Strongly interacting Fermi gases

Gora Shlyapnikov
LPTMS, Orsay, France
University of Amsterdam

Outline

- Atomic fermions. BCS-BEC crossover
- Ferromagnetism in 2-component Fermi gases
- Fermi mixtures. Experiments
- BCS-BEC crossover in Fermi mixtures

Reisenburg, July 28, 2016

Experiments ^{40}K ^{6}Li

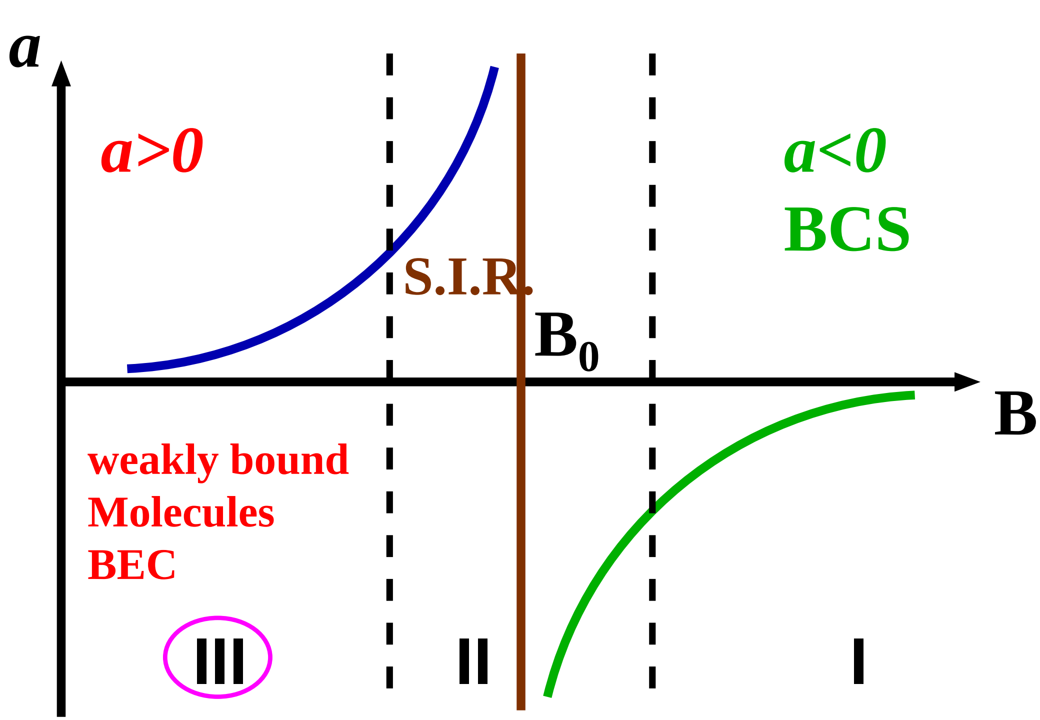
Dilute limit $nR_e^3 \ll 1$ Ultracold limit $\Lambda_T \gg R_e$

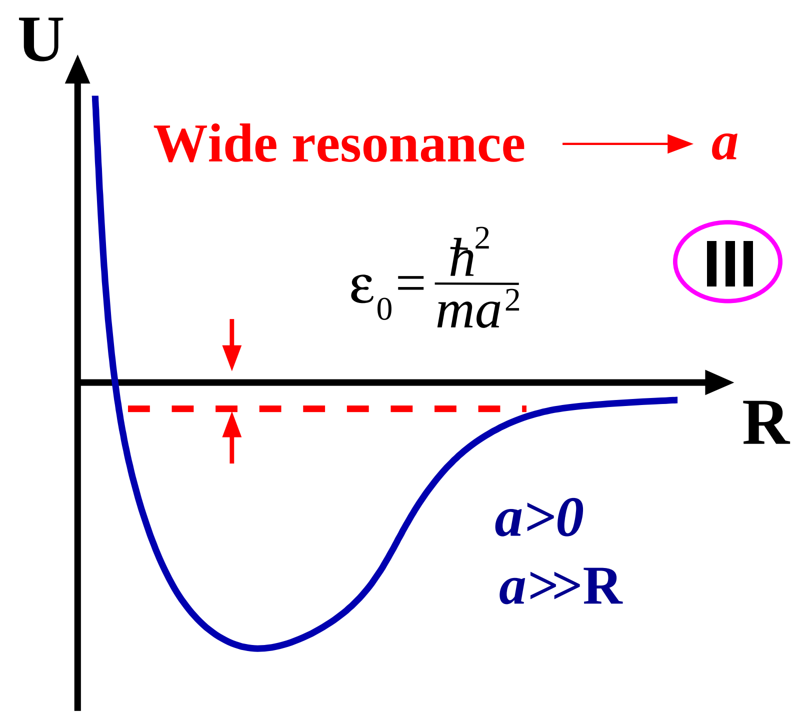
Quantum degeneracy $\rightarrow$ JILA 1998 ^{40}K

At present $n \sim 10^{13} - 10^{14}\mathrm{cm}^{-3}; \quad T \sim 1\mu\mathrm{K} - 100\mathrm{nK}$

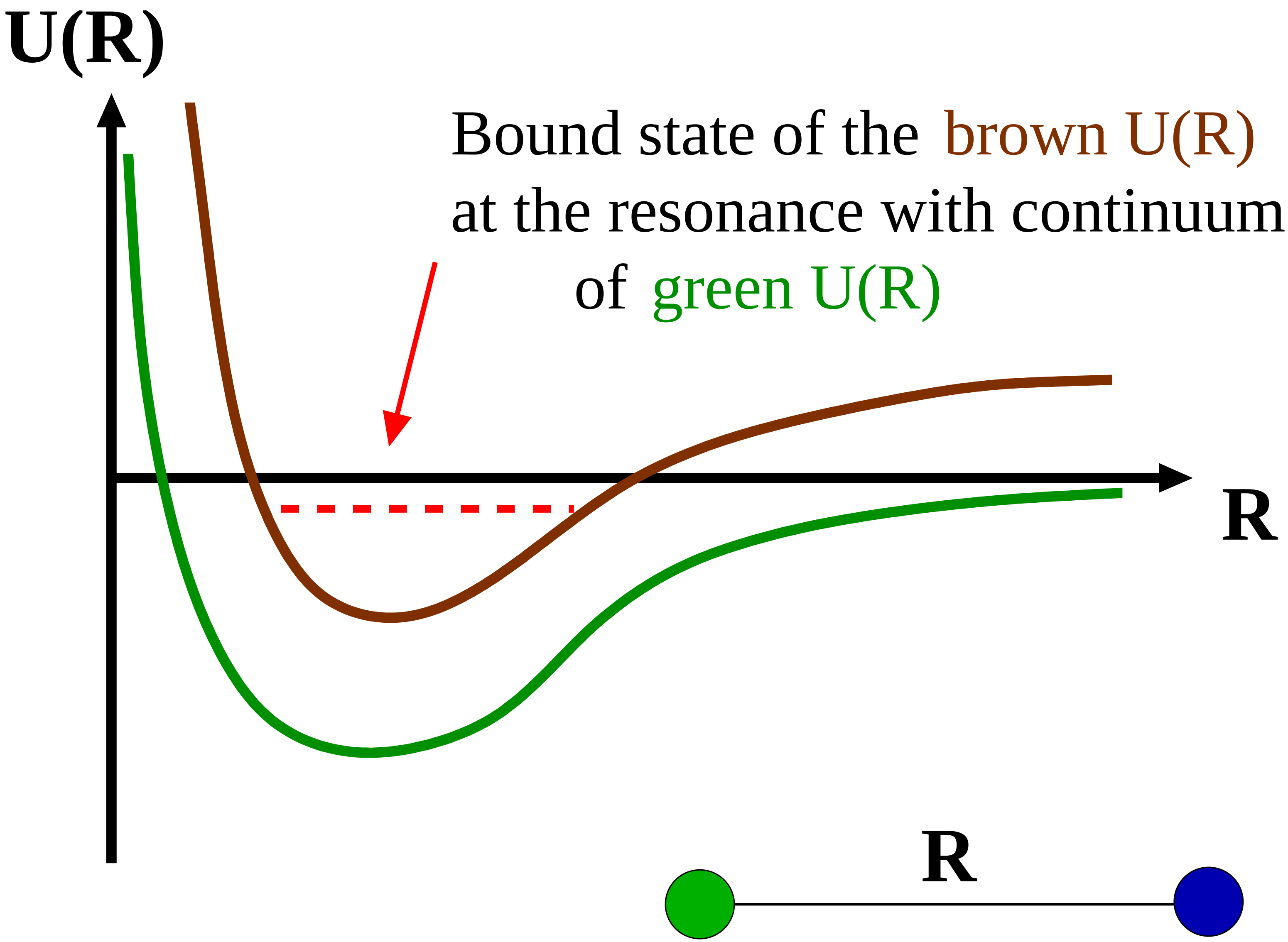
Superfluid BCS transition $\Rightarrow T_C \sim E_F \exp(-\pi/2k_F|a|)$ extremely low for ordinary attractive interaction ($a < 0;\ |a| \lesssim 10\mathrm{nm}$ and $k_F|a| \ll 1$)

