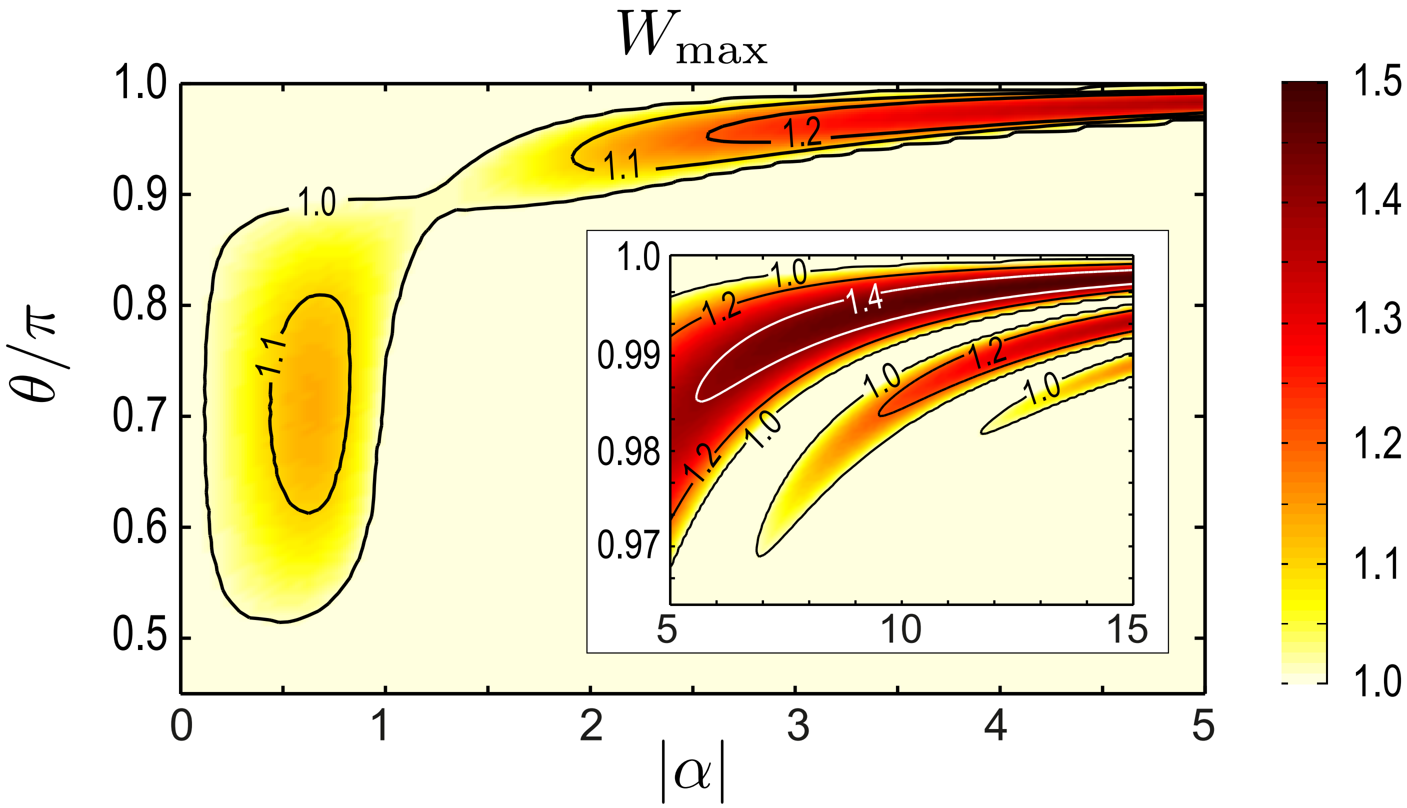
*[1] Leggett, J. Phys. Cond. Matter **14**, R415 (2002); Emary, Lambert, Nori, arXiv:1304.5133*

Violation of the LG inequality



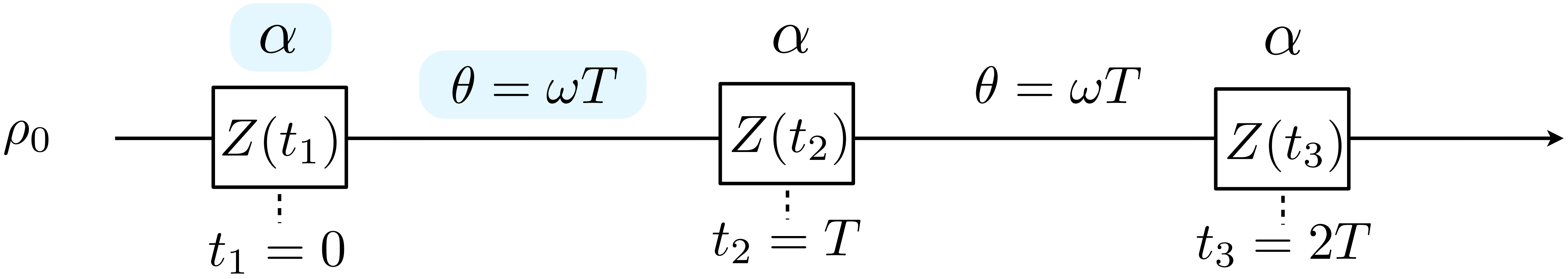
Result:



$(n_{\mathrm{th}} = 0)$

$$W = C(t_1, t_2) + C(t_2, t_3) - C(t_1, t_3) \leq 1$$

Violation of the LG inequality



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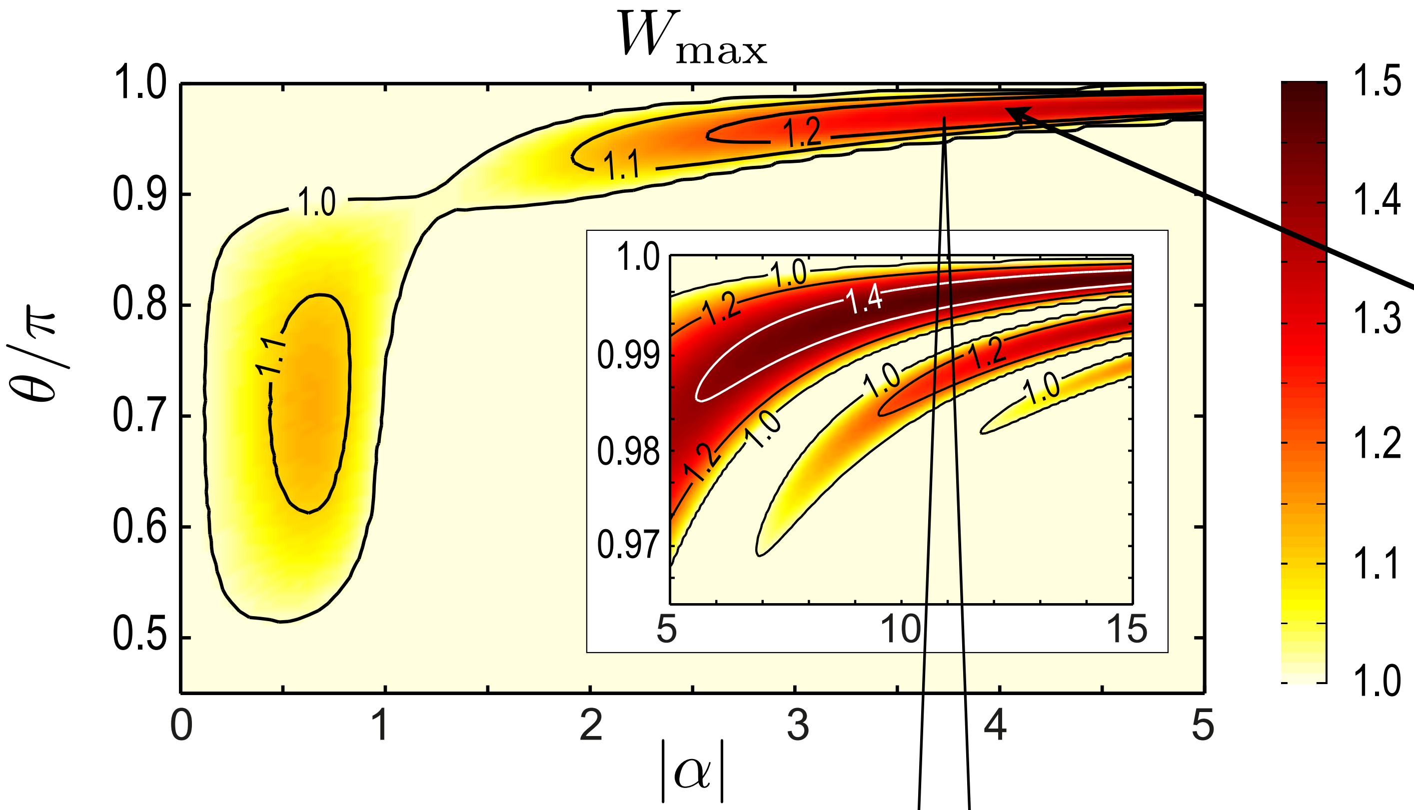

Maximal & robust violation of the LGI for large displacements:

$$W \simeq \frac{3}{2}\left(1 - \frac{\pi^2}{4|\alpha|^2}(2n_{\mathrm{th}}+1)\right)$$

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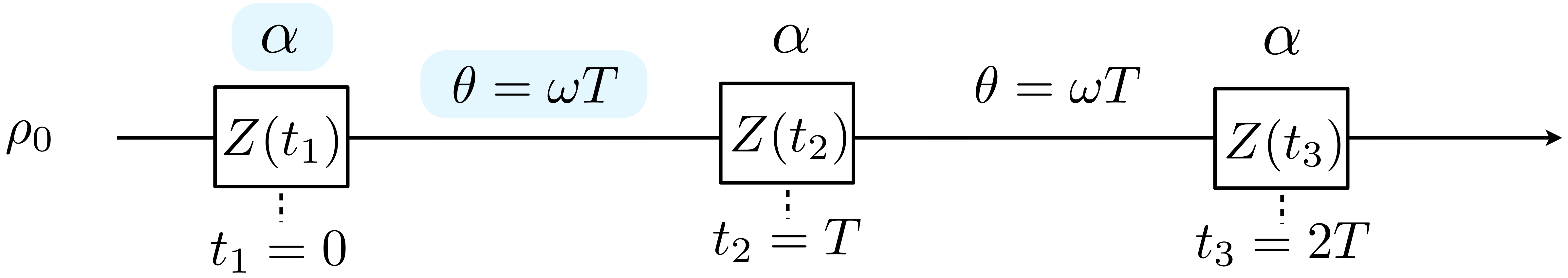

