Quantum Simulation with Rydberg Atoms

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Outline

Dissipative quantum state engineering

Rydberg atoms

Mesoscopic Rydberg gates

A Rydberg Quantum Simulator

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Dissipative quantum state engineering

Rydberg atoms

Mesoscopic Rydberg gates

A Rydberg Quantum Simulator

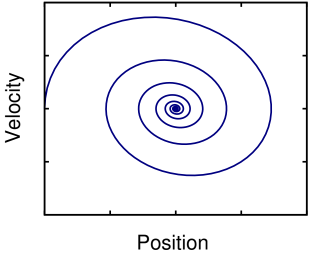
Quantum state engineering

- Tailored quantum states are an important resource (quantum simulation, quantum communication, quantum metrology, ...)
- Previously: coherent evolution (adiabatic following, quantum logic gates)
- New tool: controlled dissipation

  S. Diehl et al., Nature Phys. **4**, 878 (2008)

  F. Verstraete et al., Nature Phys. **5**, 633 (2009)

  HW et al., Nature Phys. **6**, 382 (2010)
- Engineer a suitable attractor state of the dynamics
- Inherently more robust



Quantum master equation

- Open quantum system described by a quantum master equation (Lindblad form)

$$\frac{d\rho}{dt} = -i\hbar\,[H,\rho] + \sum_n \gamma_n \left(c_n \rho c_n^\dagger - \frac{1}{2}\{c_n^\dagger c_n, \rho\} \right)$$

$\rho = \sum_i p_i \vert\psi_i\rangle\langle\psi_i\vert$	Density operator
H	Hamiltonian
c_n	Quantum jump operators (non-Hermitian)
γ_n	Decay rate

Stationary state

- Sufficient (and usually also necessary) condition: $\frac{d\rho}{dt} = 0$
- Special case: pure state $\rho = |\psi\rangle\langle\psi|$
- More general: von Neumann entropy

$$S = -\mathrm{Tr}\left\{\rho \log \rho\right\}$$

- Only Hermitian jump operators
  $\Rightarrow$ Maximally mixed state ($S = \log d$)

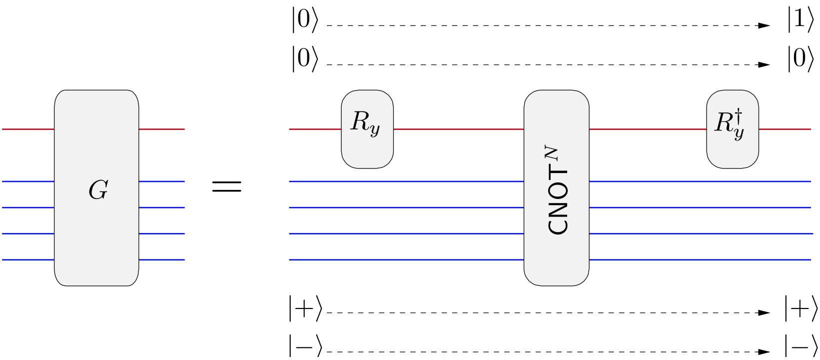
$$\rho = \begin{pmatrix} 1/d & & \\ & 1/d & \\ & & \ddots \end{pmatrix}$$

$$\frac{d\rho}{dt} = -i\hbar\,[H, \rho] + \sum_n \gamma_n \left(c_n \rho c_n^\dagger - \frac{1}{2}\{c_n^\dagger c_n, \rho\} \right)$$