FFR resonances JILA,LENS,Innsbruck,MIT,ENS,Rice,Duke,Melbourne,Tokyo,elsewhere





Feshbach resonance



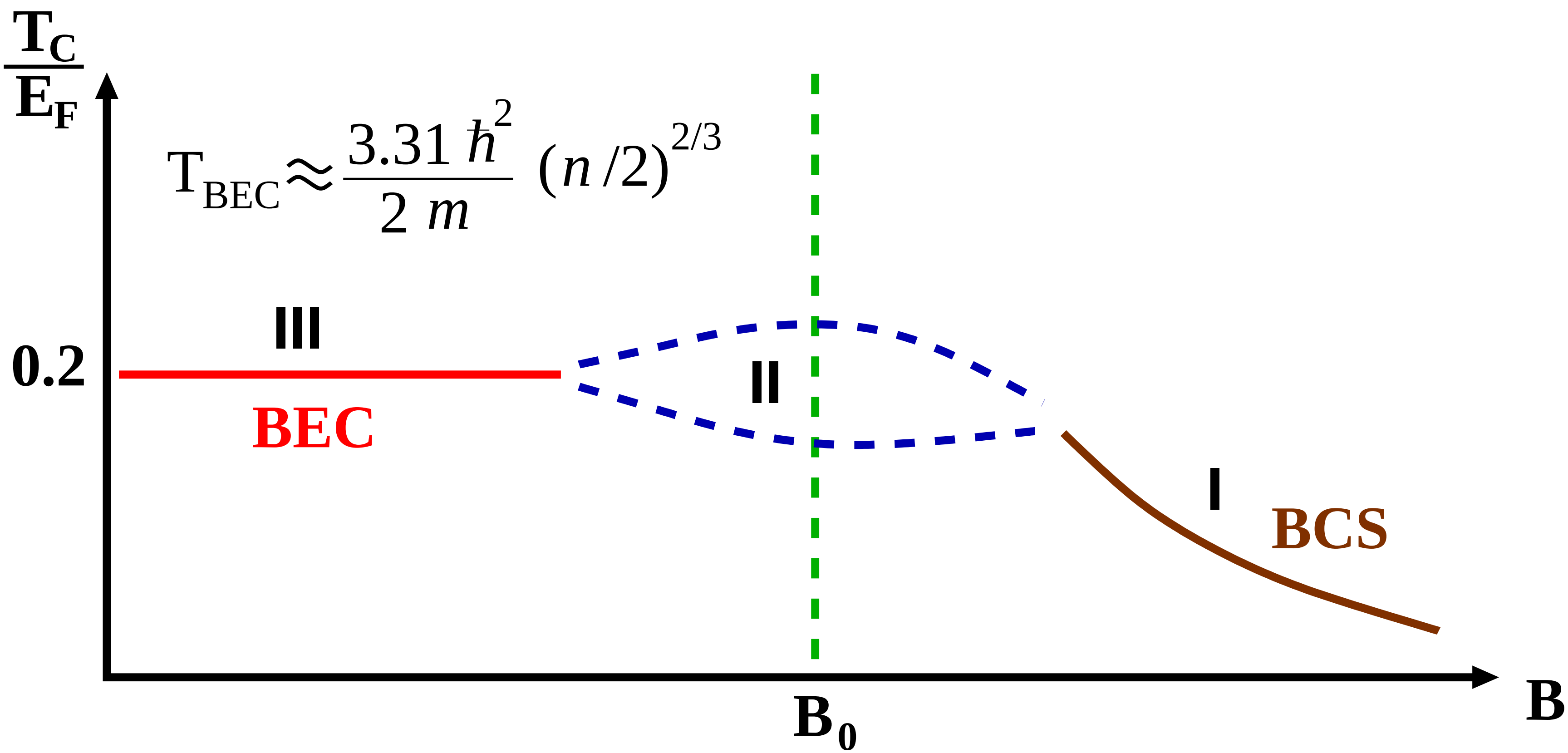
Superfluid regimes

I $k_F|a| \ll 1 \;\rightarrow$ **BCS**

II $k_F|a| > 1 \;\rightarrow$ **Strongly interacting regime**

III $na^3 \ll 1 \;\rightarrow$ **Gas of bosonic molecules**

$a \gg R_e \;\rightarrow$ BEC of weakly bound molecules



BCS-BEC crossover: Leggett, Nozieres-Schmitt-Rink

Strongly interacting regime

Wide resonance (single-channel model)

$T = 0 \quad k_F|a| \gg 1 \quad \rightarrow$ Only one distance scale $n^{-1/3}$

Only one energy scale $E_F \sim \hbar^2 n^{2/3}/m$

Universal thermodynamis (J. Ho)

Monte Carlo studies $\rightarrow \mu \approx 0.4 E_F$
(Carlson et al, Giorgini/Astracharchik, etc.)

Nature of superfluid pairing, Transition temperature, Excitations

$T_c = 0.15 E_F$ UMASS-ETH

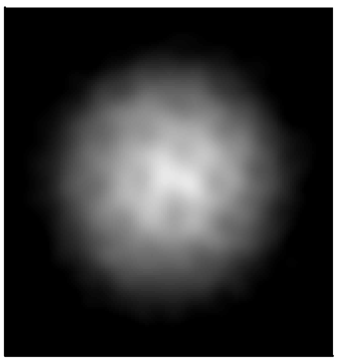
Experiments

BEC-type behavior of fermionic atom pairs (JILA, MIT), Excitation frequencies and damping rates (Innsbruck, Duke), Pairing gap (Innsbruck), Heat capacity (Duke), Study of thermodynamics at ENS

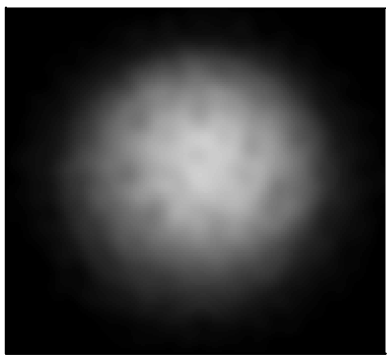
Superfluid behavior through vortex formation (MIT)

Vortex lattices

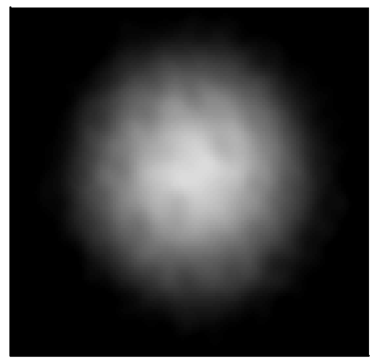
MIT, Zwierlein et al., Science 05



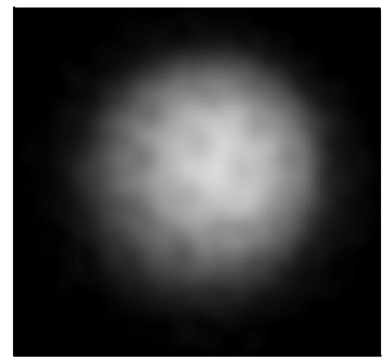
$B_f = 835$ G
$1 / k_F a = 0$



$B_f = 843$ G
$1 / k_F a = -0.13$



$B_f = 854$ G
$1 / k_F a = -0.27$

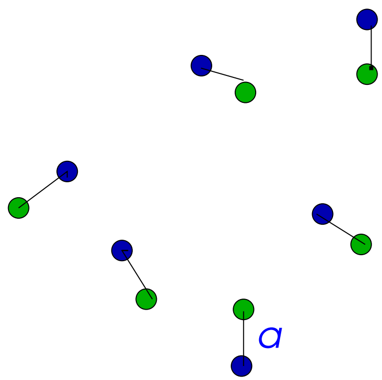


$B_f = 864$ G
$1 / k_F a = -0.39$

Direct proof of superfluidity !