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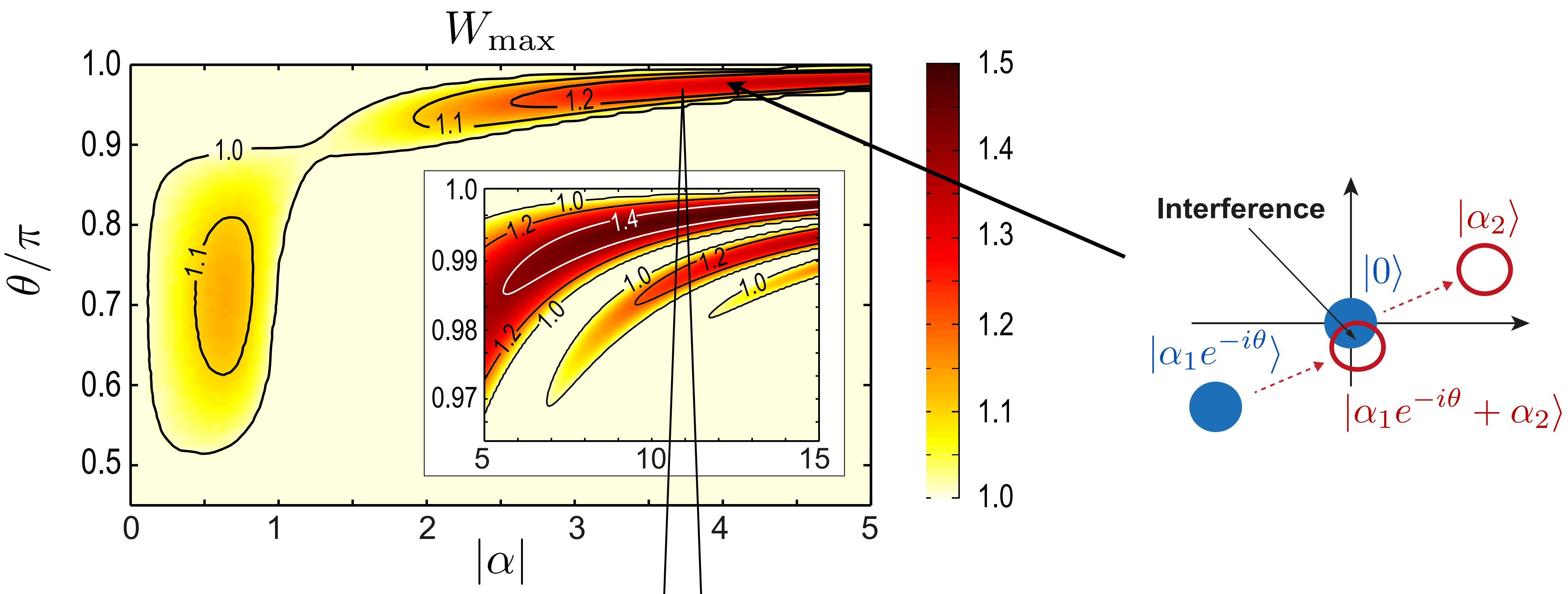

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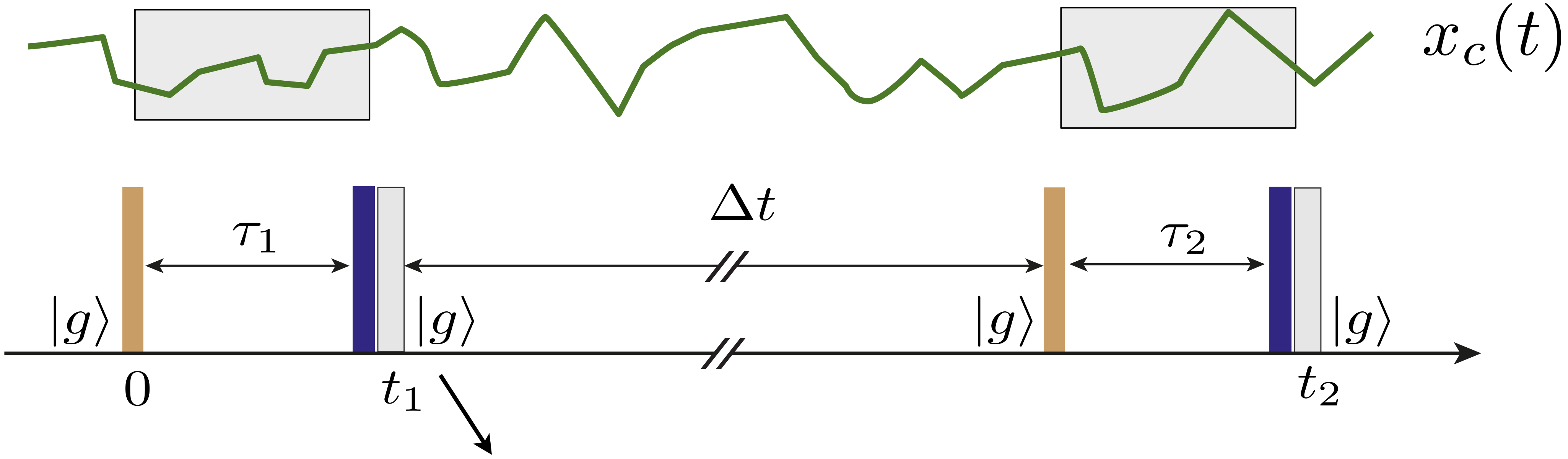
Quantum vs. classical correlations



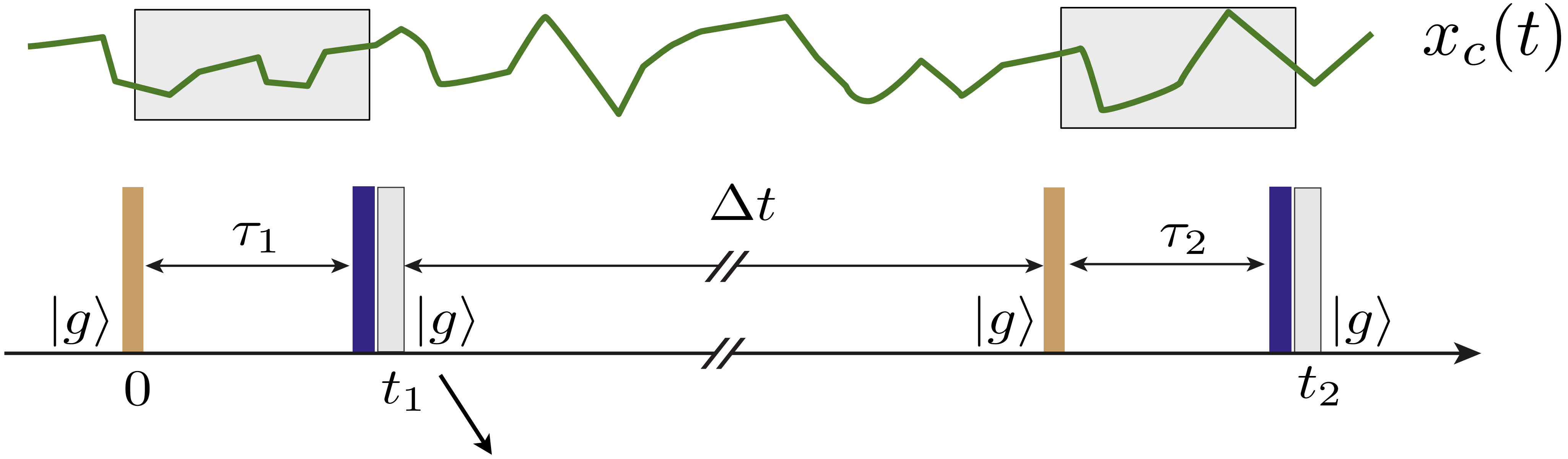
qubit reset

Qubit probing a classical field:

$$H_{\text{int}} = \sqrt{2}\lambda x_c(t)|e\rangle\langle e|$$

(random) trajectory, but with a specific value at each time

Quantum vs. classical correlations



qubit reset

Qubit probing a classical field:

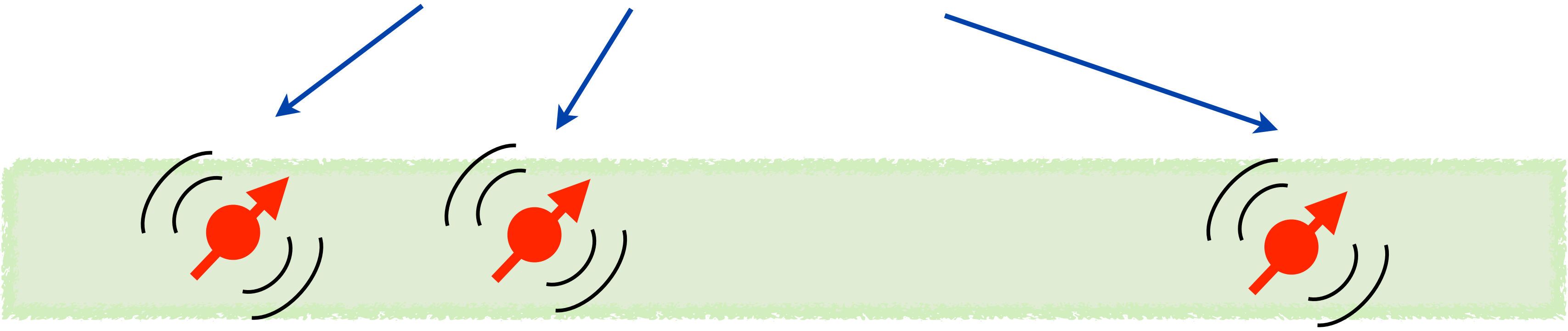
$$H_{\text{int}} = \sqrt{2}\lambda x_c(t)|e\rangle\langle e|$$

No violation !

(random) trajectory, but with a specific value at each time

Mechanical quantum transducers

Solid state spin systems



localized
electronic (nuclear) spins

“weak”
magnetic dipole
interactions

environment

“storage times”

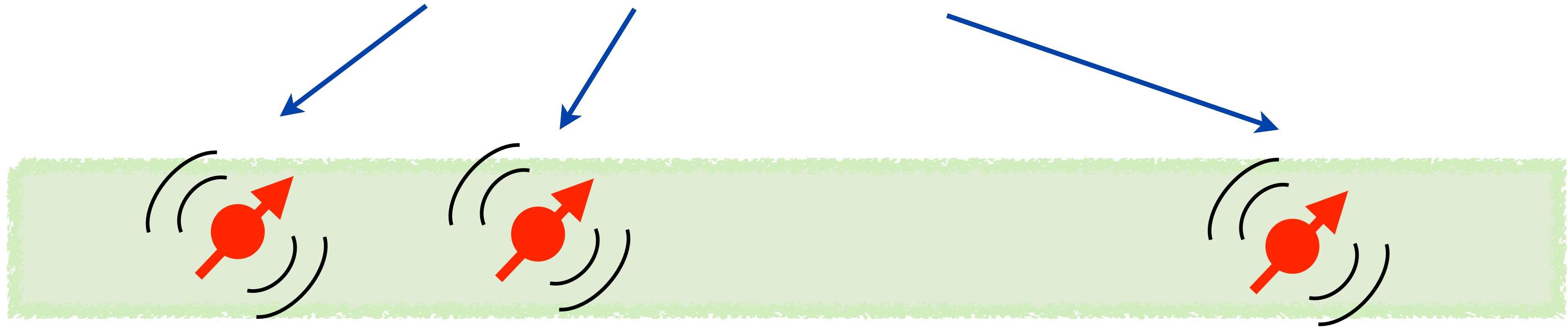
$T_2 > \ 1\,\mathrm{ms}$

(T=300 K !!!)

✓

Solid state spin systems

localized
electronic (nuclear) spins



"weak"
magnetic dipole
interactions

environment

"storage times"

$T_2 > 1\,\mathrm{ms}$

(T=300 K !!!)