Coherent quantum simulation

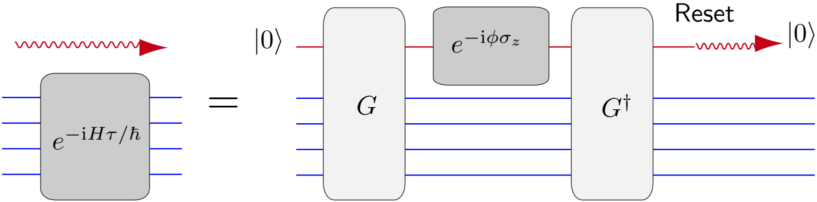
- Goal: simulate an effective plaquette interaction $H = A_p = \sigma_x^{(1)}\sigma_x^{(2)}\sigma_x^{(3)}\sigma_x^{(4)}$ using the N-body gate
- Map $|\pm 1\rangle$ eigenstates of A_p onto $|0\rangle$, $|1\rangle$ of the control atom

$$R_y = \exp(-i\pi\sigma_y/4) = \frac{1}{\sqrt{2}}\begin{pmatrix} 1 & -1 \\ 1 & 1 \end{pmatrix}$$



Coherent dynamics

- Apply the mapping G transferring the eigenvalue of A_p onto the control spin
- Write a phase $\exp(-\mathrm{i}\phi\sigma_z)$ onto the control spin
- Undo the mapping $G = G^{-1}$



- Simulates the Hamiltonian H at discrete times $t = k\tau$ (digital)
- Energy scale $E_0 = \hbar\phi/\tau$

HW et al., Nature Phys. **6**, 382 (2010)

Dissipative cooling

- Goal: cool into the ground state of $H = A_p$ (-1 eigenstate)
- Use the same mapping (ensemble $\mapsto$ control) as before
- Instead of writing a phase on the control spin: controlled spin flip of one *random* ensemble spin j

$$U = |0\rangle\langle 0|_c \otimes \mathbf{1} + |1\rangle\langle 1|_c \otimes \exp(\mathrm{i}\phi\sigma_z^{(j)})$$

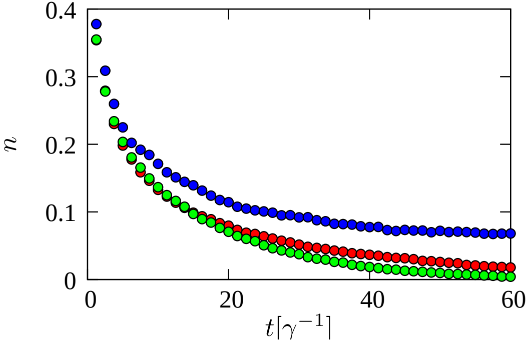
- If we do a spin flip: control atom will not end in $|0\rangle$
- Reset spin (incoherent) from $|1\rangle$ to $|0\rangle$
- Discrete Markovian master equation

$$\hat{\rho}(t+\tau) = \hat{\rho}(t) + \gamma\left(c\hat{\rho}c^\dagger - \frac{1}{2}\left\{c^\dagger c, \hat{\rho}\right\}\right)$$

- Rate $\gamma = \phi^2/\tau$, jump operator $c = \sigma_z^{(j)}(1 + A_p)/2$

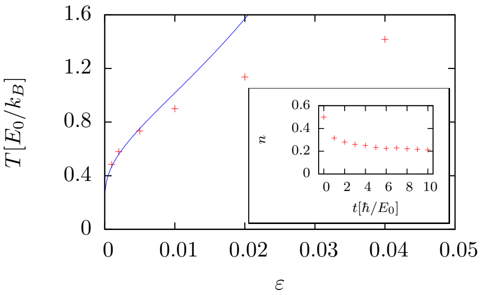
Scaling up

- Cooling: random walk of the anyons
- Averaging over 10^3 realizations of the dynamics
- Imperfections: residual anyon density n



Linear response theory

- Gate error probability ε: probability to end up in a state orthogonal to the desired one
- Toric code: gate errors create anyons



HW, Mol. Phys. **111**, 1753 (2013)

- Uncorrelated errors ⇒ Effective temperature

$$T \approx -\frac{2E_0}{k_B \log n}$$

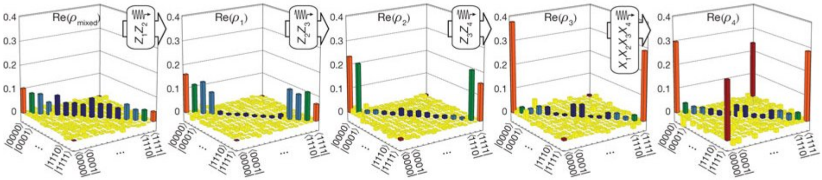
- Anyon density n within linear response: $n = 14\varepsilon$