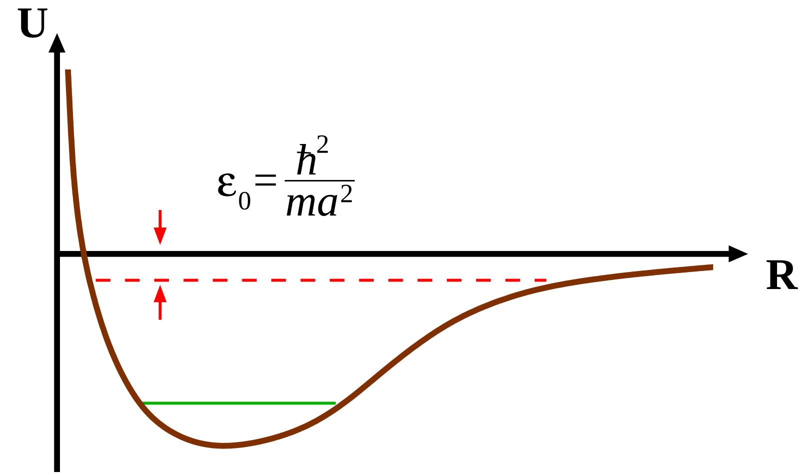
Gas of bosonic molecules (dimers) at $a > 0$

Weakly bound dimers $\rightarrow$ Highest rovibrational state $\rightarrow$ Collisional relaxation



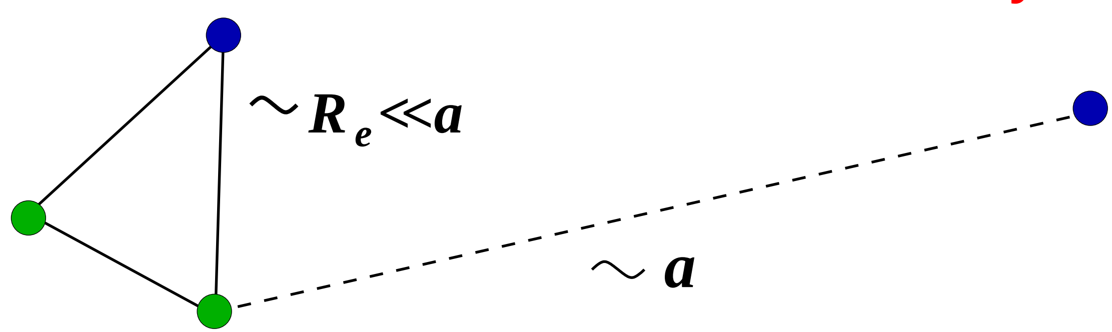
Size of a molecule

a



Elastic interaction $a_{dd} = 0.6a \rightarrow$ BEC stability

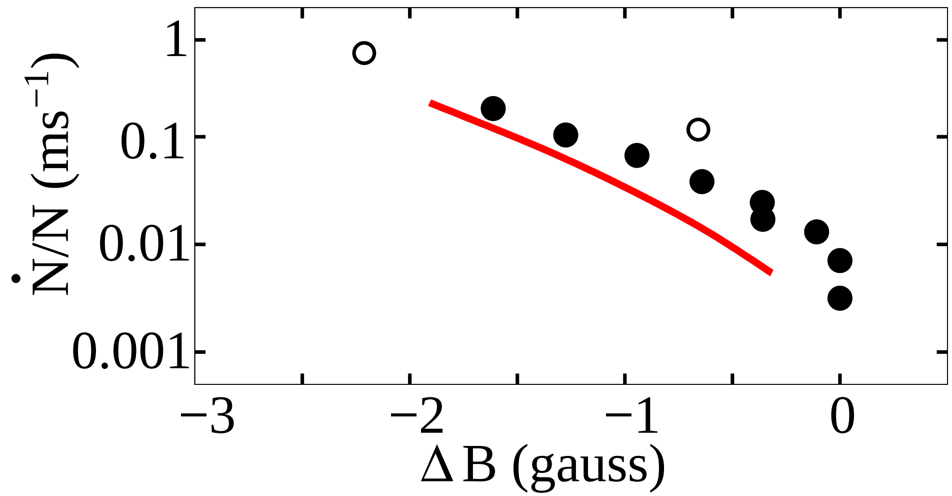
Remarkable collisional satability

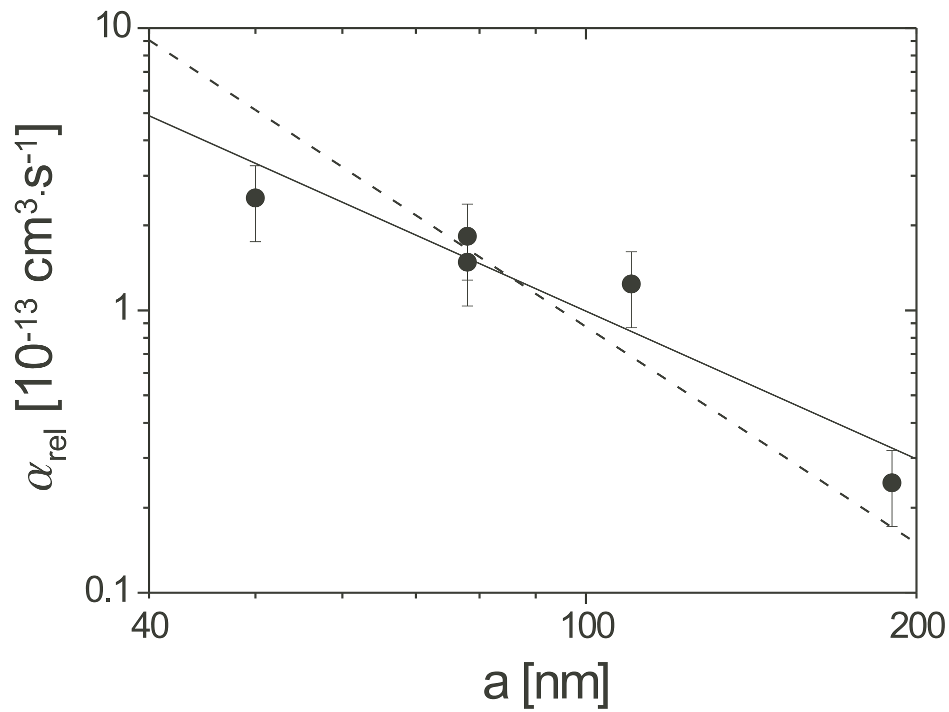


$$\alpha_{rel} \sim (k_{eff} R_e)^{2?} \sim (R_e/a)^{2?} \Rightarrow C(\hbar R_e/m)(R_e/a)^s; \quad s = 2.55$$

$\tau \sim (\alpha_{rel} n)^{-1} \sim$ seconds (Petrov et al 2003)

Suppressed collisional relaxation





Bose-Einstein condensates of molecules

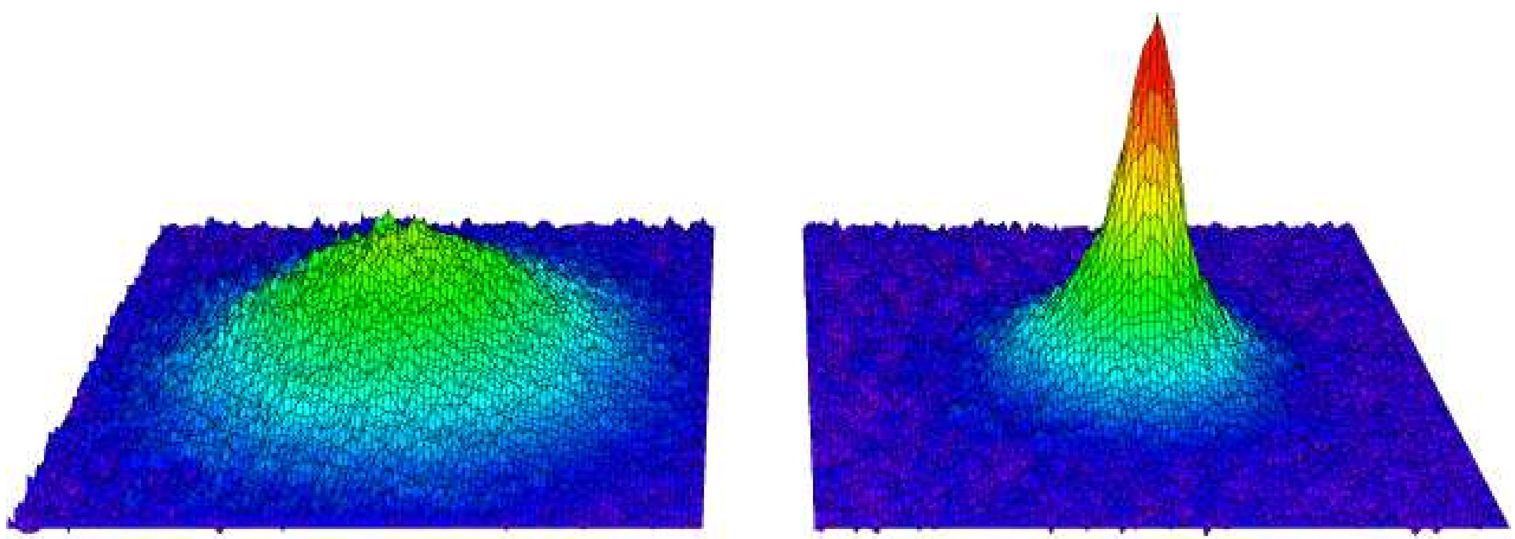
Suppressed relaxation **Fast elastic collisions** $a_{dd} = 0.6a$