✓

other spins

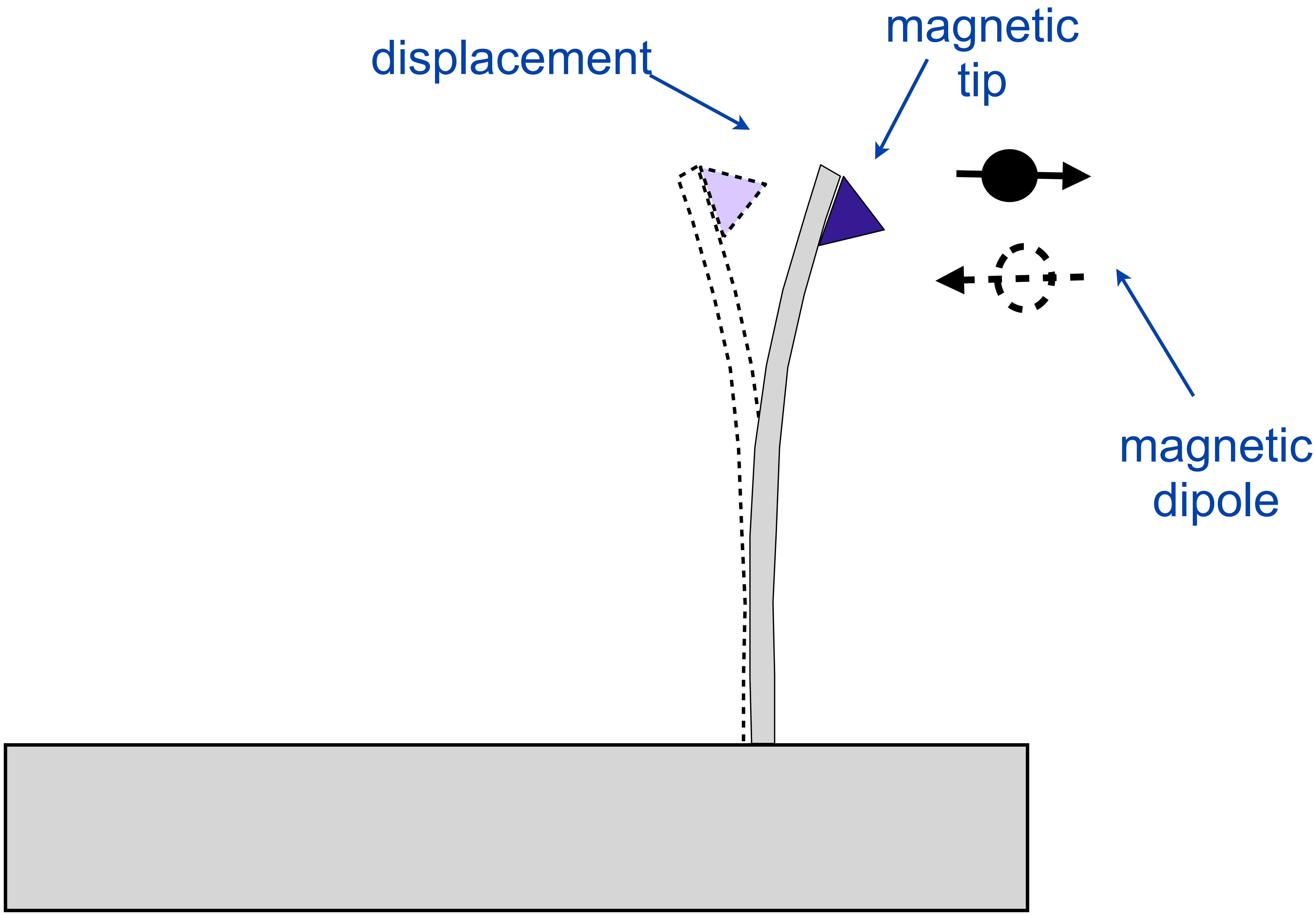
gate operations

(d < 10 nm)

?

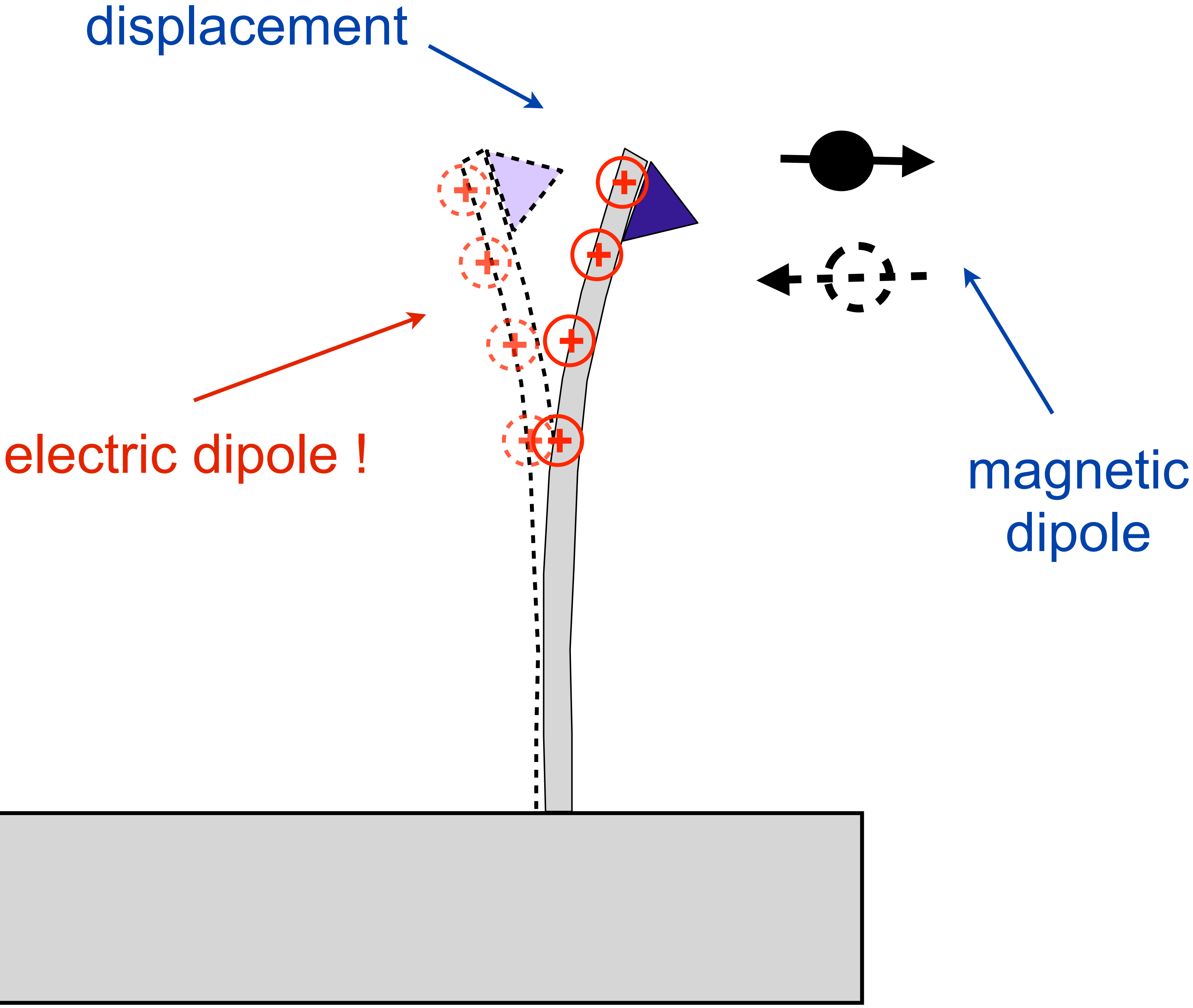
Jelezko, Wrachtrup, Nature Physics (2010)

Mechanical quantum transducer



Mechanical quantum transducer

add charge !

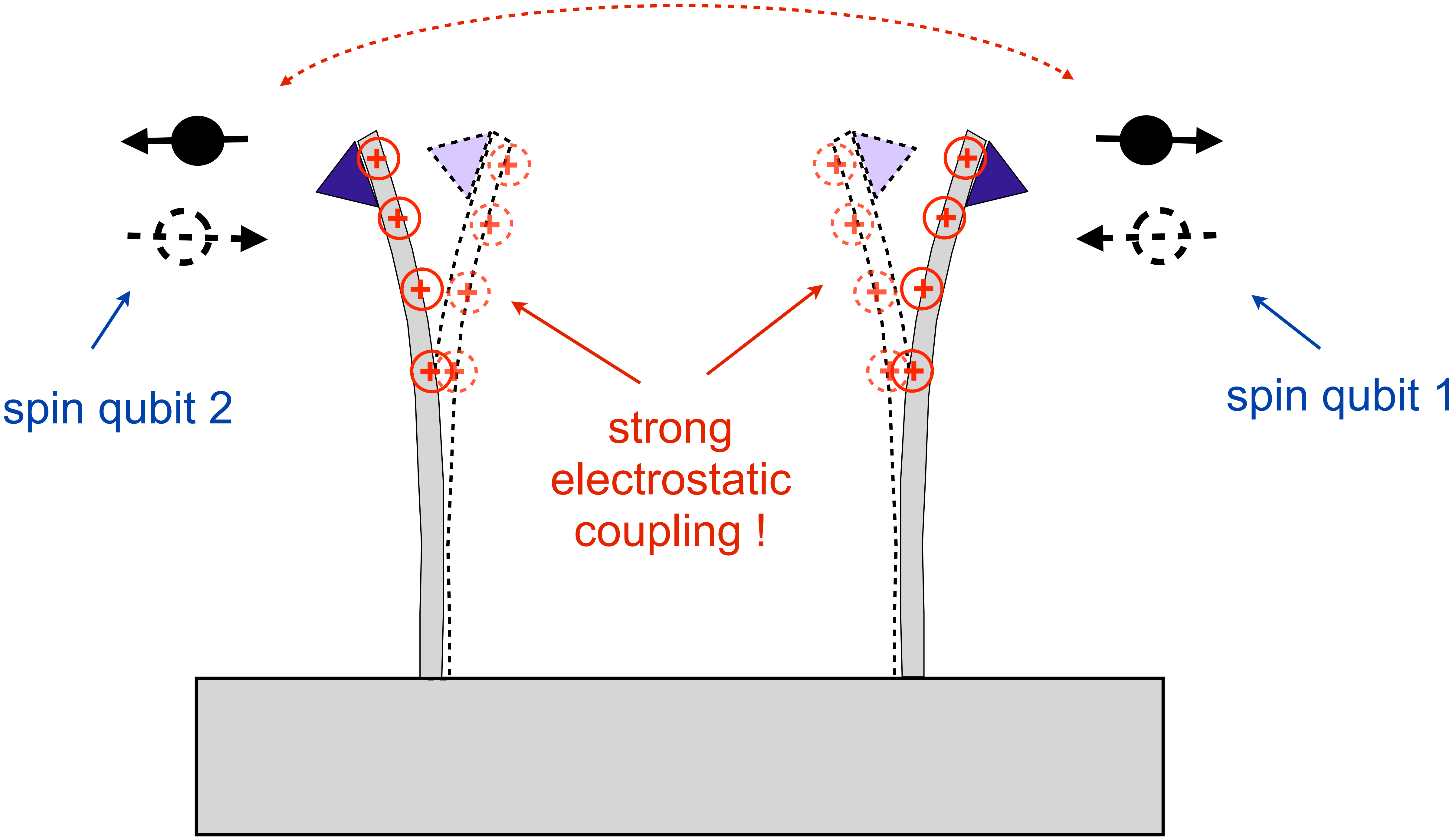


displacement

electric dipole !