- ⇒ Effective temperature benchmarks the quantum simulator

Experimental realization

- Proof of principle experiment with trapped ions
- N-body Mølmer-Sørensen gate

  A. Sørensen and K. Mølmer, Phys. Rev. Lett. **82**, 1971 (1999)
- Four ensemble spins + 1 control ion
- Minimal instance of a toric code Hamiltonian (1 plaquette)



J. Barreiro et al., Nature **470**, 486 (2011)

Outline

Rydberg states

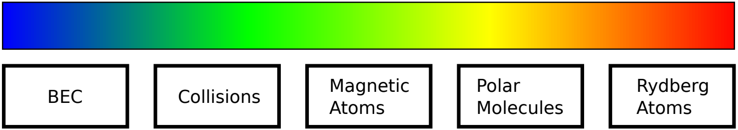
- Electronic excitations with large principal quantum number n
- Hydrogen-like wavefunctions

Rb ground state $5s$:

Rydberg state $35s$:

- Long ($\propto n^3$) life-time ($100\,\mu\mathrm{s}$)
- Large ($\propto n^2$) diameter ($100\,\mathrm{nm}$)
- Highly sensitive to electric fields
- Strong dipolar ($\propto n^4$) or van der Waals ($\propto n^{11}$) interactions

Interaction strength:

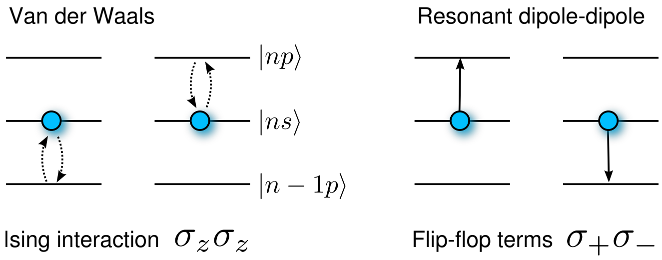


Quantum defect

- Correction to the hydrogenic energy levels

$$E = -\frac{Rhc}{(n - \delta_{l,j})^2}$$

- Depends only on the angular quantum numbers
- Lifts l degeneracy

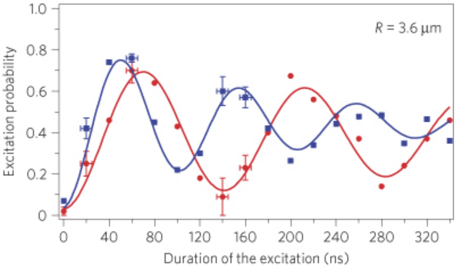


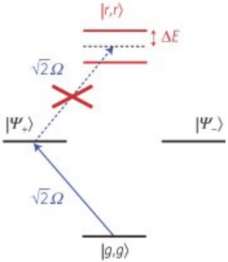
Rydberg blockade

- Strong interaction between Rydberg states shift the doubly excited state
- Rabi oscillations are enhanced by a factor $\sqrt{N}$

  D. Jaksch et al., PRL **85**, 2208 (2000)

  M. D. Lukin et al., PRL **87**, 037901 (2001)





A. Gaëtan et al., Nature Phys. **5**, 115 (2009)

see also M. Saffman

- $\Omega_{\text{eff}} = (1.38 \pm 0.03)\Omega$

Outline

Single qubit gates

- Encode quantum information into two hyperfine ground states
- Resonant microwave driving

$$H = \hbar\omega|1\rangle\langle 1| + \hbar\Omega\cos(\omega t)(|0\rangle\langle 1| + |1\rangle\langle 0|)$$