$$^6\mathrm{Li}_2 \rightarrow \frac{\alpha_{rel}}{\alpha_{el}} \leq 10^{-4}$$

Efficient evaporative cooling $\rightarrow$ BEC

JILA, Innsbruck, MIT, ENS, Rice, Duke

Largest diatomic molecules in the world $\sim$ 2000 Å



Is it the end of the story?

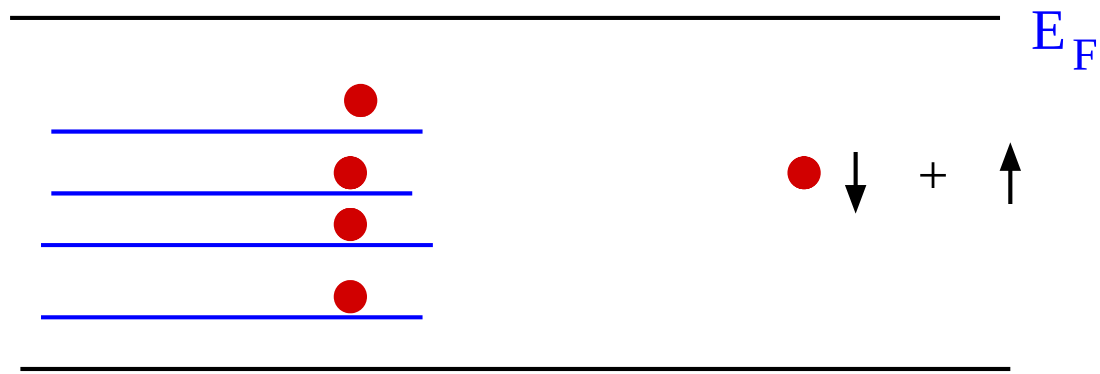
No! $\Rightarrow$ Ferromagnetism in 2-component Fermi gases

Xi-Wen Guan/Yuzhu Jiang, Denis Kurlov/Florian Schreck, GS

Non-interacting fermions

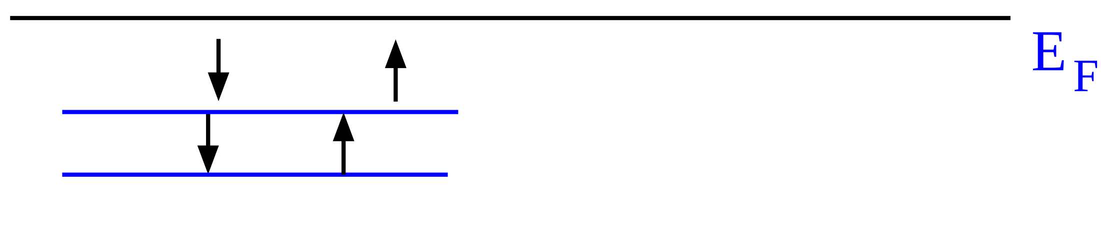
3D case   ferro state of many $\uparrow$ and $\downarrow$ fermions

Each particle is in a superposition state $(\uparrow + \downarrow)/\sqrt{2}$



$$\int_0^{k_F} \frac{4\pi k^2 dk}{(2\pi)^3} = n; \quad k_F = (6\pi^2 n)^{1/3}; \; E_F = \frac{\hbar^2 k_F^2}{2m}; \quad E_{kin} = \frac{3}{5} E_F N$$

3D case   para state:statistical mixture of $\uparrow$ and $\downarrow$ fermions



$$2\int_0^{k_F} \frac{4\pi k^2 dk}{(2\pi)^3} = n; \quad k_f = (3\pi^2 n)^{1/3};$$ E_F and E_{kin} smaller by $2^{2/3}$

Stoner mechanism

Introduce a strong intercomponent repulsion

Works only in the para state

$$E_{int} = \frac{N_\uparrow N_\downarrow}{V} g; \qquad g = \frac{4\pi\hbar^2}{m} a$$

$$E_{para} > E_{ferro} \Rightarrow \quad a > \frac{0.54}{n^{1/3}} \approx \frac{2}{k_F} \text{ for } N_\uparrow = N_\downarrow$$

The true number is a bit different

Itinerant ferro states in condensed matter

Large amount of work

Responsible for properties of transition metals (cobalt, nickel, iron)

Extensive discussions of the character of the ferromagnetic transition

Experiments with cold atoms

^{6}Li Ketterle group

Strongly interacting regime

No ferromagnetism!

Large $a > 0 \Rightarrow$ weakly bound dimers of $\uparrow$ and $\downarrow$ fermions

The rapidly formed dimerized phase has the lowest energy

Pekker et al, 2011

2D and 1D

In 2D one expects the same physics as in 3D

1D non-interacting fermions

ferro $(\uparrow + \downarrow)/\sqrt{2}$

$$k_F = \pi n; \quad E_F = \frac{\pi^2 n^2 \hbar^2}{2m}$$

para $k_F = \dfrac{\pi n}{2}; \quad E_F = \dfrac{\pi^2 n^2 \hbar^2}{8m}$

$E_{kin} = E_F N/3;$ Para energy is lower by a factor of 4

Infinite contact repulsion $\Rightarrow$ All spin configurations are degenerate

No Stoner mechanism!