magnetic dipole

Mechanical quantum transducer



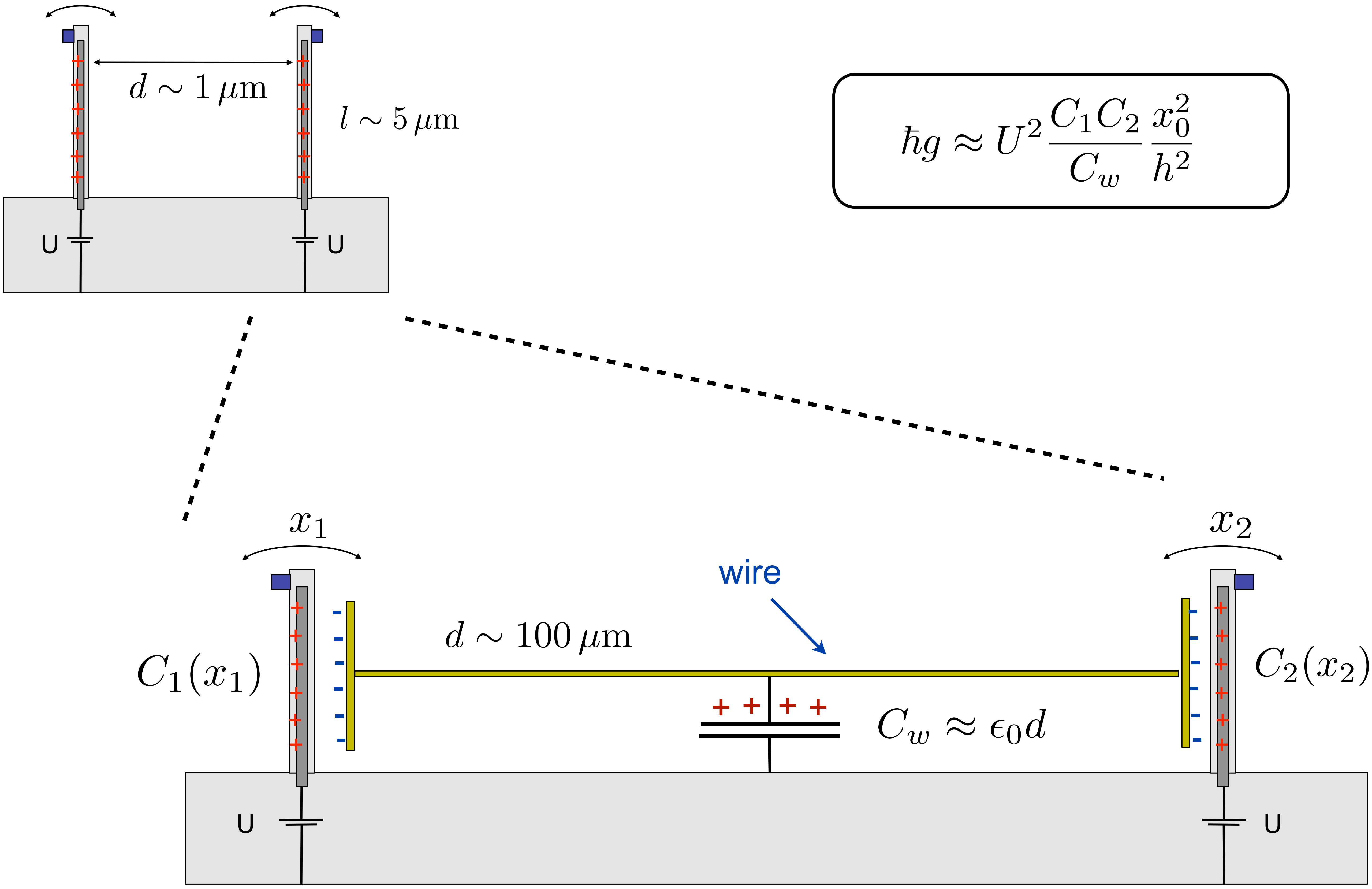
spin qubit 2

strong electrostatic coupling !

spin qubit 1

long-range spin-spin interactions !

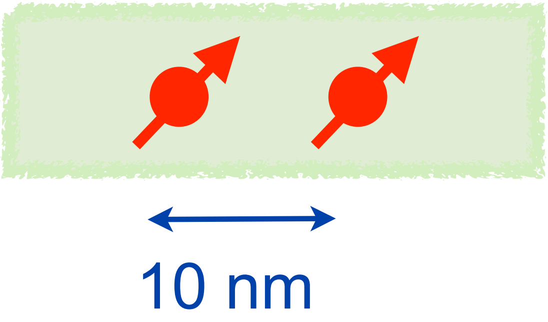
Wiring up mechanical resonators



$$\boxed{\hbar g \approx U^2 \frac{C_1 C_2}{C_w} \frac{x_0^2}{h^2}}$$

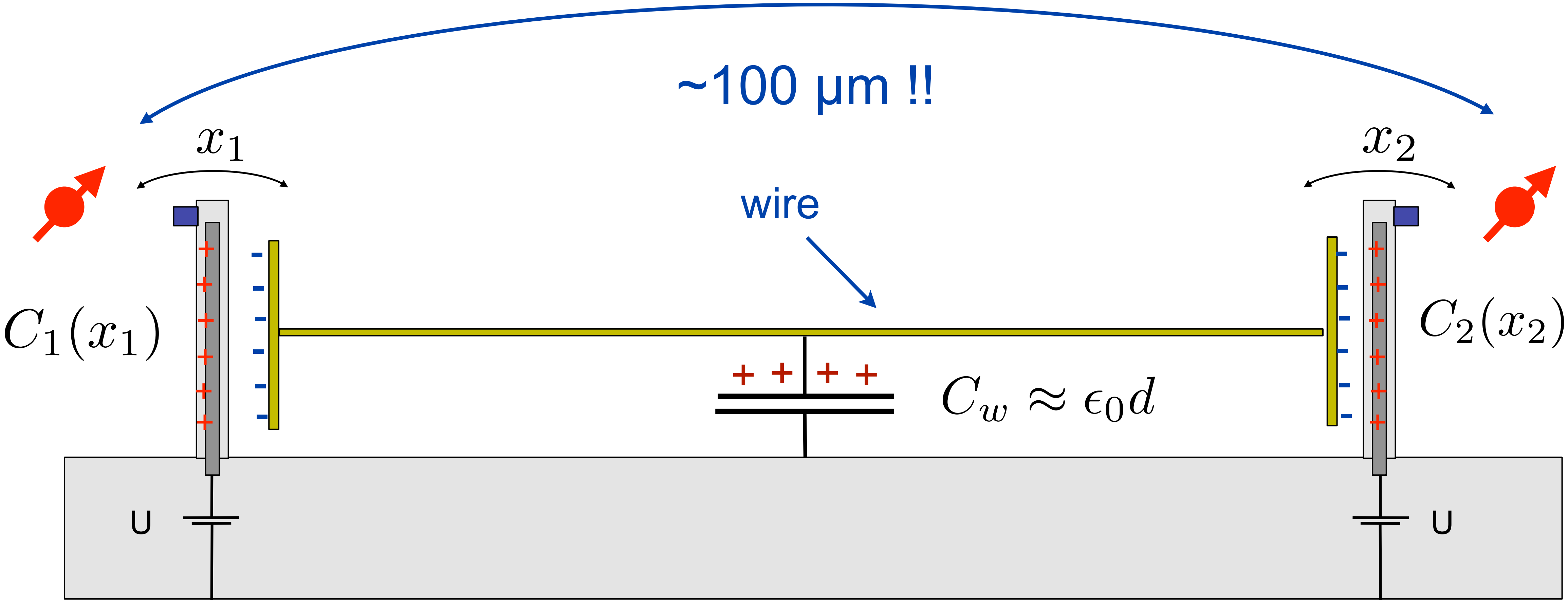
Wiring up mechanical resonators

direct, magnetic:

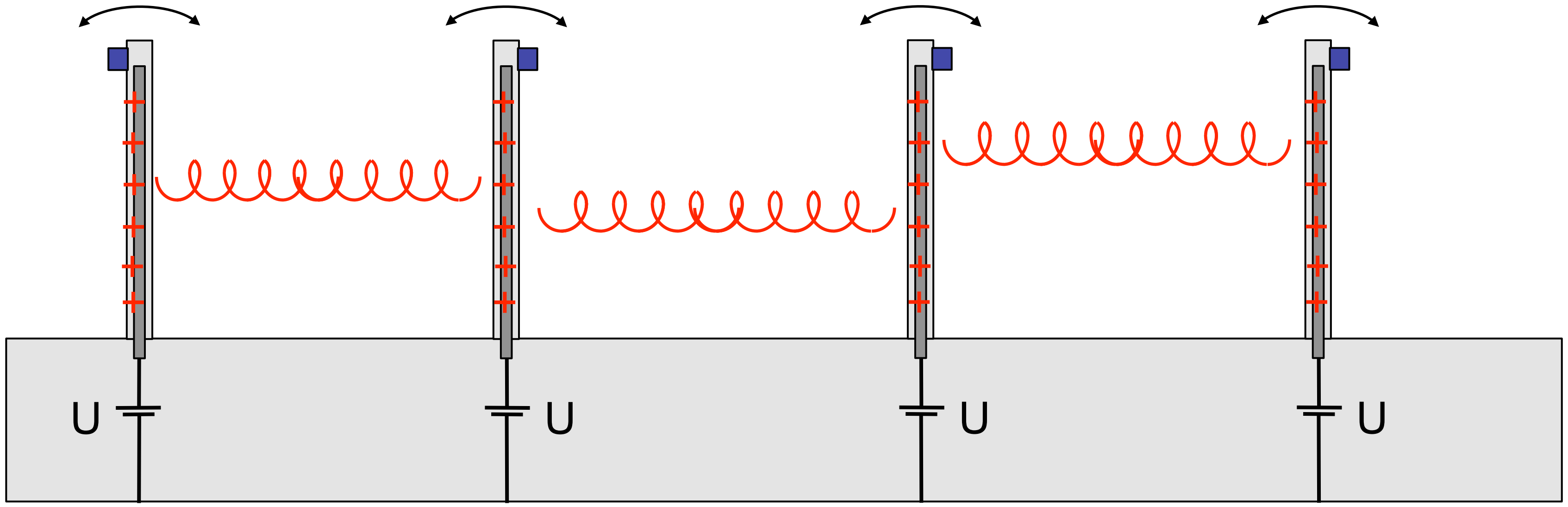


$$\boxed{\frac{g}{2\pi} \approx 90\,\mathrm{kHz}\frac{U^2[V]}{d[100\,\mu\mathrm{m}]}}$$

indirect:



Wiring up mechanical resonators

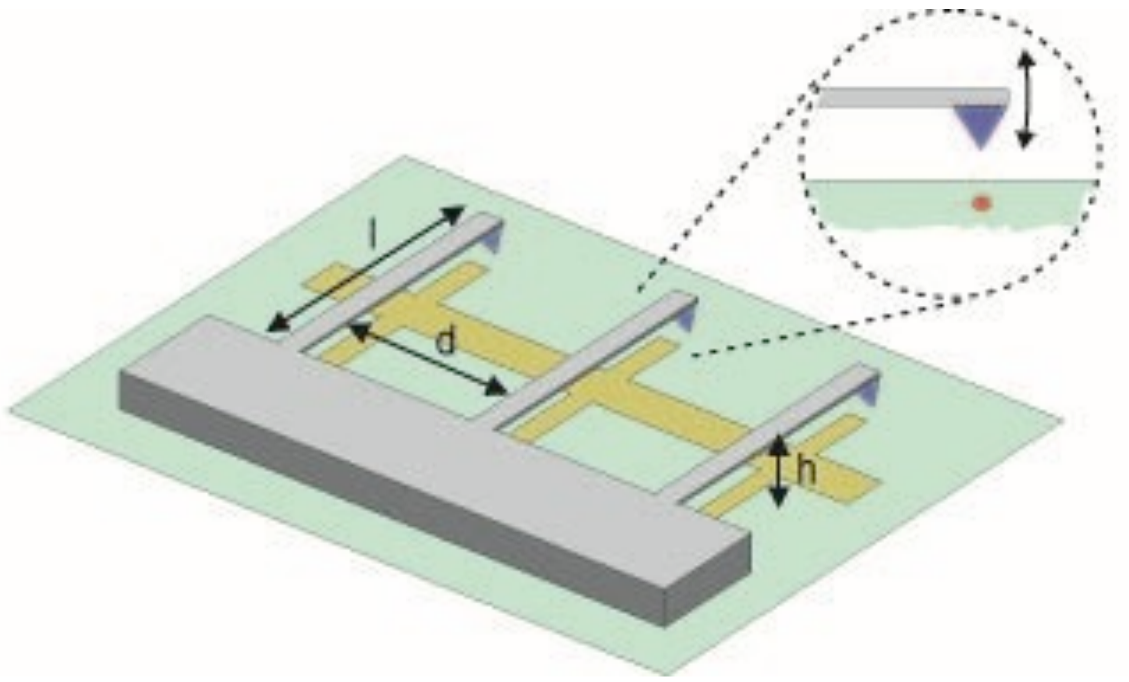


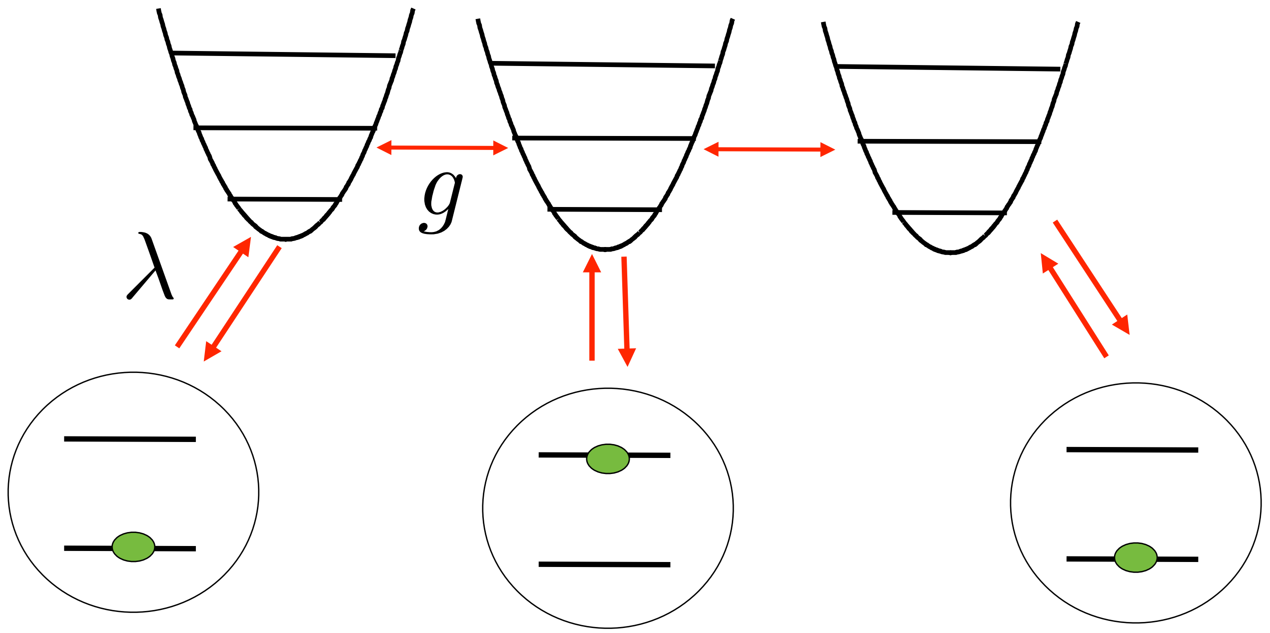
Coupled resonator chain:

$$
\begin{aligned}
H_{\mathrm{phon}} &= \sum_i \omega_i a_i^\dagger a_i + \sum_{i,j} \frac{g_{ij}}{2} (a_i^\dagger + a_i)(a_j^\dagger + a_j) \\
&= \sum_n \omega_n a_n^\dagger a_n
\end{aligned}
$$

→ collective phonon modes

Electro-mechanical quantum bus





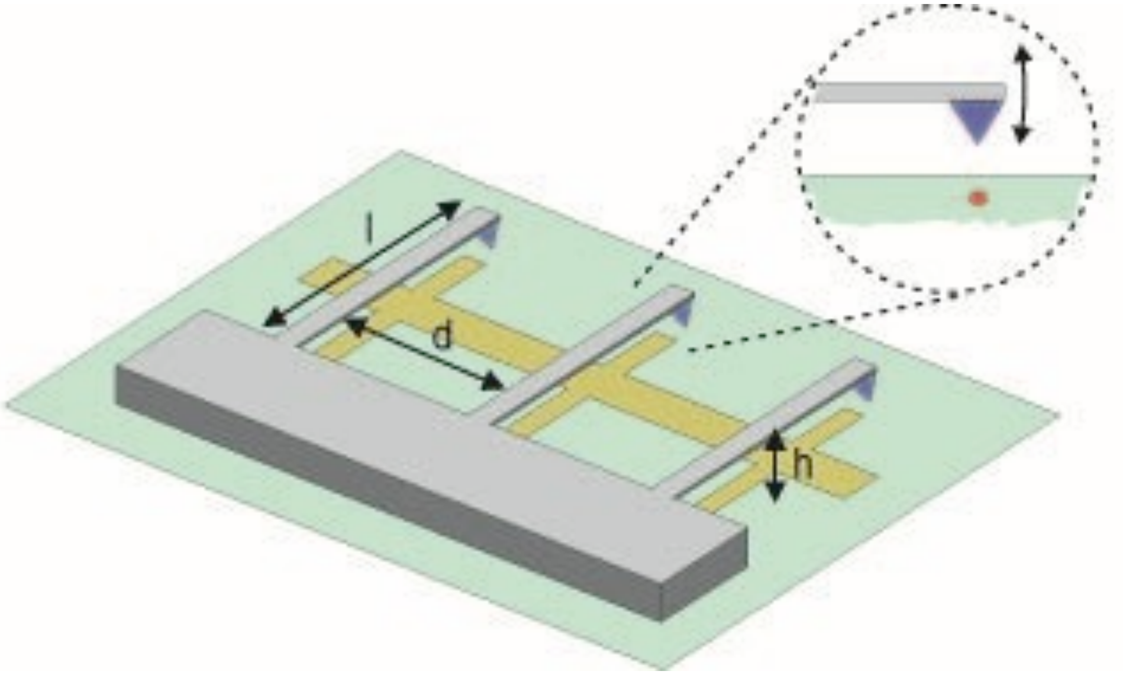
$$H = \sum_i \frac{\Omega_i(t)}{2}\sigma_x^i + \sum_n \omega_n a_n^\dagger a_n + \frac{1}{2}\sum_{i,n}\lambda_{n,i}(a_n^\dagger + a_n)\sigma_z^i$$

individual
spin control

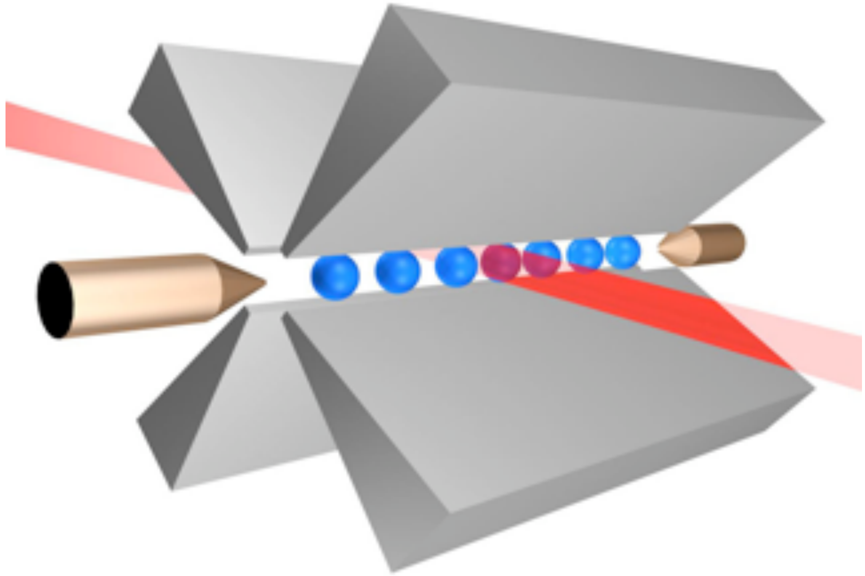
collective
phonon modes

spin-phonon
coupling

Electro-mechanical quantum bus



⇔



trapped ion
quantum computer [1]

mechanical resonators ⇒ *“artificial, massive ions”*

Phonon-mediated spin-spin interactions

$$H = \sum_n \omega_n a_n^\dagger a_n + \frac{1}{2} \sum_{i,n} \lambda_{i,n} (a_n^\dagger + a_n) \sigma_z^i$$

Phonon-mediated spin-spin interactions

$$H = \sum_n \omega_n a_n^\dagger a_n + \frac{1}{2} \sum_{i,n} \lambda_{i,n} (a_n^\dagger + a_n) \sigma_z^i$$

Polaron transformation
(≡ displaced oscillator basis)

$$\Big\Downarrow$$

$$U = e^{\sum_{i,n} \frac{\lambda_{n,i}}{\omega_n} (a_n^\dagger - a_n) \sigma_z^i}$$

$$\tilde{H} = \sum_n \omega_n a_n^\dagger a_n \boxed{- \sum_{i \neq j} M_{ij} \sigma_z^i \sigma_z^j}$$

free
phonons

$$M_{ij} = \frac{1}{4} \sum_n \frac{\lambda_{n,i} \lambda_{n,j}}{\omega_n}$$

(phonon frequencies,
mode functions)

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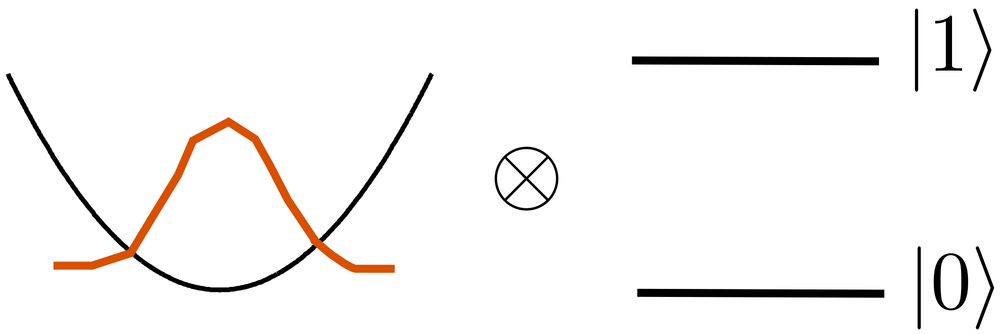
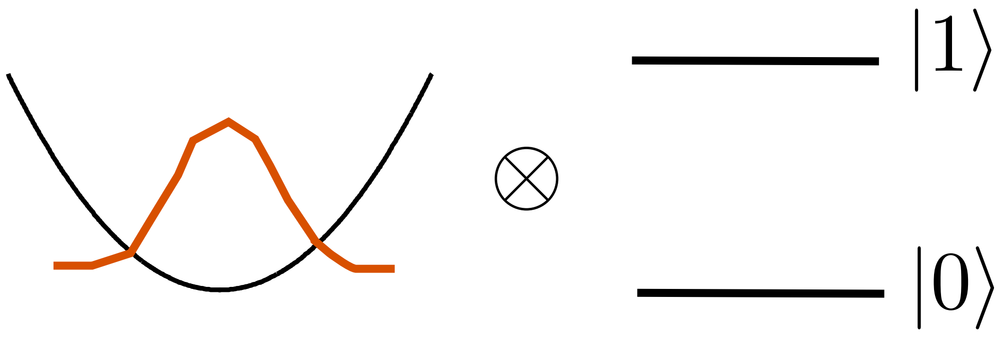
see also proposals by C. Wunderlich, I. Cirac, etc. for trapped ion QC