- Rotating frame

$$|1\rangle \to |1\rangle \exp(i\omega t)$$

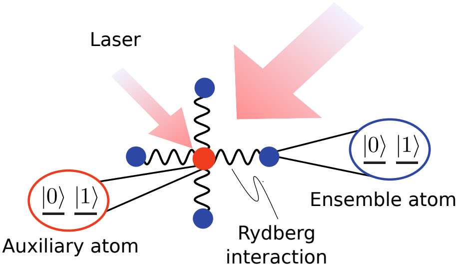
$$H = \frac{\hbar\Omega}{2}(1 + \exp(2i\omega t))(|0\rangle\langle 1| + |1\rangle\langle 0|)$$

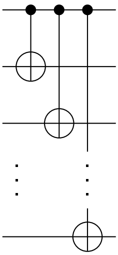
- Rotating wave approximation: neglect fast oscillating term
- Unitary time-evolution operator

$$U = \exp(-iHt/\hbar) = \begin{pmatrix} \cos(\Omega t) & -i\sin(\Omega t) \\ -i\sin(\Omega t) & \cos(\Omega t) \end{pmatrix}$$

- Rotation about x axis; rotation about z axis using detuning

Many-body quantum gates

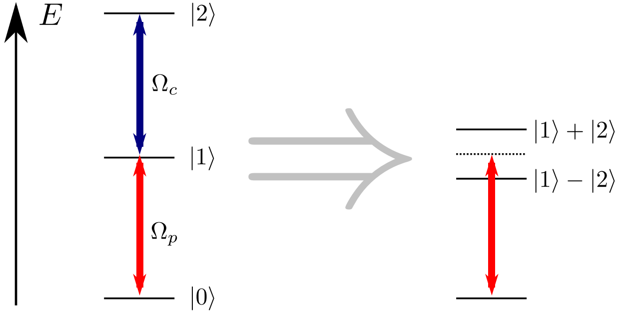




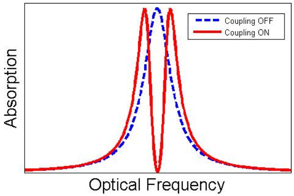
- Introduce auxiliary atoms to mediate interactions
- Controlled-NOT^N plus single qubit gates
- Single-step implementation based on Electromagnetically Induced Transparency
  M. Müller, I. Lesanovsky, HW, H.P. Büchler, P. Zoller, PRL **102**, 170502 (2009)

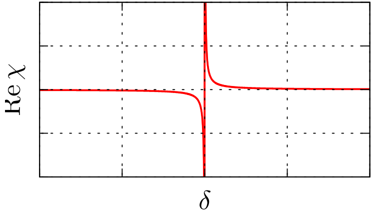
Electromagnetically Induced Transparency

- Destructive interference effect in a multi-level system



- Linear susceptibility $\chi \sim \rho_{01}$

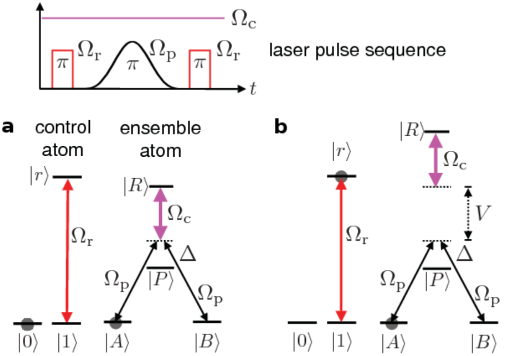




Pulse sequence

- Ensemble atoms start in $|A\rangle$
- 3 laser fields: Ω_r, Ω_c (strong), Ω_p (weak)

1. π pulse on control atom $|1\rangle \mapsto |r\rangle$
2. Adiabatic Raman pulse from $|A\rangle$ to $|B\rangle$
   a. Control in $|0\rangle$: transfer is blocked (EIT)
   b. Control in $|r\rangle$: transfer is enabled
3. π pulse on control atom