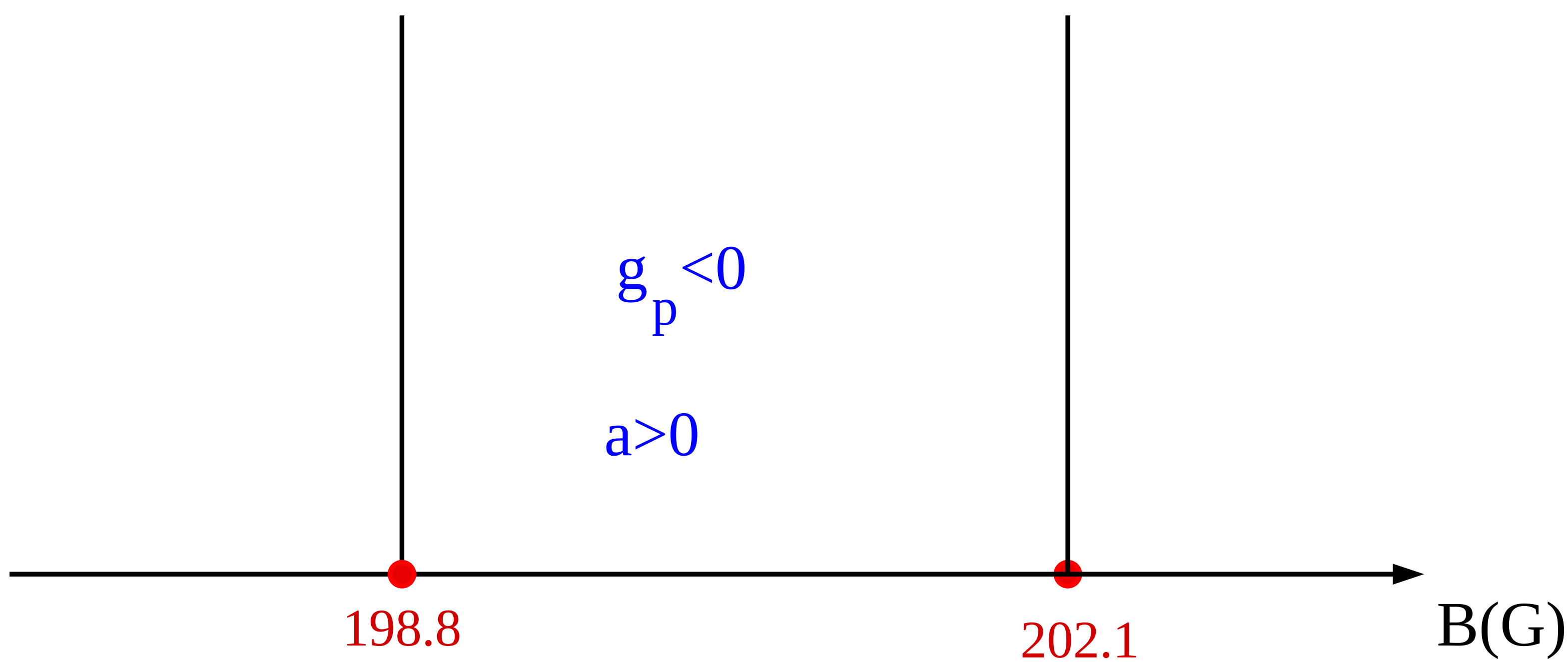
Idea

Add p-wave interaction on top of the strong s-wave repulsion

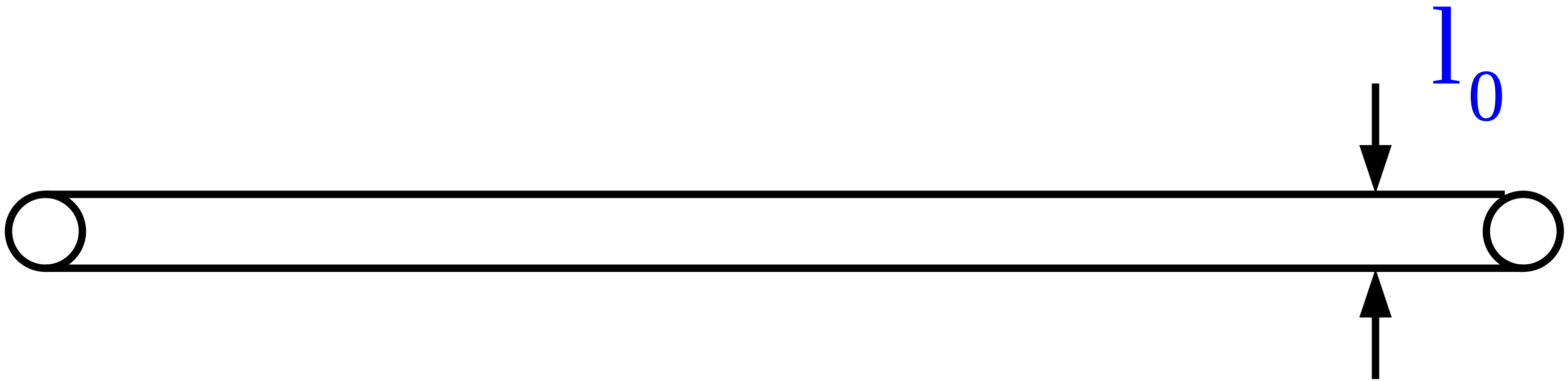
2 Feshbach resonances!

^{40}K present from nature

s-wave resonance	$(9/2,-7/2)+(9/2,-9/2)$	202.1G
p-wave resonance	$(9/2,-7/2)+(9/2,-7/2)$	198.8G



Even-wave interaction in 1D



Even-wave $g_{1D} = \frac{2\hbar^2 a}{m l_0 (l_0 - 1.03a)}$ (Olshaii, 1998)

$\omega_0 \to 100$ or 150 kHz $l_0 \simeq 50$ or 40 nm

$B \to 199$G leads to $a \approx 40$ nm

$$g_{1D} \to \infty$$

Odd-wave interaction in 1D

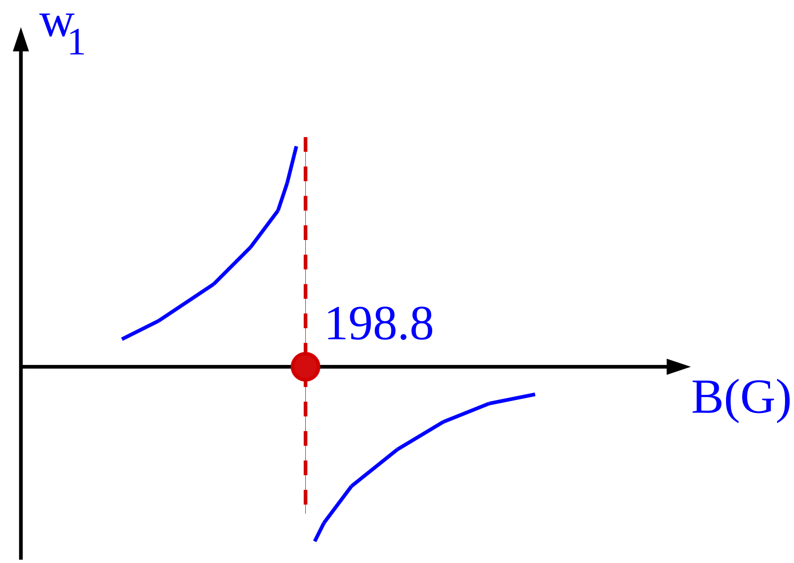
$$f(k',k) = \int_{-\infty}^{\infty} dx\, e^{-ik'x} V(x) \psi_k(x)$$

$$f = \frac{2\hbar^2}{m} \frac{k' k l_p}{1 + \xi_p l_p k^2}$$

$$k^3 \cot \delta_{3D} = -\frac{1}{w_1} - \alpha_1 k^2$$

$$l_p = 3 l_0 \left[l_0^3 / w_1 + 0.88 \right]^{-1}; \qquad \xi_p = \alpha_1 l_0^2 / 3$$