Phonon-mediated spin-spin interactions



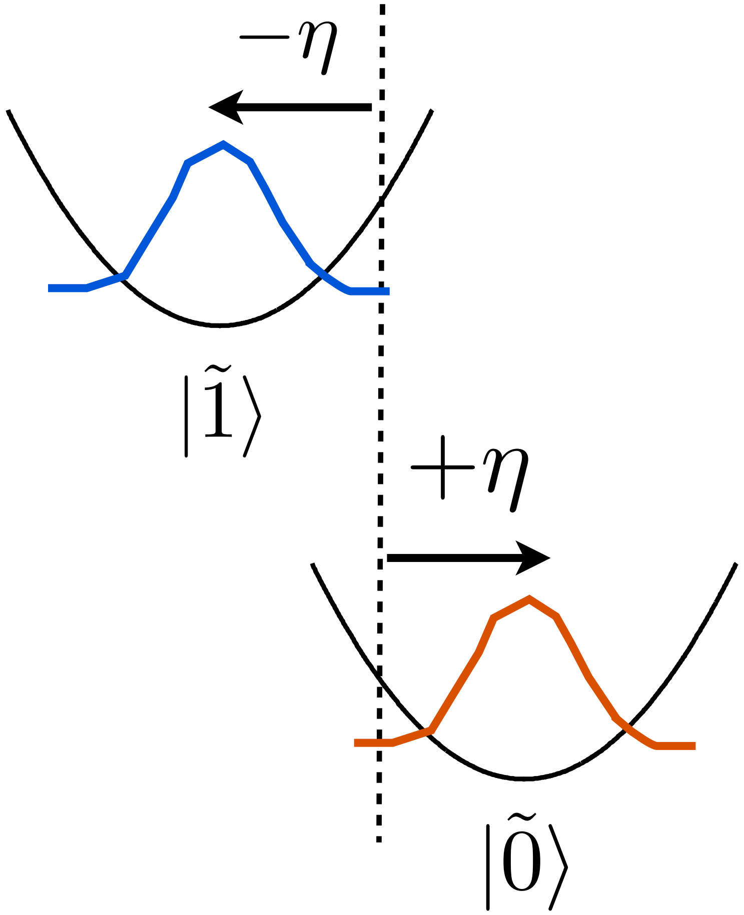
$$H = \omega_m a^\dagger a + \lambda(a + a^\dagger)\underset{\overset{\uparrow}{\pm 1}}{\sigma_z}$$

Phonon-mediated spin-spin interactions



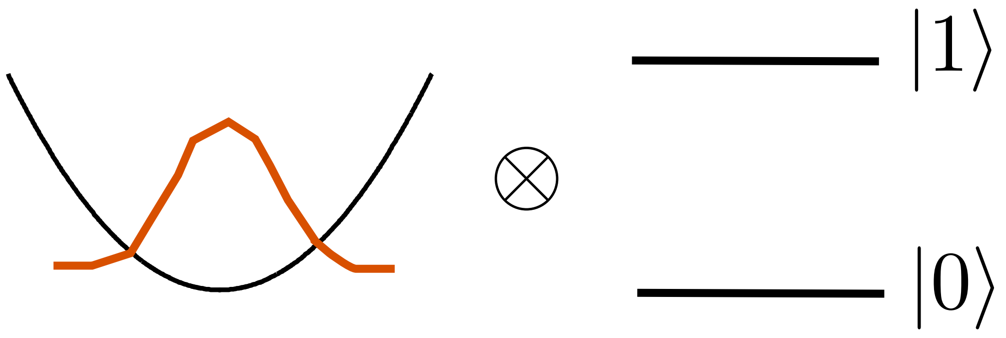
$$H = \omega_m a^\dagger a + \lambda(a + a^\dagger)\underset{\substack{\uparrow \\ \pm 1}}{\sigma_z}$$

displaced oscillator basis:



$$\boxed{\eta \equiv \frac{\lambda}{\omega_m}}$$

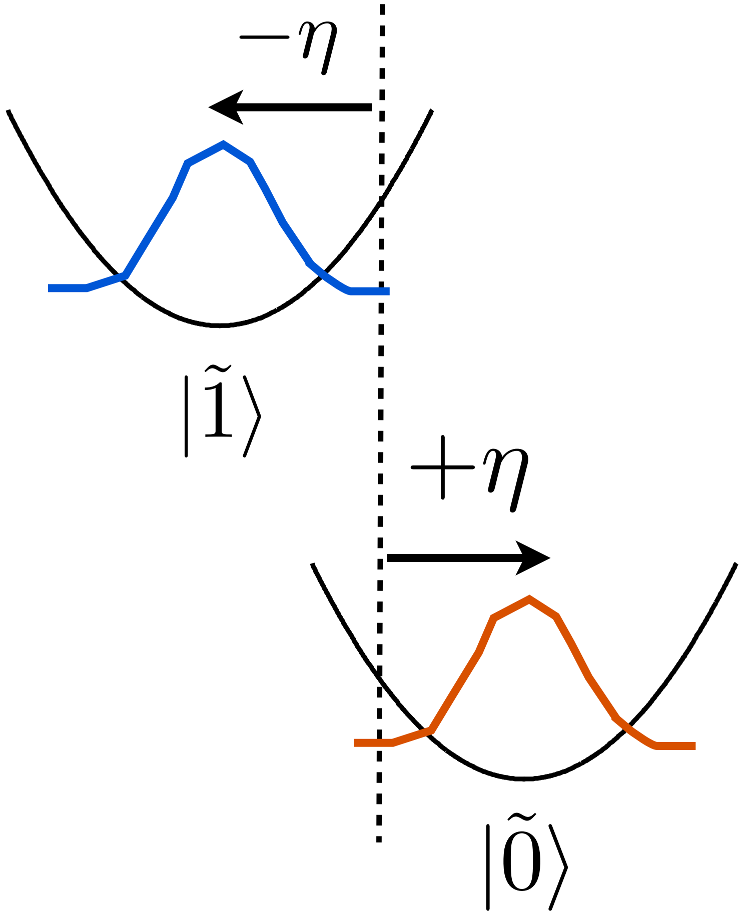
Phonon-mediated spin-spin interactions

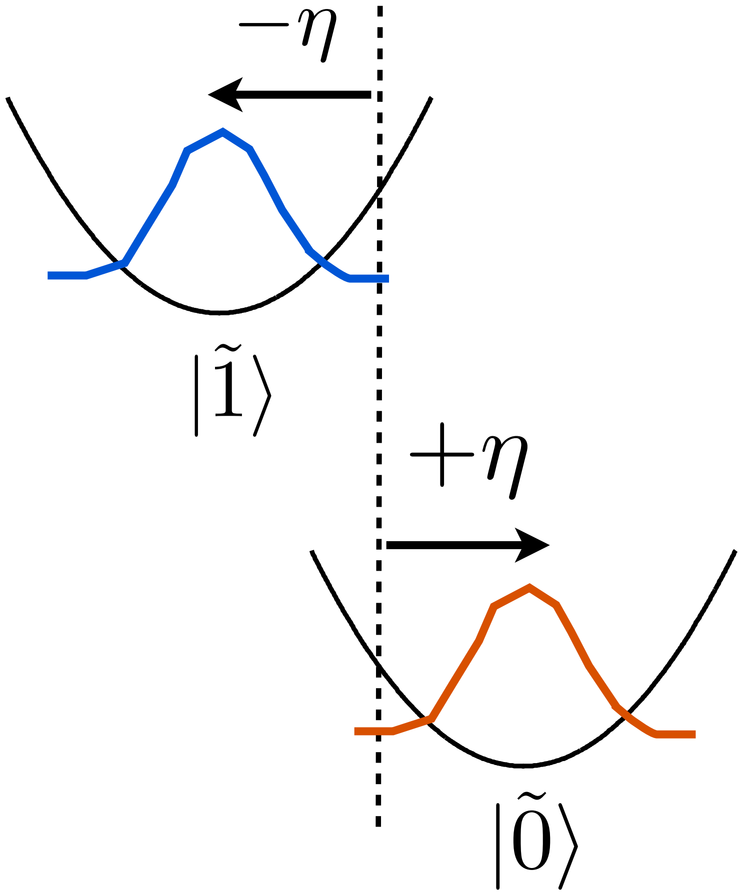


$$H = \omega_m a^\dagger a + \lambda(a + a^\dagger)\sigma_z$$

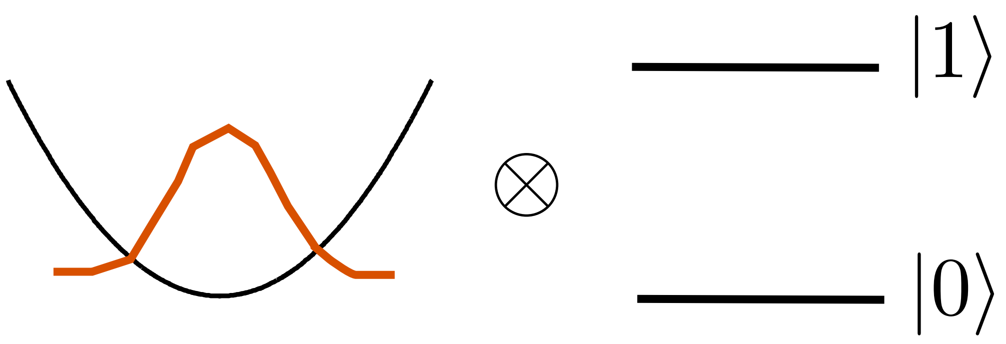
± 1

displaced oscillator basis:



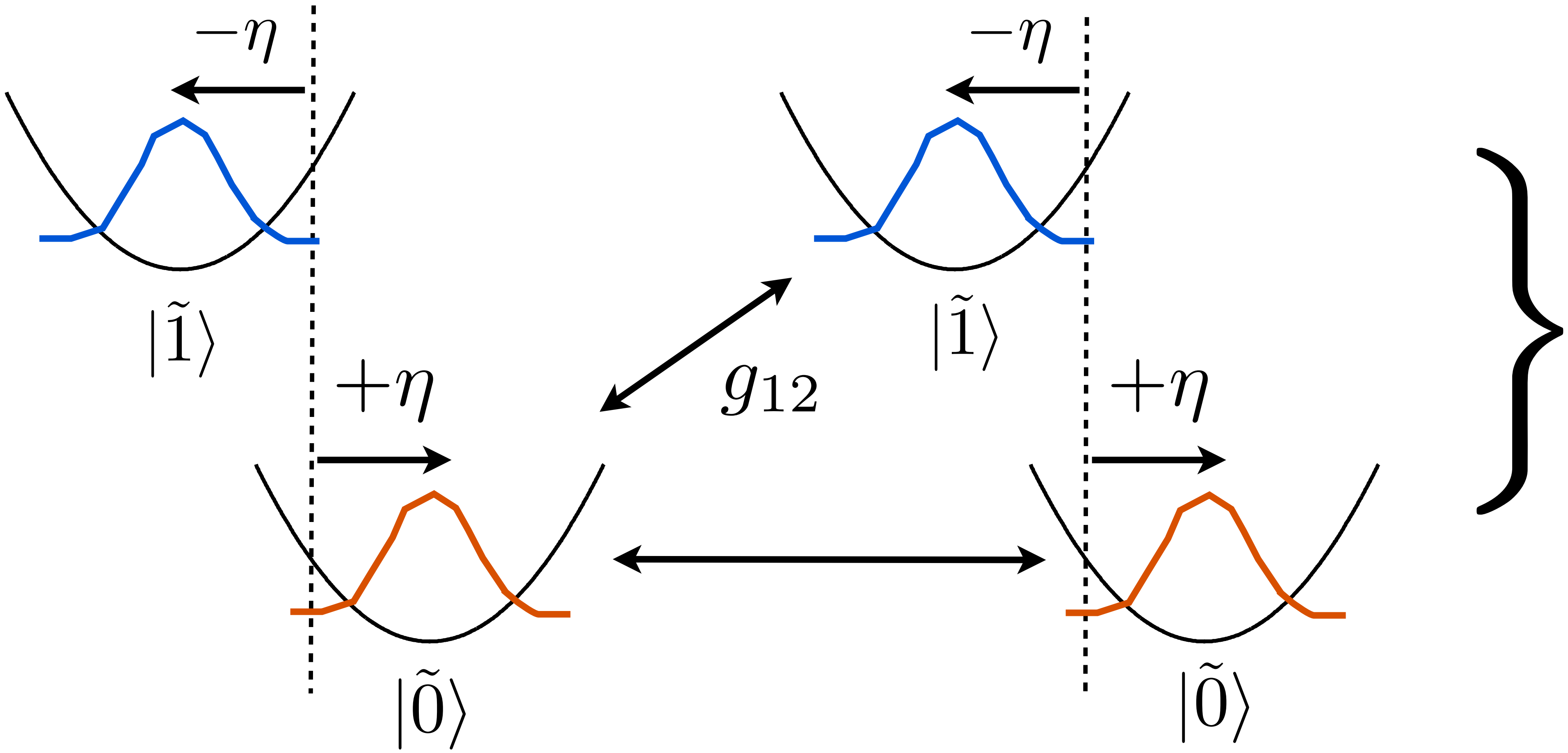


Phonon-mediated spin-spin interactions



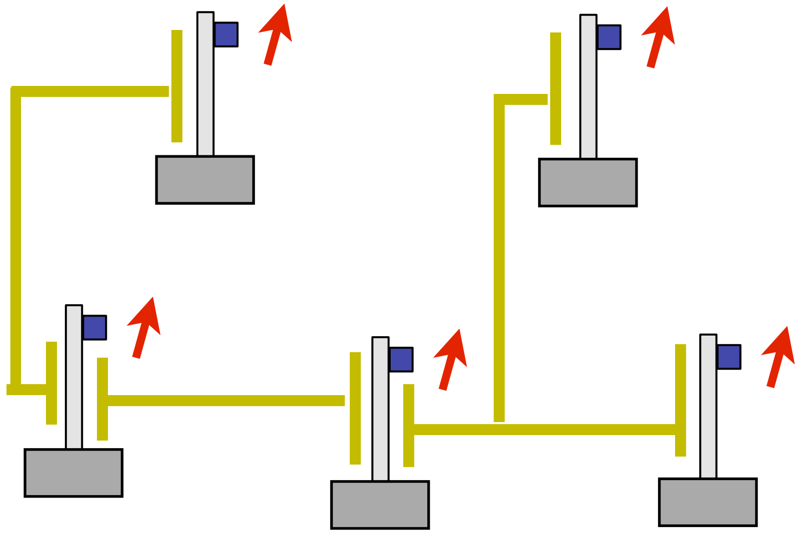
$$H = \omega_m a^\dagger a + \lambda(a + a^\dagger)\underset{\underset{\pm 1}{\uparrow}}{\sigma_z}$$

displaced oscillator basis:



effective spin-spin interactions

Wiring up spins ...

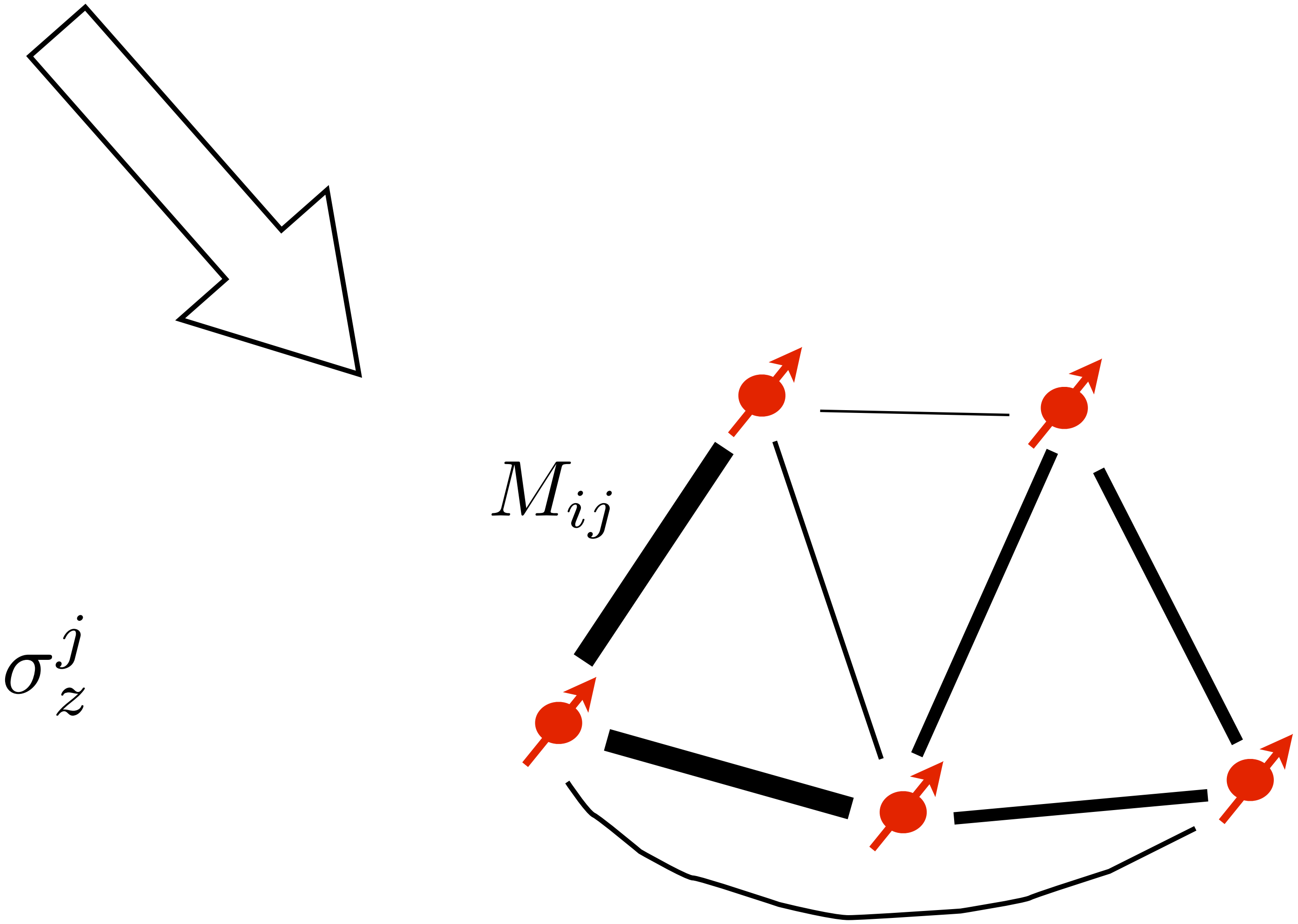


full model:

$$H = \sum_n \omega_n a_n^\dagger a_n + \frac{1}{2}\sum_i \lambda_i (a_i^\dagger + a_i)\sigma_z^i$$
$$+ \sum_{ij} g_{ij}(a_i^\dagger + a_i)(a_j^\dagger + a_j)$$

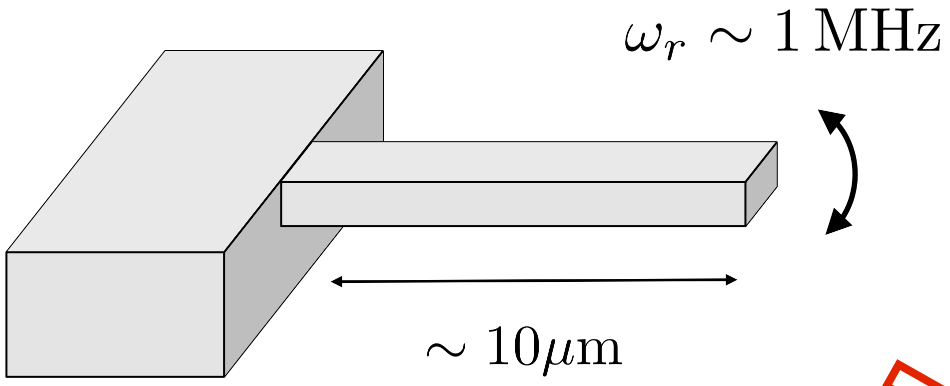
effective spin model:

$$H_{\text{spin}} = \sum_{ij} M_{ij}\sigma_z^i \sigma_z^j$$

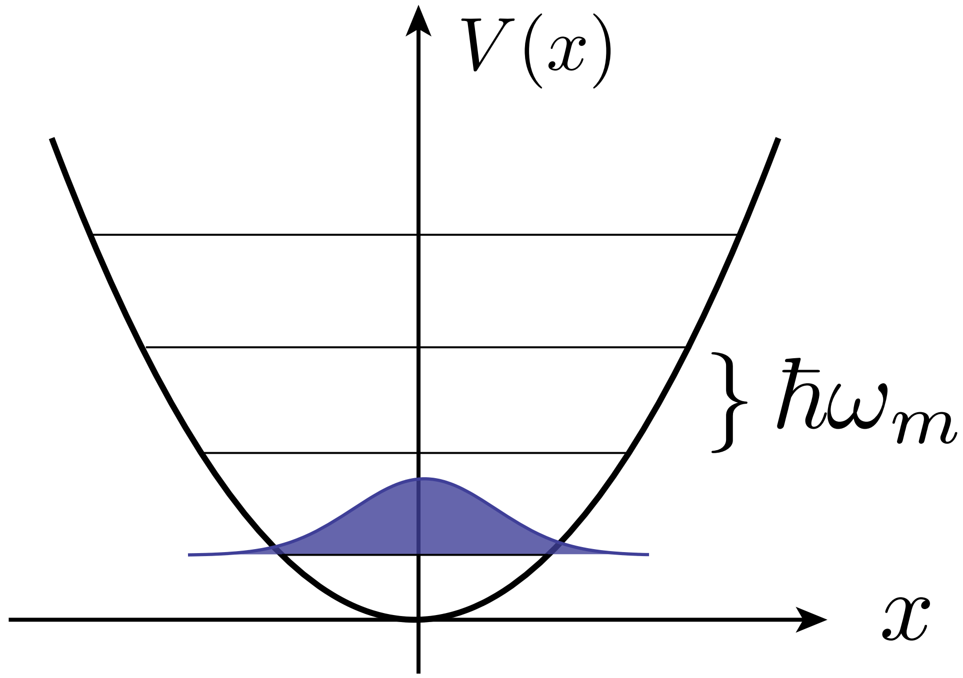


Summary & conclusions

“Quantum” mechanical systems

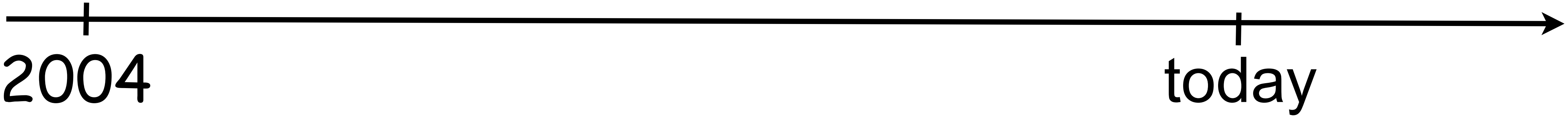


- passive / active cooling



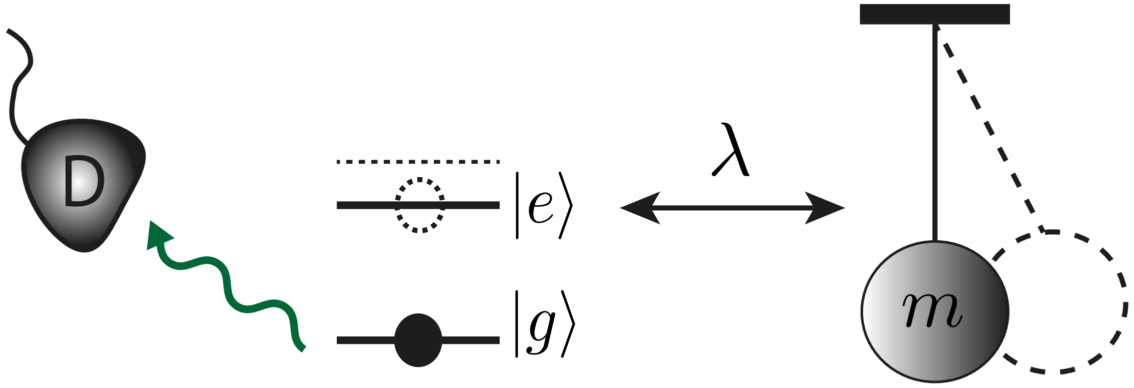
“quantum regime”