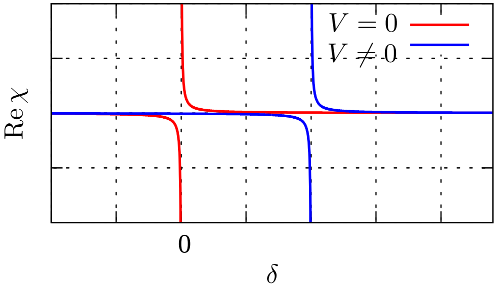


Linear susceptibility

- Effective Hamiltonian:

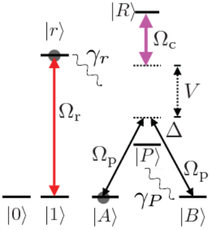
$$H \sim \sqrt{2}(\Omega_p/\Omega_c)^2 \left|+\right\rangle\left\langle+\right| + (1+V) \left|R\right\rangle\left\langle R\right| + \Omega_p/\Omega_c\Big(\left|+\right\rangle\left\langle R\right| + \mathrm{h.c.}\Big)$$

- Rydberg-Rydberg interaction shifts the EIT resonance
- Two-photon resonance ($\delta = 0$) is no longer dark



Limiting factors

- Incoherent processes
  - Radiative decay of the $|P\rangle$ level: $\Delta \gg \gamma_P$
  - Radiative decay of the control atom: $\Omega_p \gg \gamma_r$
- EIT condition
  - Sharp EIT resonance: $\Delta \gg \Omega_c$
  - Interaction shift: $V \gg \Omega_c$
- Adiabatic Raman pulse
  - $\Omega_c \gg \Omega_p$



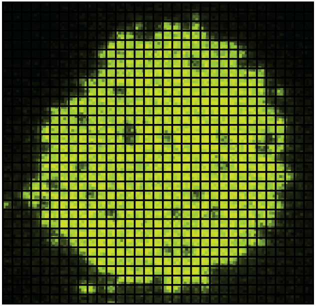
- Effect of ensemble-ensemble interaction V_{ee} depends on N
- Transfer is enhanced by a repulsive interaction
- Blocking is reduced: superatom calculation gives a phase shift

$$\phi < N(N-1)\phi_0 \left(1 - \frac{2}{V_{ee}}\right) \; ; \; \phi_0 = 35\pi/48(\Omega_p/\Omega_c)^2$$

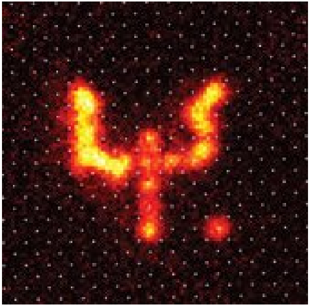
Outline

Current status

- Mott insulator of ultracold atoms in optical lattices with more than 1,000 atoms
- Arbitrary lattice geometries
- Single site addressing
- Need to achieve strong interactions between the atoms
- Laser excitation to Rydberg states



(M. Greiner, Harvard)



(I. Bloch, MPQ)

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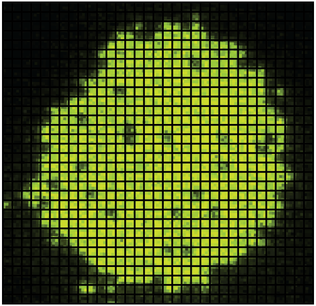

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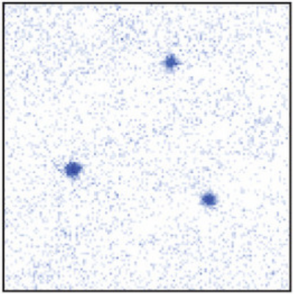
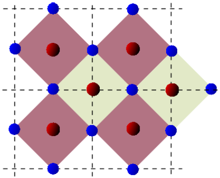


Nature **491**, 87 (2012)

(I. Bloch, MPQ)