for ^{40}K we have $\alpha_1 \simeq 4 \times 10^6$ cm $^{-1}$

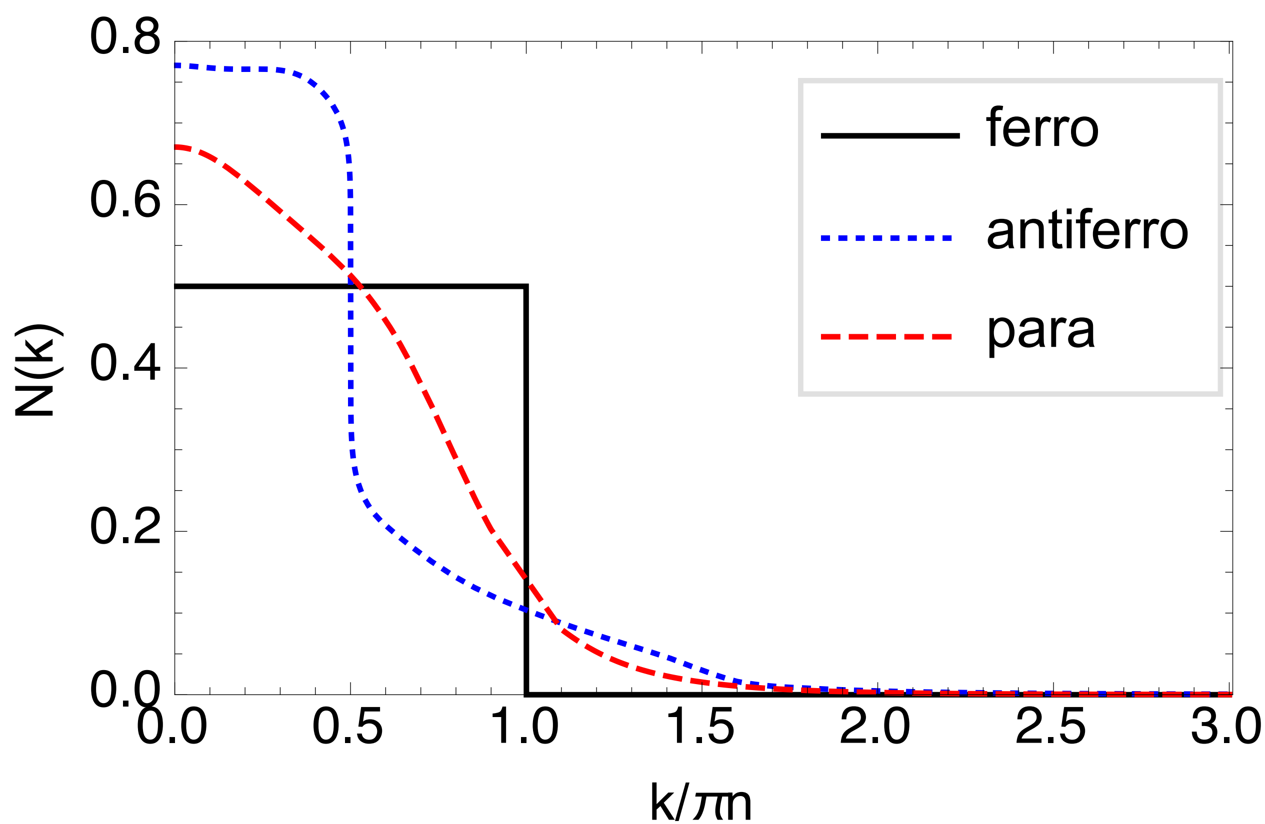


Many-body perturbative approach

$$E = E_{kin} + \tilde{E}^{(1)} + \tilde{E}^{(2)}$$

$$\tilde{E}^{(1)} = \frac{1}{L} \sum_{k_1,k_2} \tilde{f}_{odd}(k) N(k_1) N(k_2); \quad k = (k_1 - k_2)/2$$

$$\tilde{E}^{(2)} = -\frac{1}{L^2} \sum_{k_1,k_2,k_1'} \frac{4m}{\hbar^2} \frac{\tilde{f}_{odd}(k',k)\tilde{f}_{odd}(k,k')}{k_1^2 + k_2^2 - k_1'^2 - k_2'^2} \times N(k_1)\, N(k_2)\, N(k_1')$$



Many-body perturbative approach

$$\tilde{E}^{(1)} = -E_{kin}\left\{\frac{1}{2\pi}\eta + \frac{3}{16\pi}\kappa\,\eta^2\,\mathcal{I}(Q)\right\}$$

$$\tilde{E}^{(2)} = \frac{3}{4\pi^2}\eta^2\mathcal{J}(Q)E_{kin}$$

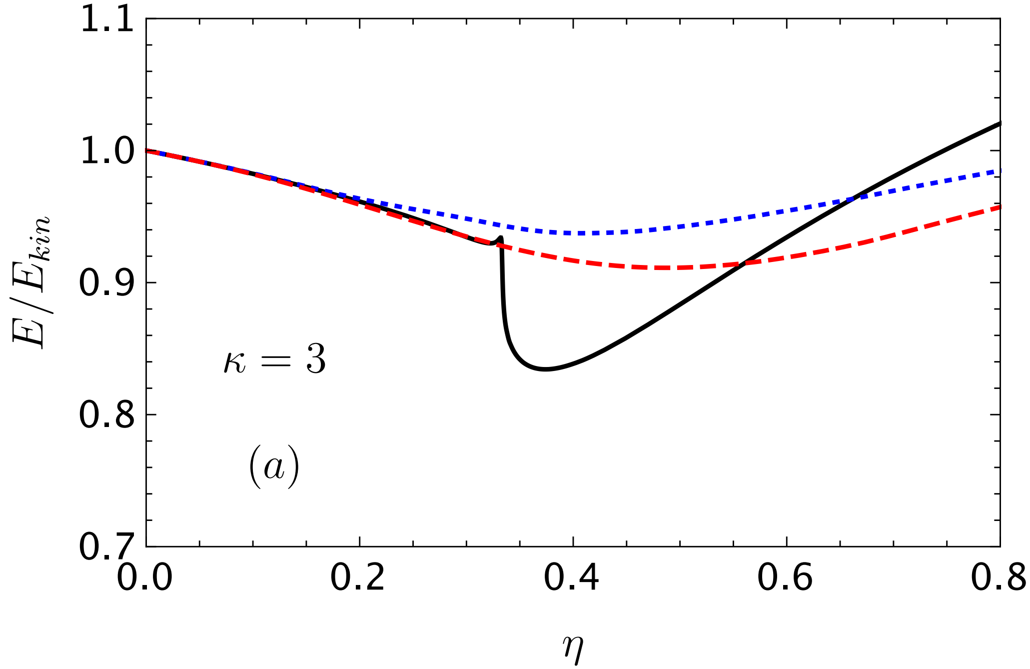
$$\eta = k_F|l_p| \quad \kappa = k_F\xi_p \quad Q = \eta\kappa$$

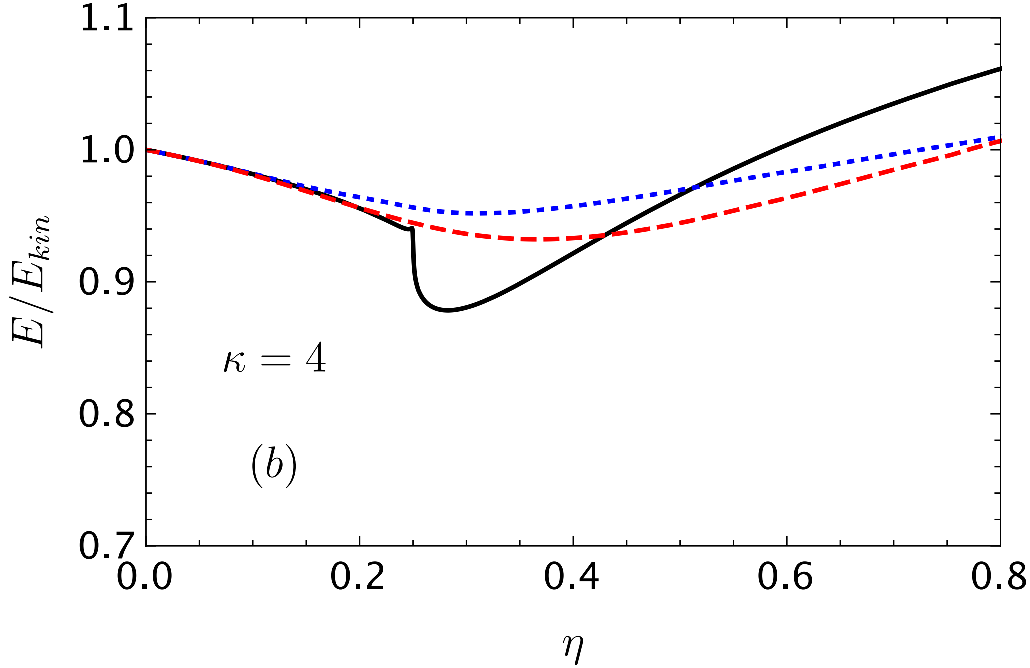
Perturbative approach requires $\eta/\pi \ll 1$

$$\mathcal{I}(Q) = \int_{-\infty}^{+\infty} dx_1 dx_2 N(x_1)N(x_2)\frac{(x_1-x_2)^4}{1-\frac{Q}{4}(x_1-x_2)^2}$$

$$\mathcal{J}(Q) = \int_{-\infty}^{+\infty} dx_1 dx_2 dx_3 \frac{N(x_1)N(x_2)N(x_3)}{x_1-x_3} \times \frac{(x_1-x_2)^2}{\left(1-\frac{Q}{4}(x_1-x_2)^2\right)}\frac{(x_1+x_2-2x_3)^2}{\left(1-\frac{Q}{4}(x_1+x_2-2x_3)^2\right)}$$

Results





Prospects

$E_f \to$ ground state. What are the parameters?

$\omega_0 \simeq 120$ kHz and $n \simeq 3 \times 10^4$ cm^{-1} ($E_F \approx 540$ nK) $\quad \Rightarrow \kappa \simeq 2.6$

$$B \simeq 199\text{G} \quad \to \eta \simeq 0.43$$

$$\frac{(E_p - E_f)}{N} \approx 20 \text{ nK}$$

required temperature in experiments $T < 20$ nK

Underwater stones

3-body recombination K+K+K$\Rightarrow$ K_2+K

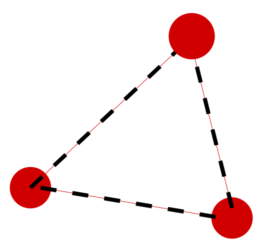
Bottleneck in 3D near the Feshbach resonance

1D behavior (Mehta, Esry, Greene, 2007)

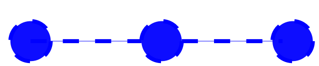
$$\alpha^{1D}_{red} \sim \alpha^{3D}_{rec} \left(\frac{E_F}{E_*}\right) \frac{1}{3\pi l_0^2}$$

Suppression by 3 or 4 orders of magnitude. Why?