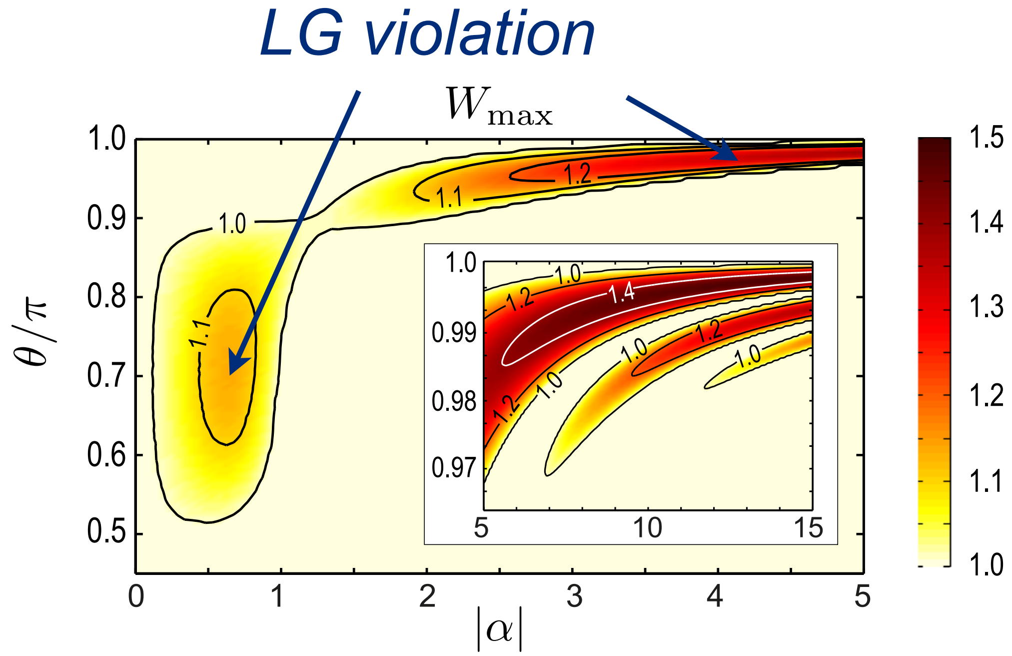
New (macroscopic) quantum degree of freedom !



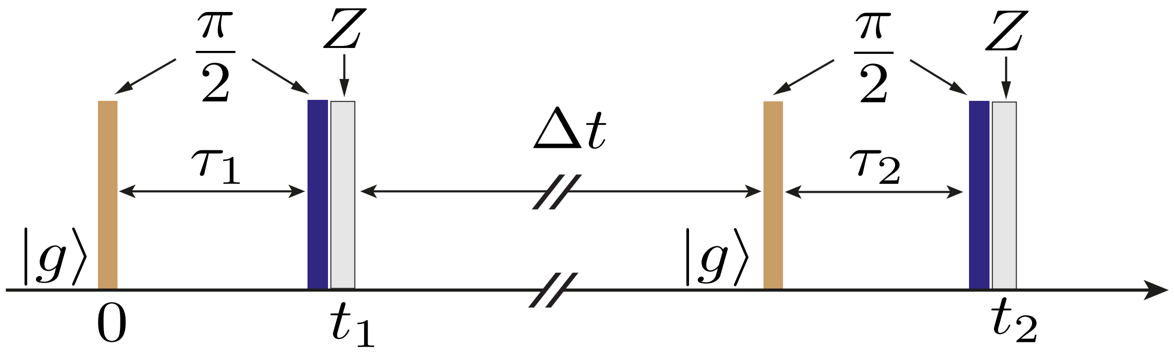
Probing QM at the macroscopic scale

- **Violation of LGI with macroscopic systems:**





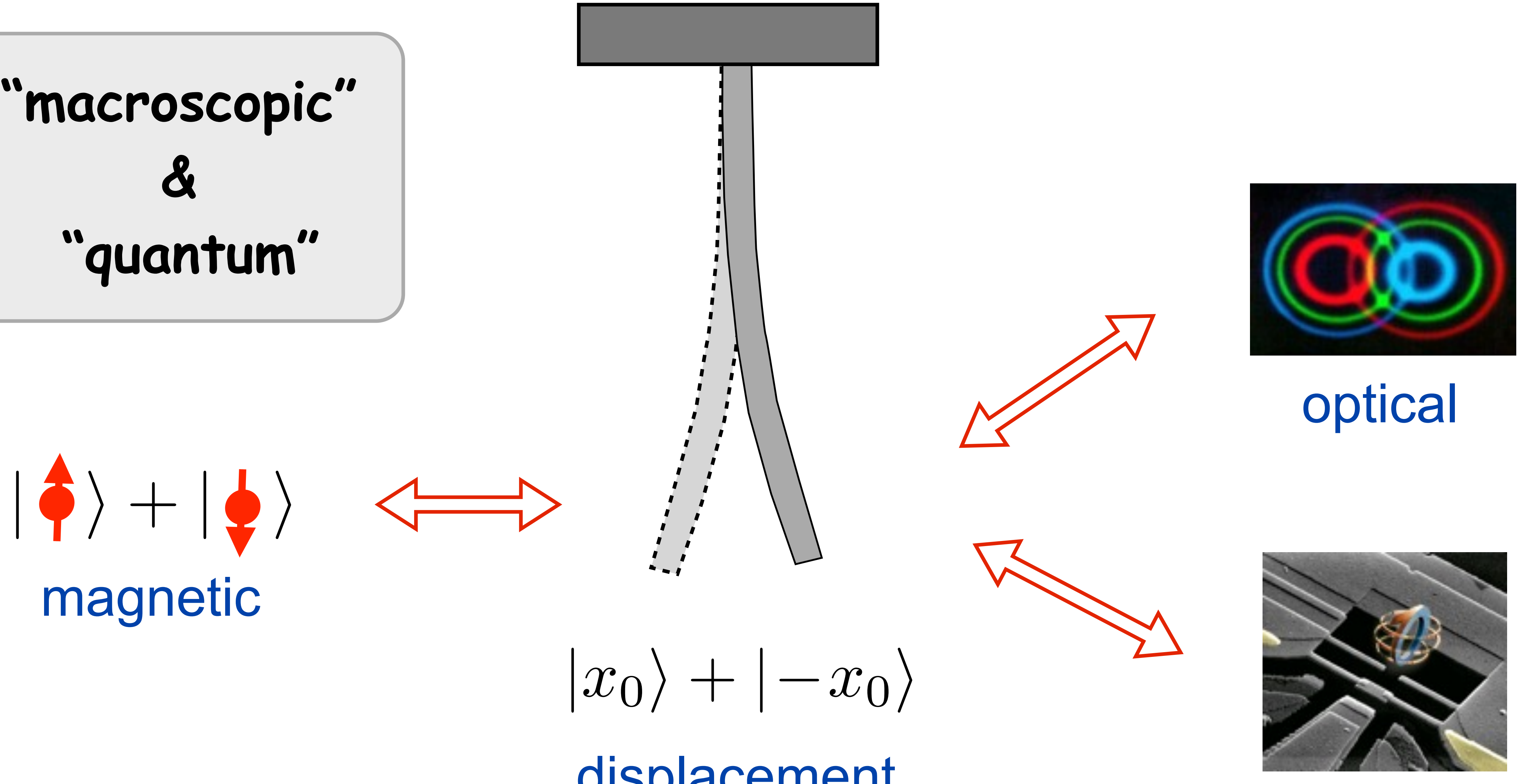
- **Ramsey correlation measurements:**
  - *modular variables*
  - *contextuality*
  - *Bell inequalities (?), ...*



*A. Asadian, C. Brukner, PR, PRL **112**, 190402 (2014)*

Mechanical quantum transducers

“macroscopic”
&
“quantum”



$|\uparrow\rangle + |\downarrow\rangle$

magnetic

$|x_0\rangle + |-x_0\rangle$

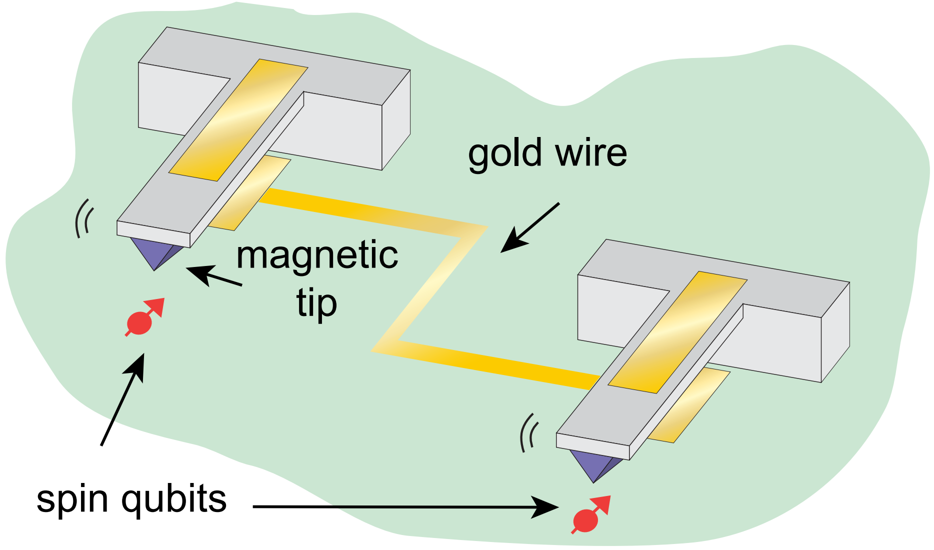
displacement

optical

electric

- Coherent interface between electric, magnetic & optical quantum system !

Mechanical quantum transducers

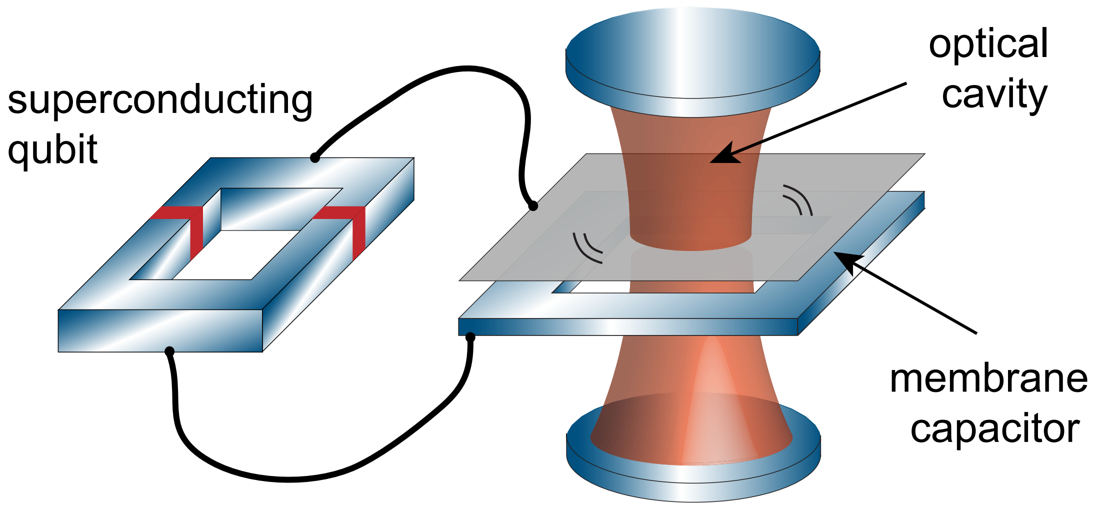


- Electro-mechanical spin-spin interactions.

PR et al, Nature Physic (2010)

- Qubit-light-interfaces.

K. Stannigel et al, PRL (2010)
A. H. Safavi-Naeini, O. Painter, NJP (2011)



Experiments: *Caltech, NIST/JILA, Copenhagen, Santa Barbara, ...*