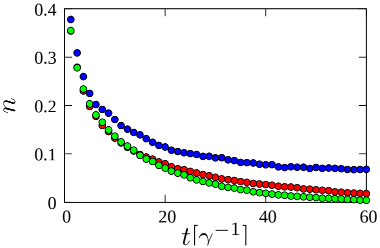
Example I: Toric code

- Control atoms (red) and ensemble atoms (blue) on a 2D lattice
- Plaquette interaction $A_p = \sigma_x^n$ (light red), site interaction $B_s = \sigma_z^n$ (green)



- Toric code Hamiltonian (Kitaev, 2003)

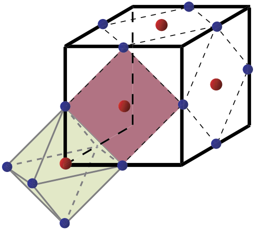
$$H = -E_0 \left(\sum_p A_p + \sum_s B_s \right)$$

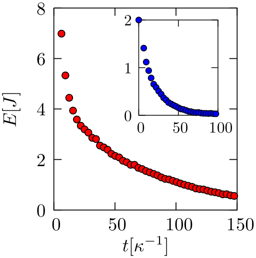


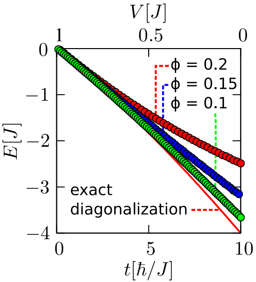
- Rydberg quantum simulator: $E_0 \approx 100\,\text{kHz}$
- Cooling: random walk of the anyonic excitations

Example II: Lattice Gauge Theory

- $H = U\sum_{o}\left(\sum_{i\in o}\hat{\sigma}_z^{(i)}\right)^2 - J\sum_{p}B_p + V\sum_{p}B_p^2$
- Ring-exchange $B_p = \hat{\sigma}_+\hat{\sigma}_-\hat{\sigma}_+\hat{\sigma}_- + \mathrm{h.c}$ via gate sequence
- Low-energy sector $(U \gg J, V)$: three spins up/down on each octahedron
- $V = J$: Rokhsar-Kivelson point (non-stabilizer state)
- $V < J$: Spin liquid phase with Coulombic $1/r$ interactions







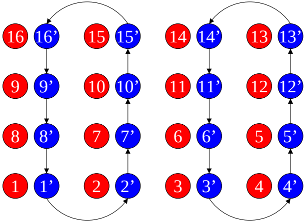
Example III: Fermi-Hubbard model

- 2D Fermi-Hubbard model is believed to be realized in high-temperature cuprate superconductors

$$H = -t \sum_{<ij>\sigma} c^\dagger_{i\sigma} c_{j\sigma} + U \sum_i n_{i\uparrow} n_{i\downarrow}$$

- Mapping fermions onto spins: Jordan-Wigner transformation
- Problem in 2D: Wigner strings (highly nonlocal interactions)
- Solution: Introduce auxillary fermion field

  Verstrate, Cirac, J. Stat. Mech. **2005**, P09012 (2005)



HW, M. Müller, H. P. Büchler, I. Lesanovsky, Quant. Inf. Proc. **10**, 885 (2010)

$$H_{\text{aux}} = -V \sum_{\{i,j\}\sigma} P_{i',j'} P_{j'+1,i'-1}$$

$$P_{i',j'} = (d_{i'\sigma} + d^\dagger_{i'\sigma})(d_{j'\sigma} - d^\dagger_{j'\sigma})$$

- Results in local six-body interactions

Summary and outlook

- Mesoscopic Rydberg gate based on Electromagnetically Induced Transparency
- Universal quantum simulation on a 100 kHz timescale
- Dissipative state preparation
- Simulation of complex spin models
- Robust against real experimental errors

HW et al., Nature Phys. **6**, 382 (2010)
HW, M. Müller, H. P. Büchler, I. Lesanovsky, Quant. Inf. Proc. **10**, 885 (2010)
HW, Mol. Phys. **111**, 1753 (2013)

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PhD and Postdoc Positions Available!

- Freigeist project "Quantum States on Demand"
- Quantum state engineering
- Dissipative many-body quantum dynamics