Identical fermions in 3D and in 1D: antisymmetrize the wavefunction

3D 

$\psi \sim k^2$ $\alpha \sim T^2$ or E_F^2

1D 

$\psi \sim k^3$ $\alpha \sim T^3$ or E_F^3

Underwater stones

Two-body relaxation

Slow not very close to the p-wave (odd-wave) resonance (J. Bohn)

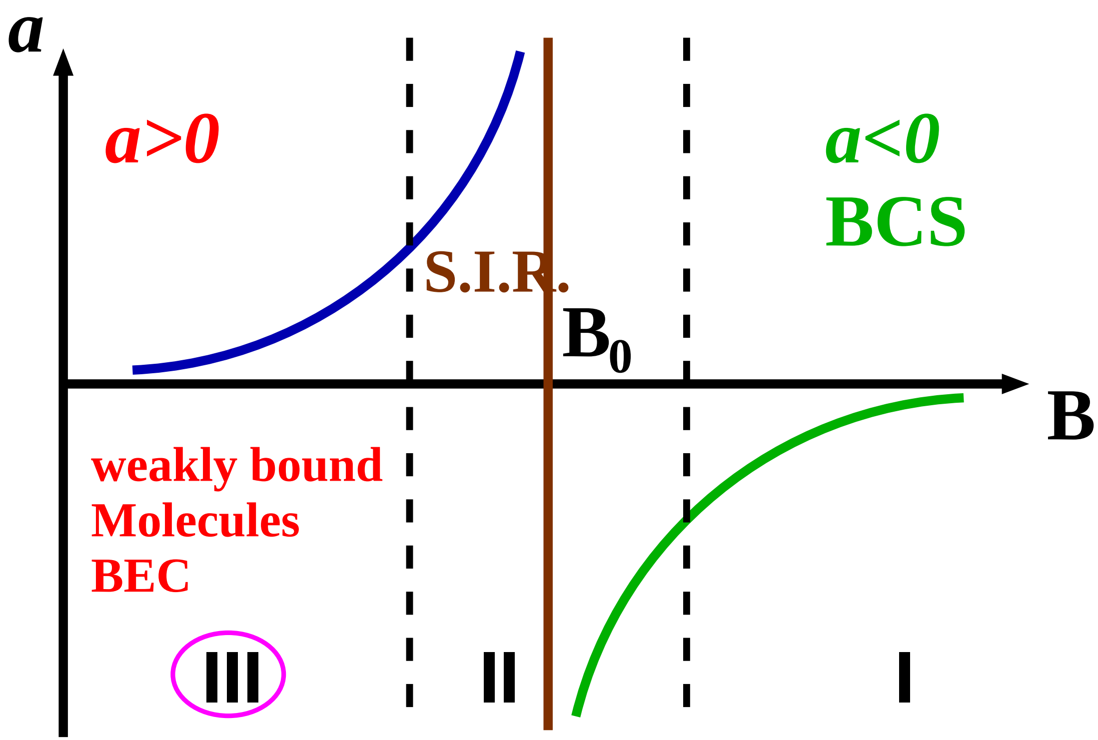
Formation of odd-wave dimers in pair interactions
(the binding energy goes to the formation of holes in the Fermi sea)

On the attractive side of the odd-wave resonance weakly bound dimer states are absent and this mechanism does not work

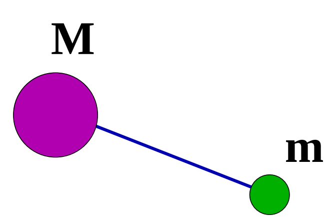
In the non-ferro phases the even-wave (s-wave) interactions are present and the decay is faster. However, infinite contact repulsion reduces the decay rate

Mixtures of different fermionic atoms

Heavy and light fermions $^6\mathrm{Li}^{40}\mathrm{K}$ $^6\mathrm{Li}^{171}\mathrm{Yb}$



$a > 0 \Rightarrow$ weakly bound molecules BEC $a < 0 \Rightarrow$ BCS pairing



Where we are and what novel phyics is expected?

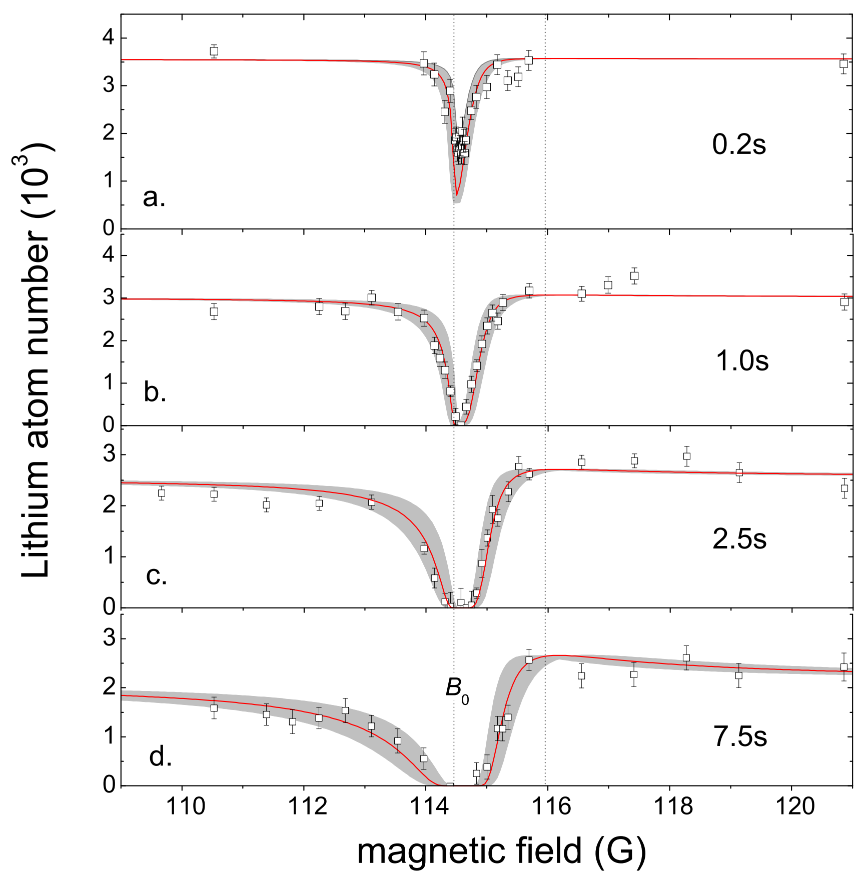
Experiments with Li-K mixtures

Insbruck, Amsterdam, Singapore, ENS, MIT

Relatively narrow Feshbach resonances (Amsterdam, Insbruck, Singapore)

Formation of molecules. Quantum degeneracy $T \sim E_F$ (Innsbruck, Singapore)

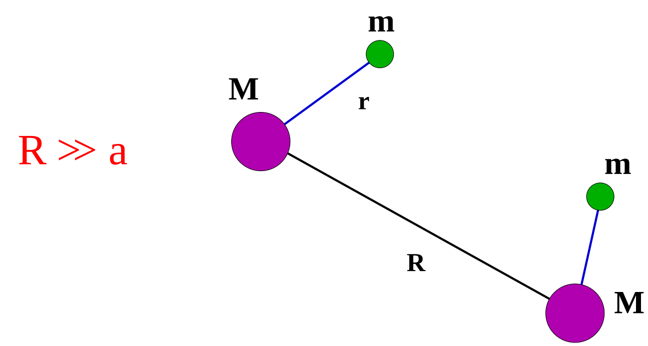
The widest resonance Li$\{1/2, +1/2\}$-K$\{9/2, +9/2\}$ $\Rightarrow$ $B = 114.7G; \Delta B \approx 1.5G$



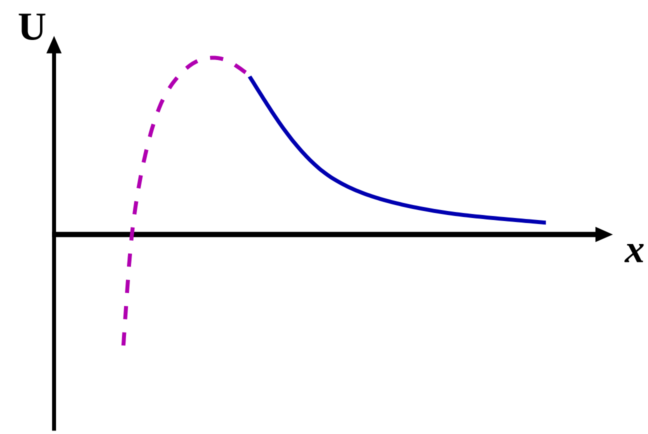
Long-range intermolecular repulsion

Wide resonance. Molecules of heavy and light fermions

Born-Oppenheimer picture



$$U(R) = 2\left(\frac{\hbar^2}{maR}\right)\exp(-2R/a)$$



$$P \sim \exp\left(-0.9\sqrt{\frac{M}{m}}\right)$$

$M >>> m \rightarrow$ Collisional stability independent of a

Molecule-molecule scattering amplitude $a_{dd} \approx a \ln \sqrt{(M/m)}$

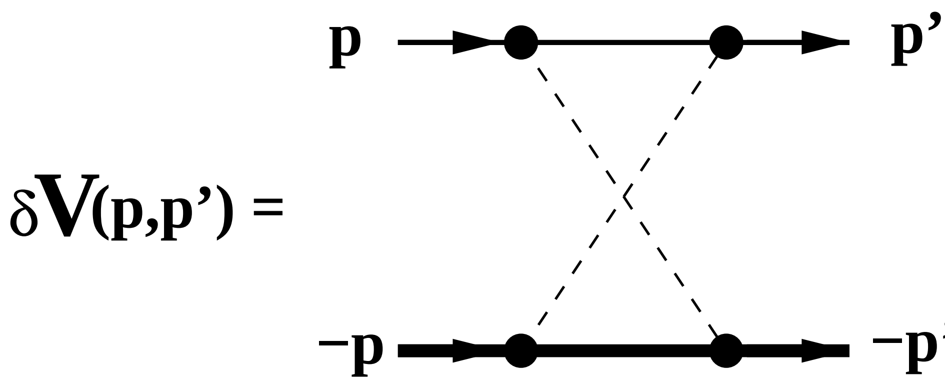
BCS regime for atomic Fermi gas at $a < 0$

Superfluid pairing between heavy and light fermions. Li-Yb?

Transition temperature in the simple BCS approach $T_c \sim ? \exp(-\pi/2k_F|a|)$

Effective interaction between heavy and light fermions in the medium

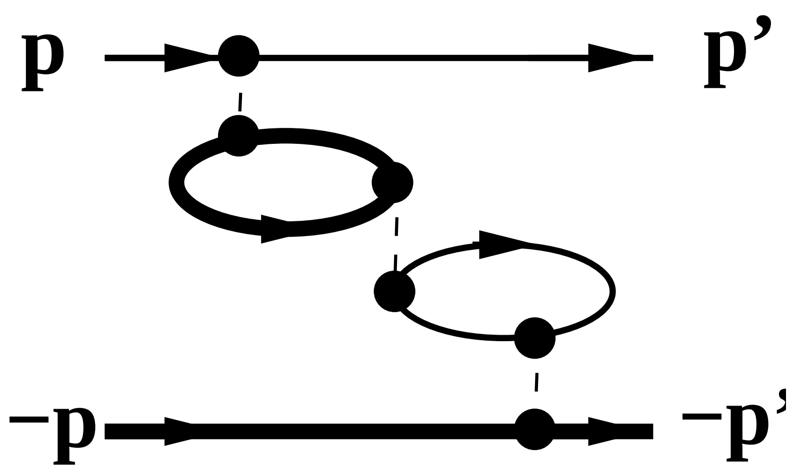
Second order contribution $\sim g^2\nu \sim g(k_F|a|)$



$T_c = 0,825 E_M \exp(-\pi/2k_F|a|)$ M.Baranov, C.Lobo, G.S. (2008)

Small parameter of the theory

Third order processes. For example:



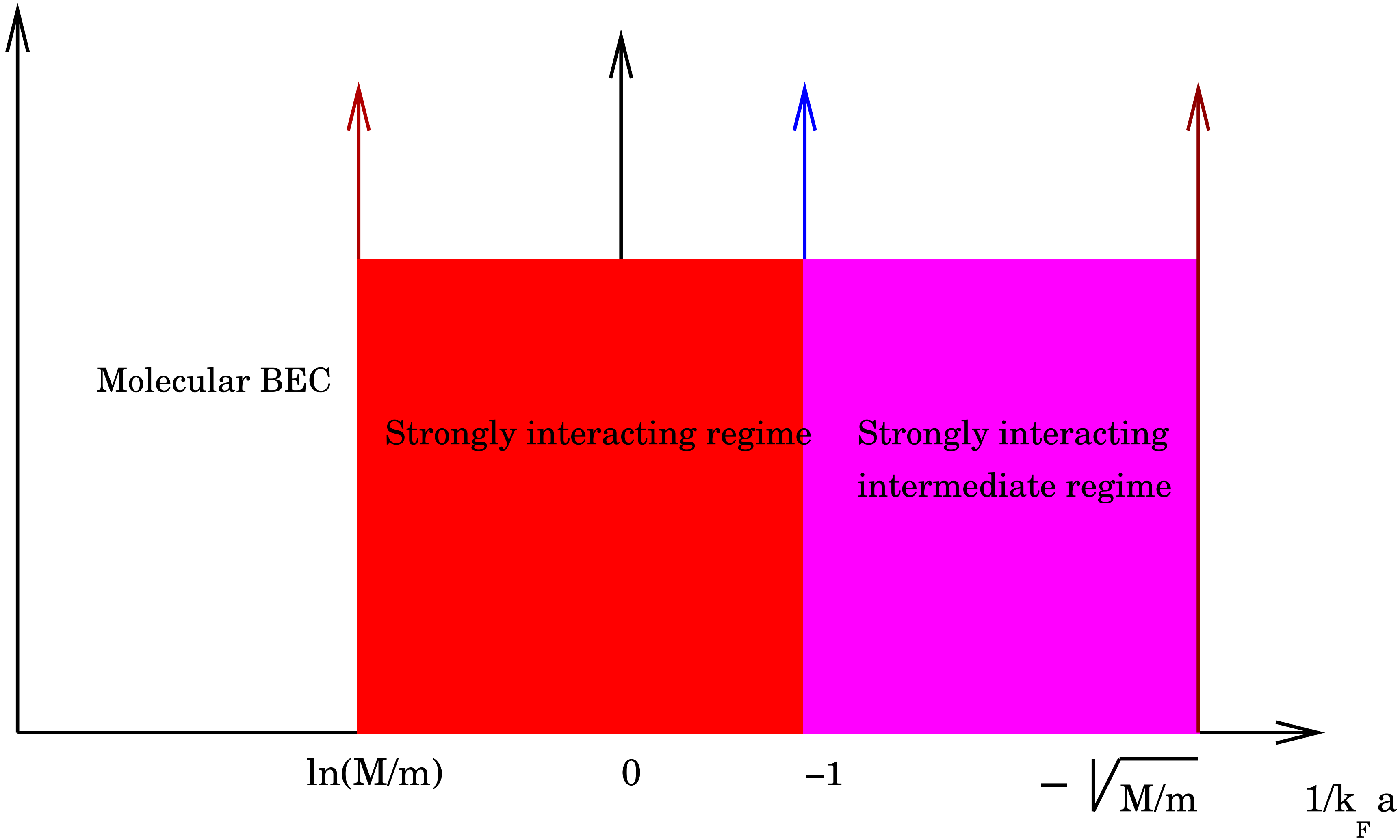
$$g_{eff} \sim g^3 \nu_M \nu_m; \quad \nu_M = Mk_F/(2\pi^2\hbar^2); \quad \nu_m = mk_F/(2\pi^2\hbar^2)$$

$$g = 4\pi\hbar^2 a/m \Rightarrow \quad g_{eff}/g \sim (k_F a)^2 M/m$$

Small parameter of the theory

$$k_F|a|\sqrt{\frac{M}{m}} \ll 1$$

How wide is the strongly interacting regime ?



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Novel types of superfluid pairing?

Conclusions

Still a lot to do with strongly interacting fermions

Thank you for attention